Optimal Dynamic Forest Fire Management Adapted to Stochastic Weather (Marginal Revision 220703. No results have been adjusted.)

by

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Empirical data and support

by

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Prague

This presentation is based on the following open access articles and documents. More details can be obtained from the links found below.

[1] Lohmander P. (2020a). **Dynamics and control of the CO_2 level via a differential equation and alternative global emission strategies**. Int Rob Auto J. 2020;6(1):7–15. DOI: 10.15406/iratj.2020.06.0019, https://medcraveonline.com/IRATJ/IRATJ-06-00197.pdf

[2] Lohmander, P. (2020b)., **Optimization of continuous cover forestry expansion under the influence of global warming**, International Robotics & Automation Journal, Volume 6, Issue 3, 2020, 127-132. https://medcraveonline.com/IRATJ/IRATJ-06-00211.pdf ,

https://medcraveonline.com/IRATJ/IRATJ-06-00211A.pdf

[3] Lohmander, P. (2020c). **Fundamental principles of optimal utilization of forests with consideration of global warming**, Central Asian Journal of Environmental Science and Technology Innovation, Volume 1, Issue 3, May and June 2020, 134-142. doi: 10.22034/CAJESTI.2020.03.02 http://www.cas-press.com/article_111213.html http://www.cas-press.com/article_111213_5ab21574a30f6f2c7bdc0a0733234181.pdf

[4] Lohmander P. (2020d). **Adaptive mobile firefighting resources: stochastic dynamic optimization of international cooperation**. Int Rob Auto J. 2020;6(4):150–155. DOI: 10.15406/iratj.2020.06.00213, https://medcraveonline.com/IRATJ/IRATJ-06-00213.pdf

[5] Lohmander, P. (2020e). **Forest fire expansion under global warming conditions: -Multivariate estimation, function properties and predictions for 29 countries**. Cent. Asian J. Environ. Sci. Technol. Innov., 5, 262-276, https://www.cas-press.com/article_122566_c3544cd0c21d5c077f72e985a77d30e9.pdf

[6] Lohmander, P. (2021a). **Optimization of Forestry, Infrastructure and Fire Management**. Caspian Journal of Environmental Sciences, 19: 287-316, https://cjes.guilan.ac.ir/article_4746_197fe867639c4cc5e317b63f9f9d370b.pdf

[7] Lohmander P. (2021b). **Optimization of distance between fire stations: effects of fire ignition probabilities, fire engine speed and road limitations, property values and weather conditions**. Int Rob Auto J. 2021;7(4):112–120. DOI: 10.15406/iratj.2021.07.0023 , https://medcraveonline.com/IRATJ/IRATJ-07-00235.pdf

[8] Lohmander, P. (2021c). **Global Stability via the Forced Global Warming Equation, Fire Control with Joint Fire Fighting Resources, and Optimal Forestry**, KEYNOTE at ICASE 2021: International Conference on Applied Science & Engineering, March 31, 2021., http://www.Lohmander.com/PL_ICASE_2021_Abstract.pdf, http://www.Lohmander.com/PL_ICASE_2021_KEYNOTE.pdf

[9] Mohammadi, Z., Lohmander, P., Kašpar, J. et al. (2021). **The effect of climate factors on the size of forest wildfires** (case study: Prague-East district, Czech Republic). J. For. Res. https://doi.org/10.1007/s11676-021-01413-w

[10] Lohmander, P., Mohammadi, Z., Kaspar, J., Tahry, M., Bercak, R., Holusa, J., Marusak, R., (2022). **Future forest fires as a functions of climate change and attack time for central Bohemian region**, Czech Republic, Annals of Forest Research 65(1): 17-30 (in press).
https://www.afrjournal.org/index.php/afr/issue/archive

[11] Lohmander, P., **Rational Control of Global Warming Dynamics via Emission Reductions and Forestry Expansion** (Submitted).

Abstract

Optimal Dynamic Forest Fire Management Adapted to Stochastic Weather

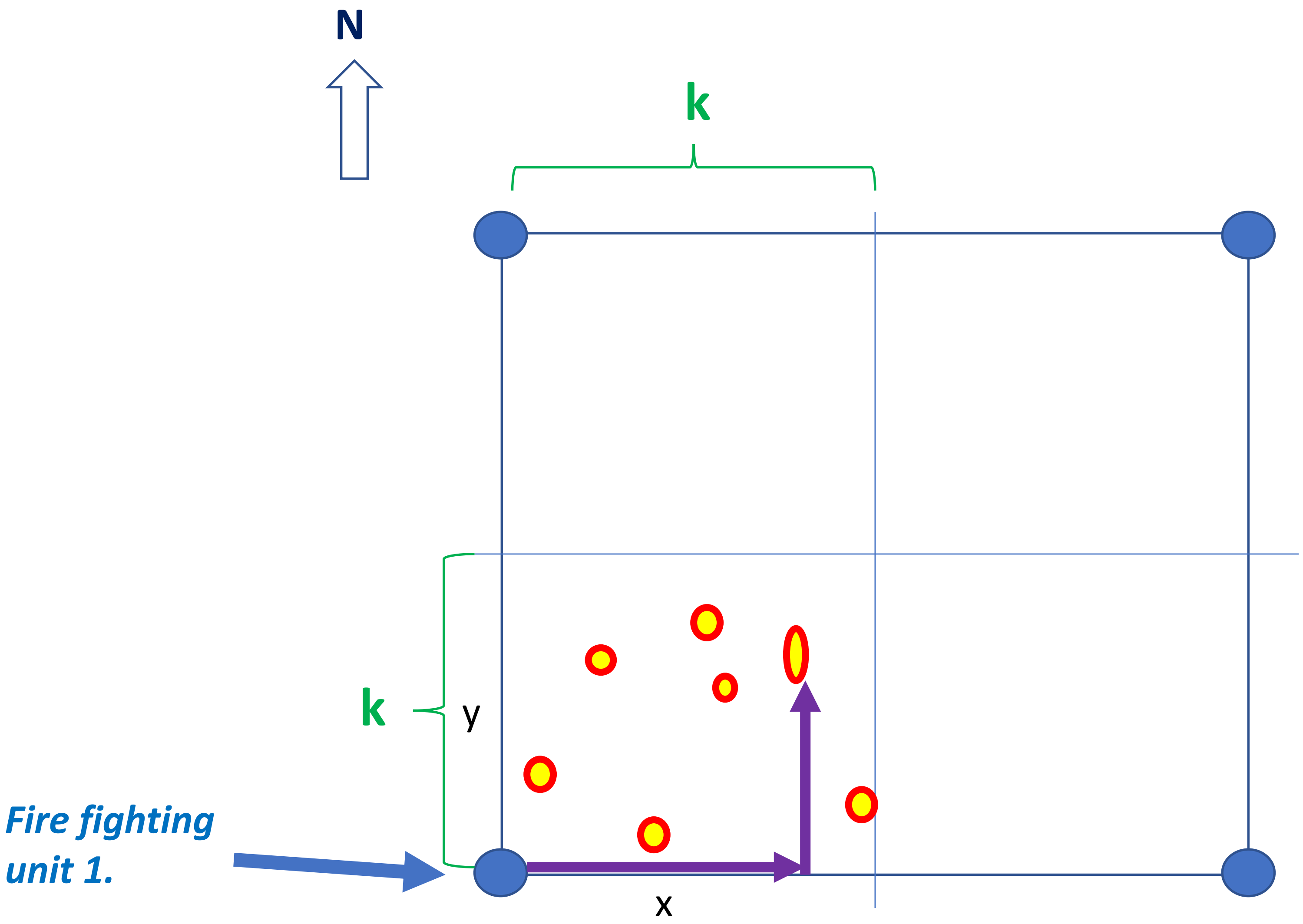
By Peter Lohmander. Empirical data and support: Zohreh Mohammadi.

Global warming is a major problem of our planet, which can be solved, [1], [2], [3], [8], [11]. Forest fires represent growing and severe direct safety problems in large parts of the world [5], [9]. Furthermore, the fires send large amounts of CO_2 to the atmosphere, which increase the speed of global warming. Recent research has estimated the size of a forest fire as a nonlinear function of the air temperature, relative humidity, wind speed and the time it takes for the fire brigade to reach the fire and start the fire suppression, [10]. Several ways to handle the forest fire problems have been suggested, [4], [6]. The sizes of fires can be reduced if the fire brigades reach the fires more rapidly. Central decisions in fire management optimization are to determine the numbers of fire brigade units, and their locations, at different points in time. A first study in this direction is [7]. If the spatial pattern of fire brigade locations is held constant and the number of fire brigade units per area unit increases, the capacity cost of the fire brigades increases. This represents an investment in firefighting capacity. Simultaneously, the expected distance to forest fires in random locations decreases, and the expected fire sizes are reduced. The reduced expected costs of burned forests can be interpreted as an expected economic revenue caused by the investment in firefighting capacity. In this paper, the optimal level of this investment is determined. The expected values of air temperature, relative humidity and wind speed, are determined as deterministic functions of time, during a typical year, in a region of Czech Republic. Furthermore, probability density functions and probability functions of the stochastic residuals of these weather components are derived. The air temperature residuals and the relative humidity residuals have considerable negative correlation. First, labor employment constraints are applied, that require that the numbers of fire brigade units at different points in time are determined before the true weather component residuals are known. Optimality conditions of the dynamically changing firefighting capacity levels are analytically determined. The solutions are found to give unique maxima. Comparative statics analysis is used to determine the directions of change of the optimal capacity levels under the influence of alternative parameter changes. The expected fire sizes can also be numerically approximated via random numbers with relevant correlations, based on Cholesky factorization. Finally, a generalized stochastic dynamic programming version of the dynamic investment decision problem is presented, based on a very flexible labor market, where the number of fire brigades rapidly can be adapted to the sequentially revealed weather situation.

Contents:

- The firefighting capacity optimization problem
- General solution to the firefighting capacity optimization problem
- Comparative statics analysis of the optimal solution: How is the optimal solution affected by the parameters?
- Fire growth as a function of weather conditions
- Background to empirical weather and fire data
- Dynamic and stochastic properties of air temperature, relative humidity and wind speed
- Conclusions

The firefighting capacity optimization problem



Forest fires have a spatially uniform probability density function.

Fire fighting units (blue balls) have initial locations in a regular ”infinite” network with roads in directions N-S and E-W.

The distances between roads is small and fire fighting units can always use roads in the two directions to reach the different fires. (See the purple arrows.)

The decision problem is to determine the optimal value of k, where the distance between the nearest neighbours is L = 2k.

The optimal value of k is affected by many different parameters, some of which are functions of the season, for instance air temperature, relative humidity and wind speed. These parameters affect fire growth.

$$\min_{k} C(k;.) = C_I\left(k\right) + C_F(k)$$

Fire fighting groups are located in a regular network with roads in directions South-North and East- West. The distance in one of these directions, between two neighbour groups, is L = 2k.

Expected total costs as a function of k.

Expected cost of fire fighting capacity ”investment” and use of these resources

Expected cost of destroyed and damaged forests including costs of CO_2 emissions

$$C_F(k) = c_D \times N \times B\big(A, H, W, t(k,m)\big)$$

Expected cost per burned ha.

Expected number of fires per time unit.

Expected size of burned forest, per fire.

A = Air temperature (Stochastic)
H = Relative humidity (Stochastic)
W = Wind speed (Stochastic)
t(k,m) = expected time of fire life.
m = fire life time before the fire fighting unit leaves the initial location and starts to move towards the fire.

Expected cost of destroyed and damaged forests including costs of CO_2 emissions

$$C_I(k) = c_u \times U$$

Expected cost of
fire fighting capacity
”investment”
and use of these
Resources.

Expected cost
per unit.

Number of units per
10000 square km

$$U = \frac{1}{L^2} = \frac{1}{(2k)^2} = \frac{1}{4}k^{-2}$$

$$B\left(A, H, W, t(k,m)\right) = B_1(k,m) \times B_2\left(A, H, W\right)$$

Expected size of burned forest, per fire.

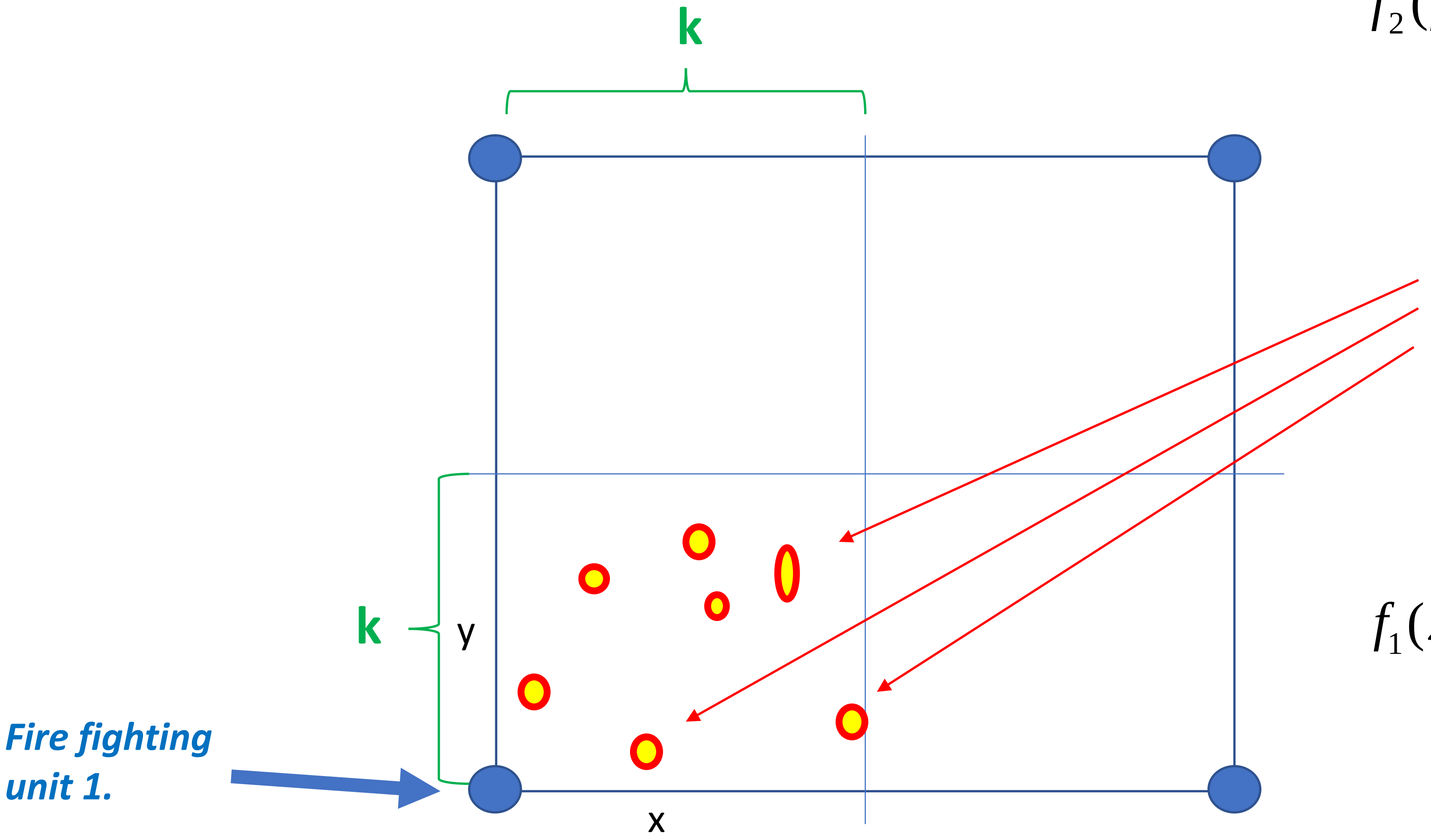
This will be shown to hold in general cases.

This has been estimated with empirical data.

$$B_1(k,m) = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$$

Why is $B_1(k,m) = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$ **?**

$$f_2(y) = \begin{cases} 0 & for \quad y < 0 \\ k^{-1} & for \quad 0 \le y \le k \\ 0 & for \quad k < y \end{cases}$$



$$f_1(x) = \begin{cases} 0 & for \quad x < 0 \\ k^{-1} & for \quad 0 \le x \le k \\ 0 & for \quad k < x \end{cases}$$

Convolution

$$f_1(x) = \begin{cases} 0 & for \quad x < 0 \\ k^{-1} & for \quad 0 \le x \le k \\ 0 & for \quad k < x \end{cases}$$

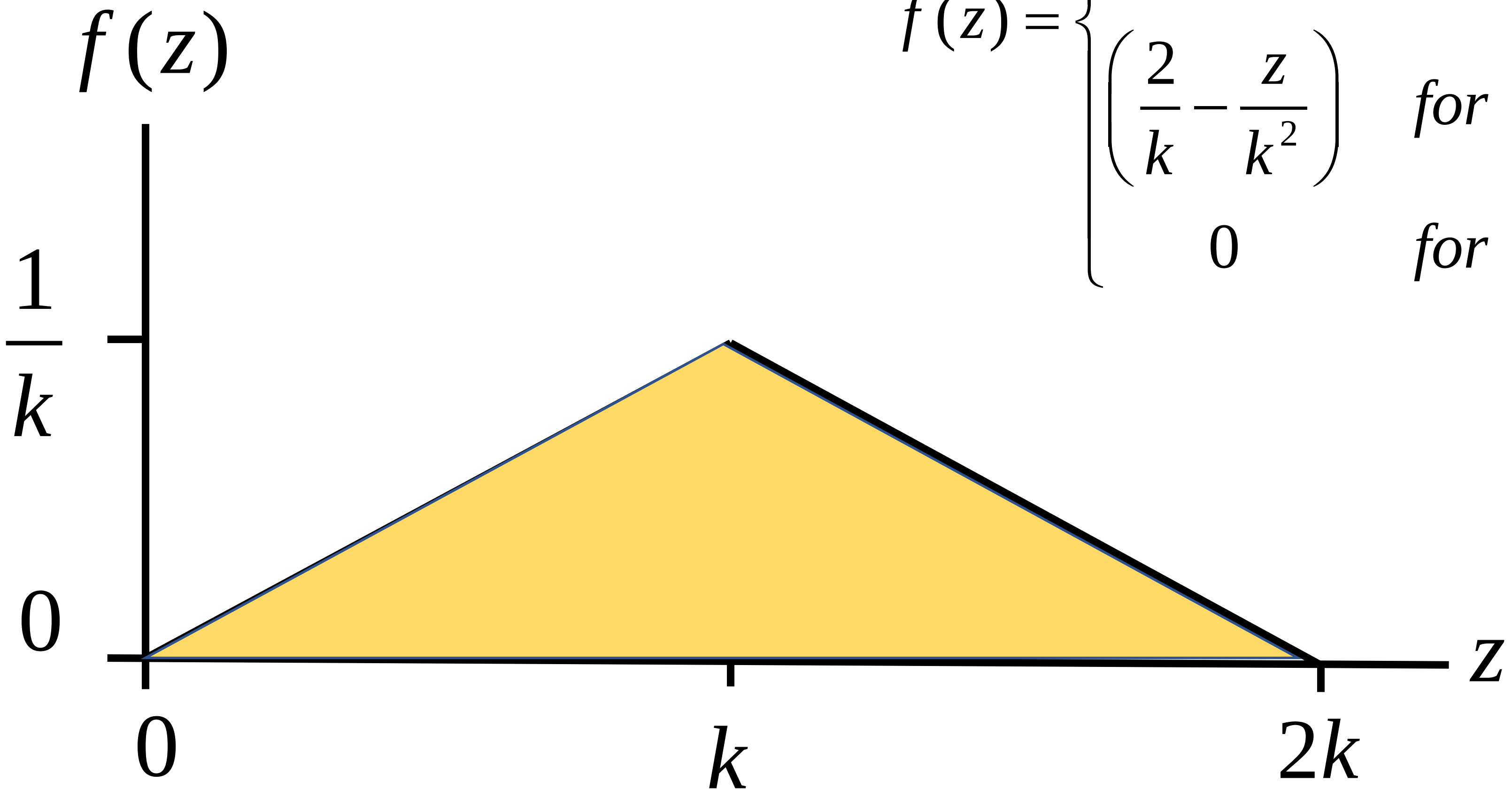
$$f_2(y) = \begin{cases} 0 & for \quad y < 0 \\ k^{-1} & for \quad 0 \le y \le k \\ 0 & for \quad k < y \end{cases}$$

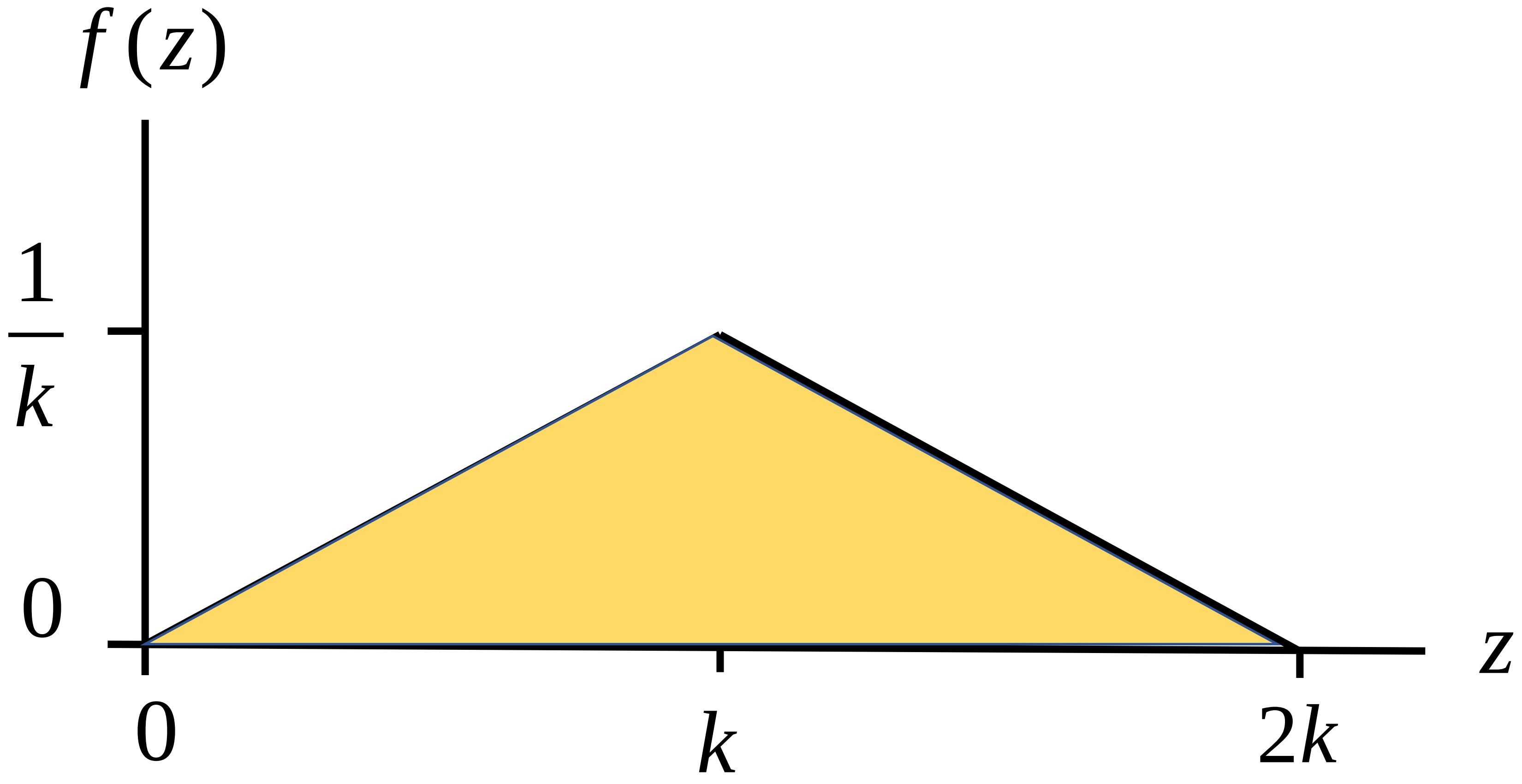
$$z = x + y$$

$$f(z) = \int_0^{2k} f_1(x)\, f_2(z-x)\, dx$$

$$f(z)=\begin{cases} 0 & for \quad z<0 \\ \dfrac{z}{k^2} & for \quad 0\le z<k \\ \left(\dfrac{2}{k}-\dfrac{z}{k^2}\right) & for \quad k\le z\le 2k \\ 0 & for \quad 2k<z \end{cases}$$

$$f(z) = \begin{cases} 0 & for \quad z < 0 \\ \dfrac{z}{k^2} & for \quad 0 \le z < k \\ \left(\dfrac{2}{k} - \dfrac{z}{k^2}\right) & for \quad k \le z \le 2k \\ 0 & for \quad 2k < z \end{cases}$$





$$B_1 = \int_0^k c\left(z+m\right)^2 \frac{z}{k^2}\,dz + \int_k^{2k} c\left(z+m\right)^2 \left(\frac{2}{k} - \frac{z}{k^2}\right) dz$$

$$B_1 = \int_0^k c\left(z+m\right)^2 \frac{z}{k^2}\,dz + \int_k^{2k} c\left(z+m\right)^2 \left(\frac{2}{k} - \frac{z}{k^2}\right)dz$$

$$G = \frac{B_1}{c} = k^{-2}\int_0^k \left(z^2 + 2zm + m^2\right)z\,dz$$

$$+\,2k^{-1}\int_k^{2k} \left(z^2 + 2zm + m^2\right)dz$$

$$-\,k^{-2}\int_k^{2k} \left(z^2 + 2zm + m^2\right)z\,dz$$

$$G=\frac{B_1}{c}=k^{-2}\int_0^k\left(z^2+2zm+m^2\right)z\,dz+2k^{-1}\int_k^{2k}\left(z^2+2zm+m^2\right)dz-k^{-2}\int_k^{2k}\left(z^2+2zm+m^2\right)z\,dz$$

$$G=k^{-2}\int_0^k z^3\,dz+2mk^{-2}\int_0^k z^2\,dz+m^2k^{-2}\int_0^k z\,dz$$

$$+2k^{-1}\int_k^{2k} z^2\,dz+4mk^{-1}\int_k^{2k} z\,dz+2m^2k^{-1}\int_k^{2k} 1dz$$

$$-k^{-2}\int_k^{2k} z^3\,dz-2mk^{-2}\int_k^{2k} z^2\,dz-m^2k^{-2}\int_k^{2k} z\,dz$$

$$G = k^{-2}\int_0^k z^3\,dz + 2mk^{-2}\int_0^k z^2\,dz + m^2k^{-2}\int_0^k z\,dz + 2k^{-1}\int_k^{2k} z^2\,dz + 4mk^{-1}\int_k^{2k} z\,dz + 2m^2k^{-1}\int_k^{2k} 1dz - k^{-2}\int_k^{2k} z^3\,dz - 2mk^{-2}\int_k^{2k} z^2\,dz - m^2k^{-2}\int_k^{2k} z\,dz$$

$$G = k^{-2}\left(\left.\frac{z^4}{4}\right]_0^k\right) + 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_0^k\right) + m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_0^k\right)$$

$$+ 2k^{-1}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) + 4mk^{-1}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right) + 2m^2k^{-1}\left(\left.z\right]_k^{2k}\right)$$

$$- k^{-2}\left(\left.\frac{z^4}{4}\right]_k^{2k}\right) - 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) - m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right)$$

$$G = k^{-2}\left(\frac{z^4}{4}\right]_0^k\Bigg) + 2mk^{-2}\left(\frac{z^3}{3}\right]_0^k\Bigg) + m^2k^{-2}\left(\frac{z^2}{2}\right]_0^k\Bigg) + 2k^{-1}\left(\frac{z^3}{3}\right]_k^{2k}\Bigg) + 4mk^{-1}\left(\frac{z^2}{2}\right]_k^{2k}\Bigg) + 2m^2k^{-1}\Big(z\Big]_k^{2k}\Big) - k^{-2}\left(\frac{z^4}{4}\right]_k^{2k}\Bigg) - 2mk^{-2}\left(\frac{z^3}{3}\right]_k^{2k}\Bigg) - m^2k^{-2}\left(\frac{z^2}{2}\right]_k^{2k}\Bigg)$$

$$G = k^{-2}\left(\frac{k^4}{4} - 0\right) + 2mk^{-2}\left(\frac{k^3}{3} - 0\right) + m^2k^{-2}\left(\frac{k^2}{2} - 0\right)$$

$$+ 2k^{-1}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) + 4mk^{-1}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right) + 2m^2k^{-1}\left(2k - k\right)$$

$$- k^{-2}\left(\frac{16k^4}{4} - \frac{k^4}{4}\right) - 2mk^{-2}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) - m^2k^{-2}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right)$$

$$G = k^{-2}\left(\frac{k^4}{4}-0\right)+2mk^{-2}\left(\frac{k^3}{3}-0\right)+m^2k^{-2}\left(\frac{k^2}{2}-0\right)+2k^{-1}\left(\frac{8k^3}{3}-\frac{k^3}{3}\right)+4mk^{-1}\left(\frac{4k^2}{2}-\frac{k^2}{2}\right)+2m^2k^{-1}\left(2k-k\right)-k^{-2}\left(\frac{16k^4}{4}-\frac{k^4}{4}\right)-2mk^{-2}\left(\frac{8k^3}{3}-\frac{k^3}{3}\right)-m^2k^{-2}\left(\frac{4k^2}{2}-\frac{k^2}{2}\right)$$

$$G = \frac{1}{4}k^2 + \frac{2}{3}mk + \frac{1}{2}m^2$$

$$+\frac{14}{3}k^2 + 6mk + 2m^2$$

$$-\frac{15}{4}k^2 - \frac{14}{3}mk - \frac{3}{2}m^2$$

$$G=\frac{1}{4}k^2+\frac{2}{3}mk+\frac{1}{2}m^2+\frac{14}{3}k^2+6mk+2m^2-\frac{15}{4}k^2-\frac{14}{3}mk-\frac{3}{2}m^2$$

$$G=\left(\frac{1}{4}+\frac{14}{3}-\frac{15}{4}\right)k^2+\left(\frac{2}{3}+6-\frac{14}{3}\right)km+\left(\frac{1}{2}+2-\frac{3}{2}\right)m^2$$

$$G=\left(\frac{3}{12}+\frac{56}{12}-\frac{45}{12}\right)k^2+\left(\frac{2}{3}+\frac{18}{3}-\frac{14}{3}\right)km+\left(\frac{1}{2}+\frac{4}{2}-\frac{3}{2}\right)m^2$$

$$G=\frac{14}{12}k^2+\frac{6}{3}km+\frac{2}{2}m^2$$

$$G=\frac{7}{6}k^2+2km+m^2$$

Hence, $B_1 = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$

Important properties:

$$\frac{dB_1}{dk} = c\left(\frac{7}{3}k + 2m\right) > 0$$

$$\frac{d^2B_1}{dk^2} = \frac{7}{3}c > 0$$

$$\frac{d^2B_1}{dmdk} = 2c > 0$$

$$\frac{dB_1}{dm} = c\left(2k + 2m\right) > 0$$

$$\frac{d^2B_1}{dm^2} = 2c > 0$$

General solution to the firefighting capacity optimization problem

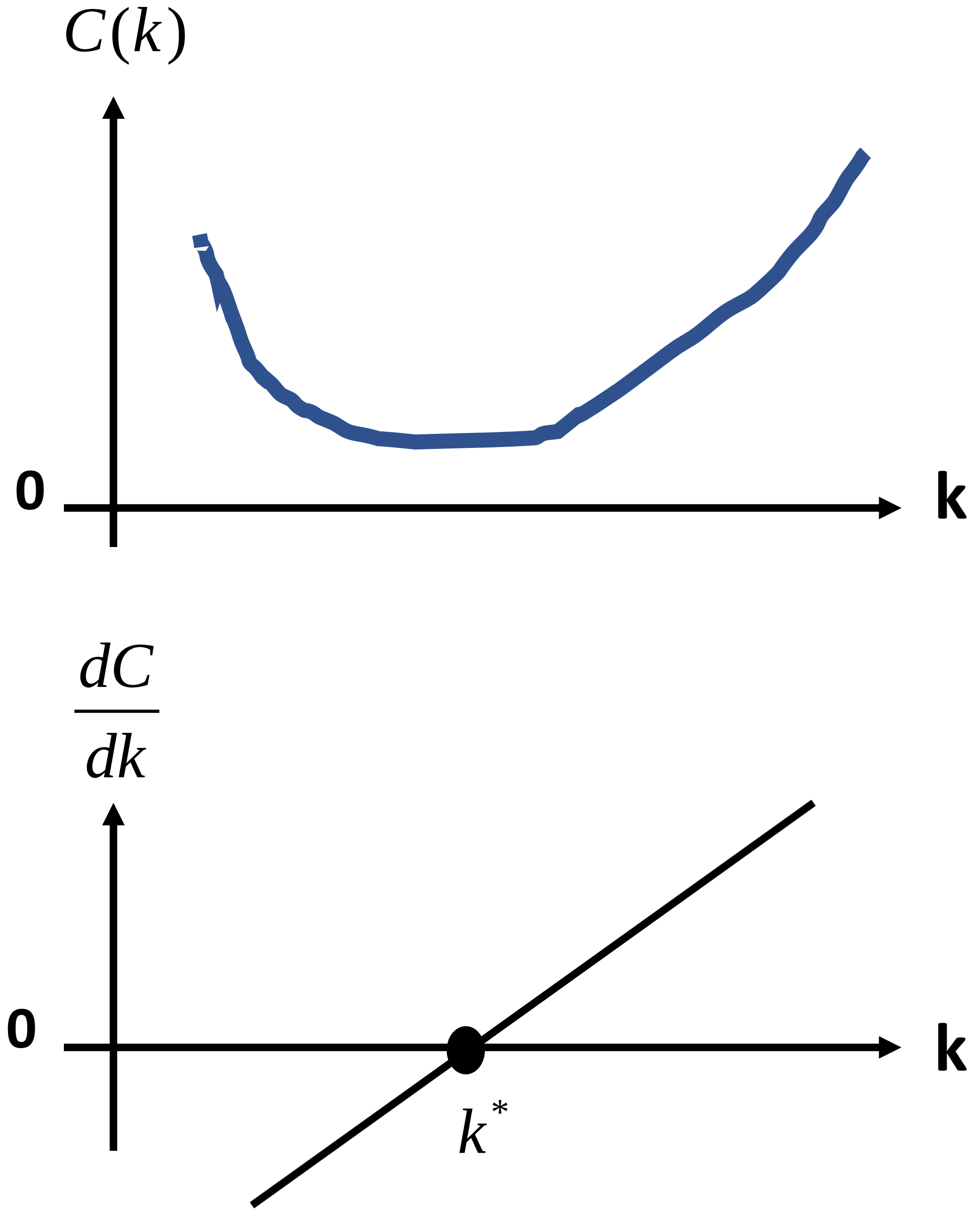
$$\min_{k} C(k;.) = C_I\left(k\right) + C_F(k)$$

$$\min_{k} C(k;.) = \frac{c_u}{4} k^{-2} + c_D N c B_1(k,m) B_2\left(A, H, W\right)$$

$$\min_{k} C(k;.) = \frac{c_u}{4} k^{-2} + c_D N c B_1(k,m) B_2\left(A, H, W\right)$$

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(.\right) = 0$$

$$\frac{d^2C}{dk^2} = \frac{3c_u}{2} k^{-4} + c_D N c \frac{d^2 B_1}{dk^2} B_2\left(.\right) > 0$$


$C(k)$
0
k
$\frac{dC}{dk}$
0
k
k^*

Comparative statics analysis of the optimal solution: How is the optimal solution affected by the parameters?

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2 \left(A, H, W \right) = 0$$

$$d\left(\frac{dC}{dk} \right) = \frac{d^2 C}{dk^2} dk^* + \frac{d^2 C}{dk dc_u} dc_u = 0$$

$$\left(d\left(\frac{dC}{dk} \right) = 0 \right) \Rightarrow \left(\frac{d^2 C}{dk^2} dk^* + \frac{d^2 C}{dk dc_u} dc_u = 0 \right)$$

$$\left(d\left(\frac{dC}{dk} \right) = 0 \right) \Rightarrow \left(\frac{dk^*}{dc_u} = \frac{-\left(\frac{d^2 C}{dk dc_u} \right)}{\left(\frac{d^2 C}{dk^2} \right)} \right)$$

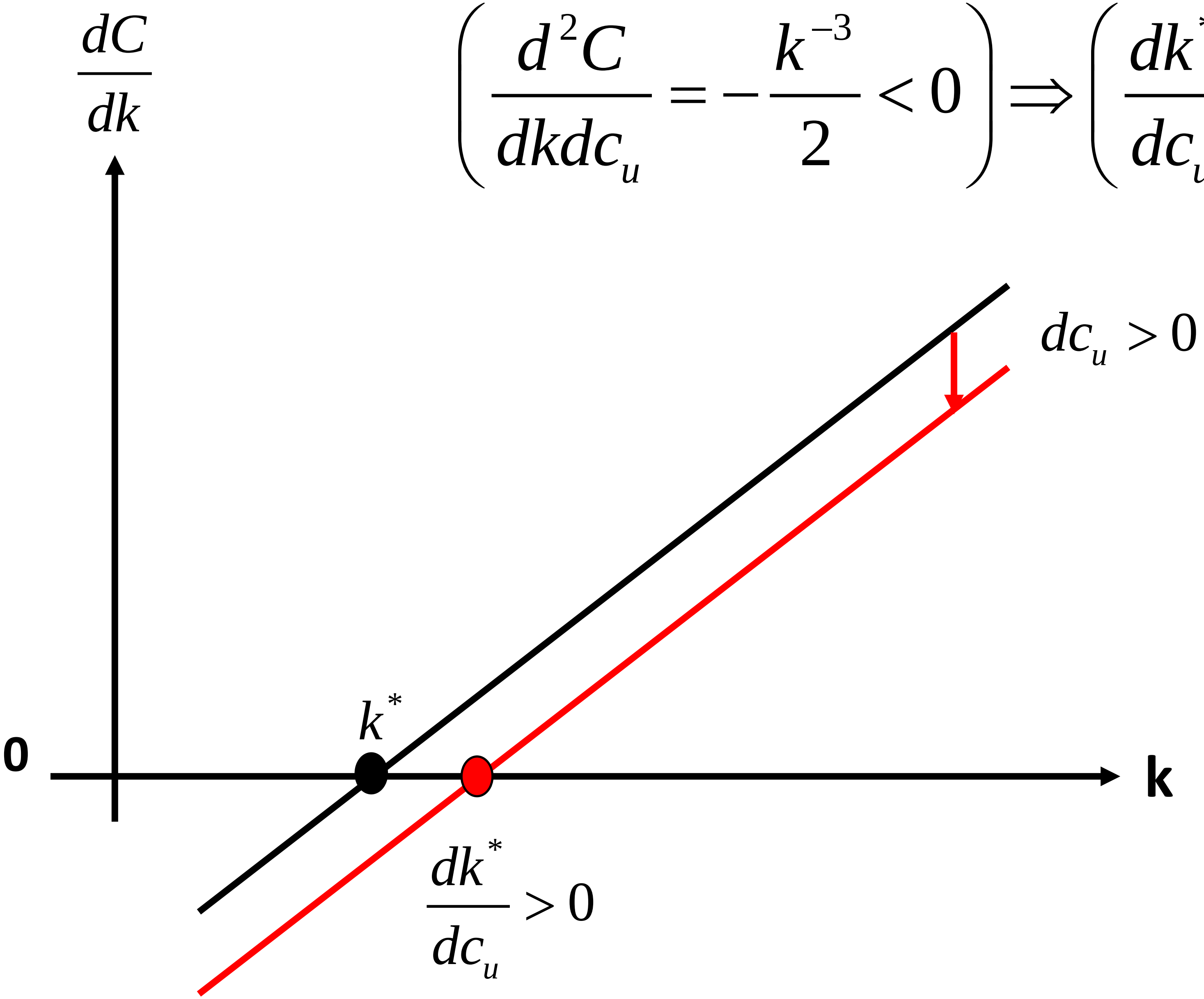
$$\left(d\left(\frac{dC}{dk}\right) = 0 \right) \Rightarrow \left(\frac{dk^*}{dc_u} = \frac{-\left(\frac{d^2C}{dkdc_u}\right)}{\left(\frac{d^2C}{dk^2}\right)} \right) \quad , \quad \frac{d^2C}{dk^2} > 0$$

$$\left(d\left(\frac{dC}{dk}\right) = 0 \right) \Rightarrow \left(\operatorname{sgn}\left(\frac{dk^*}{dc_u}\right) = -\operatorname{sgn}\left(\frac{d^2C}{dkdc_u}\right) \right)$$

$$\left(\frac{d^2C}{dkdc_u} = -\frac{k^{-3}}{2} < 0 \right) \Rightarrow \left(\frac{dk^*}{dc_u} > 0 \right)$$

If the cost per fire fighting unit increases,
the optimal distance between fire fighting units increases.

$$\left(\frac{d^2C}{dkdc_u} = -\frac{k^{-3}}{2} < 0\right) \Rightarrow \left(\frac{dk^*}{dc_u} > 0\right)$$

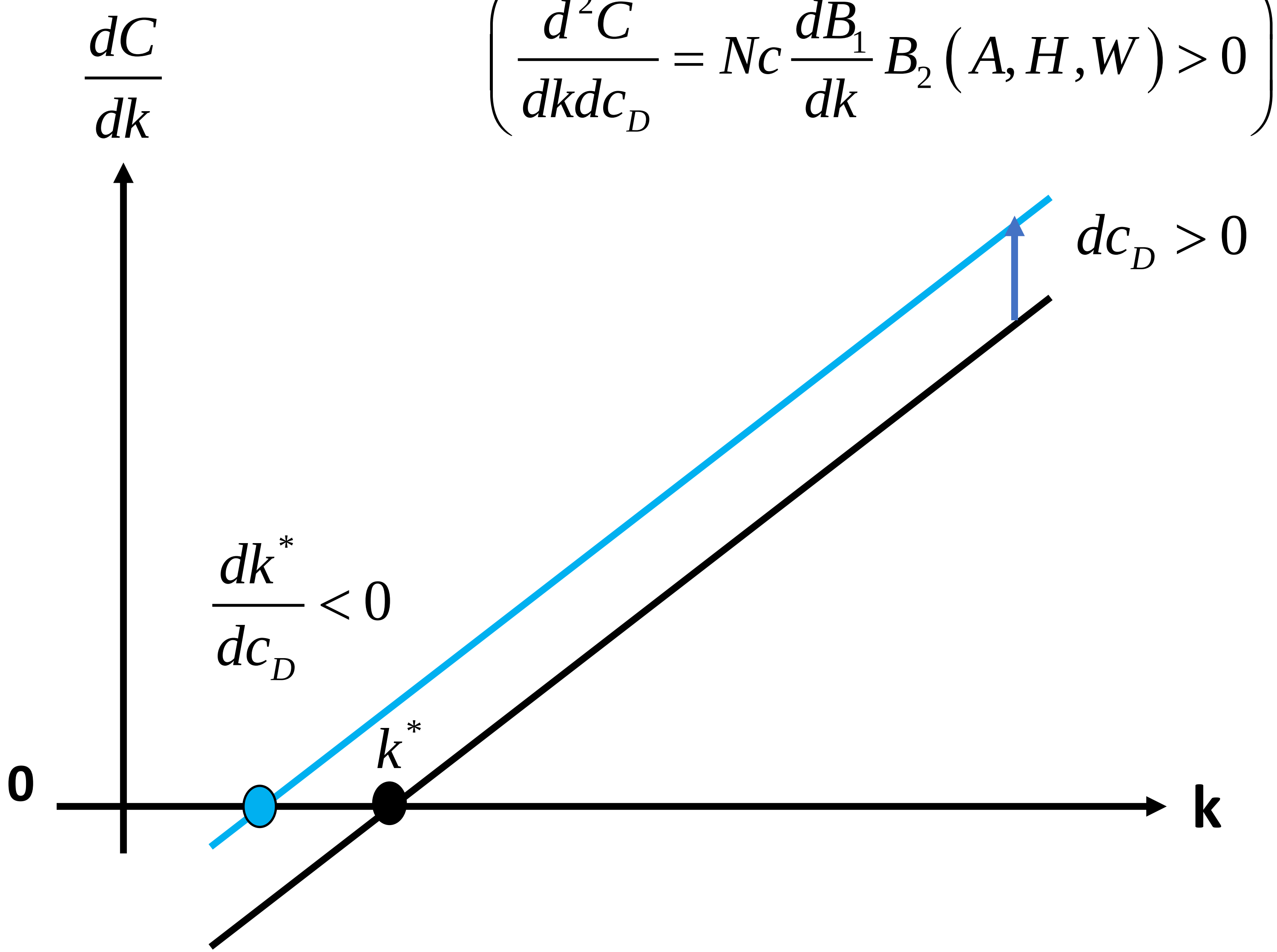


$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2(A, H, W) = 0$$

$$\left(\frac{d^2C}{dk dc_D} = N c \frac{dB_1}{dk} B_2(A, H, W) > 0 \right) \Rightarrow \left(\frac{dk^*}{dc_D} < 0 \right)$$

If the cost per fire damaged hectare of forest increases, the optimal distance between fire fighting units decreases. Note that, if the cost of CO_2 emissions is considered, the optimal value of k is lower than if we do not care about CO_2 emissions.

$$\left(\frac{d^2C}{dkdc_D} = Nc\frac{dB_1}{dk}B_2\left(A,H,W\right) > 0\right) \Rightarrow \left(\frac{dk^*}{dc_D} < 0\right)$$



$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2 \left(A, H, W \right) = 0$$

$$\left(\frac{d^2 C}{dk dN} = c_D c \frac{dB_1}{dk} B_2 \left(A, H, W \right) > 0 \right) \Rightarrow \left(\frac{dk^*}{dN} < 0 \right)$$

If the expected number of fires per area unit increases,
the optimal distance between fire fighting units decreases.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2(A, H, W) = 0$$

$$\left(\frac{d^2C}{dkdc} = c_D N \frac{dB_1}{dk} B_2(A, H, W) > 0 \right) \Rightarrow \left(\frac{dk^*}{dc} < 0 \right)$$

If the general fire speed parameter c increases,
the optimal distance between fire fighting units decreases.
Possible reasons: More open terrain and/or
larger amounts of dry grass and other fuels.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1(m,k)}{dk} B_2(A,H,W) = 0$$

$$\left(\frac{d^2C}{dkdm} = c_D N c \frac{d^2B_1}{dkdm} B_2(A,H,W) > 0 \right) \Rightarrow \left(\frac{dk^*}{dm} < 0 \right)$$

If m increases, the fires get more time to grow before fire supression starts,
and the optimal distance between fire fighting units decreases.
Possible reasons: Less efficient and/or intensive fire surveillance,
fog, smoke or worse road conditions, forcing the fire fighting units
to decrease the travel speed.

Fire growth as a function of weather conditions

The comparative statics analysis now needs some empirical information.

$$B = 5.40687 \cdot H_R^{-4.40357} e^{0.027112T} e^{1.88944 \cdot 10^{-6} W^3}$$

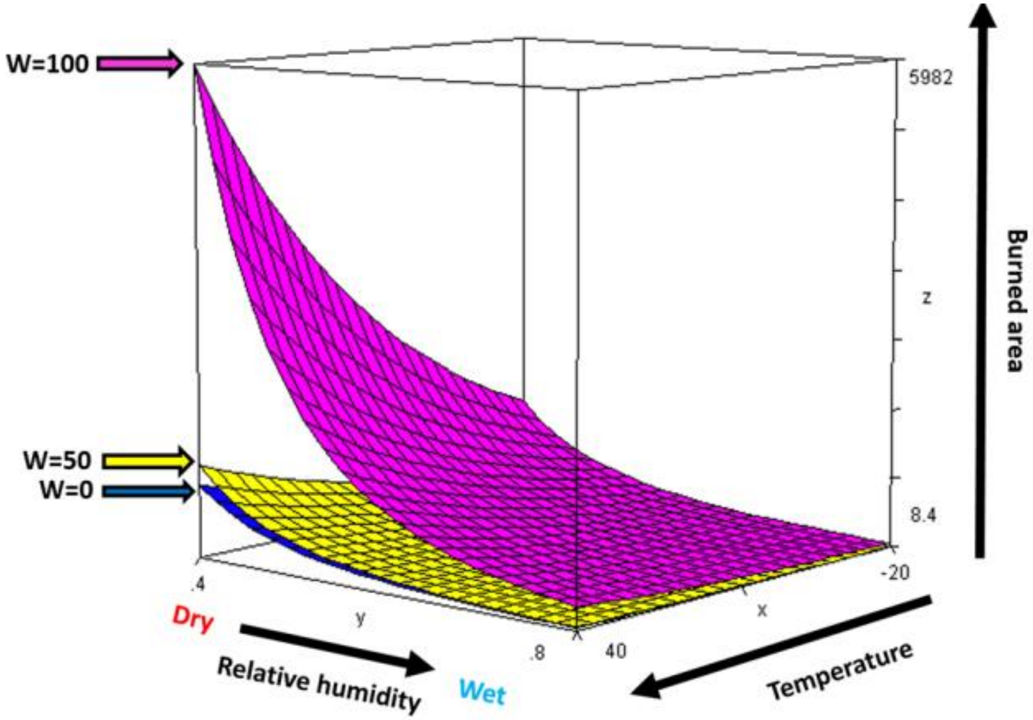


Fig. 7 Individual burned area (m^2) as a function of relative humidity (a value between 0 and 1), air temperature (°C) and wind speed (km h^{-1}) based on Eq. 19. The figure shows the simultaneous effects of relative humidity, air temperature and wind speed on forest wildfire size in the Prague-East District of the Czech Republic. Increases in air temperature and wind speed and a reduction in humidity increase the size of the wildfire

Mohammadi, Z., Lohmander, P., Kašpar, J. *et al.* **The effect of climate factors on the size of forest wildfires (case study: Prague-East district, Czech Republic)**. *J. For. Res.* (2021). https://doi.org/10.1007/s11676-021-01413-w

The empirical estimations gave these results:

$$B_2(A, H, W)$$

$$\frac{dB_2}{dA} > 0$$

$$\frac{dB_2}{dH} < 0$$

$$\frac{dB_2}{dW} > 0$$

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2 \left(A, H, W \right) = 0$$

$$\left(\frac{d^2 C}{dkdA} = c_D N c \frac{dB_1}{dk} \frac{dB_2 \left(A, H, W \right)}{dA} > 0 \right) \Rightarrow \left(\frac{dk^*}{dA} < 0 \right)$$

If the air temperature increases, the speed of fire growth increases, and the optimal distance between fire fighting units decreases.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2C}{dkdH} = c_D N c \frac{dB_1}{dk} \frac{dB_2\left(A, H, W\right)}{dH} < 0 \right) \Rightarrow \left(\frac{dk^*}{dH} > 0 \right)$$

If the relative humidity increases, the speed of fire growth decreases, and the optimal distance between fire fighting units increases.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2C}{dkdW} = c_D N c \frac{dB_1}{dk} \frac{dB_2\left(A, H, W\right)}{dW} > 0 \right) \Rightarrow \left(\frac{dk^*}{dW} < 0 \right)$$

If the wind speed increases, the speed of fire growth increases, and the optimal distance between fire fighting units decreases.

Comments on the optimimization problem

Above, we have assumed that labor employment constraints are applied, that require that the numbers of fire fighting units at different points in time are determined before the true weather component residuals are known.

Optimality conditions of the dynamically changing firefighting capacity levels have been analytically determined.

The solutions have been found to be unique minima.

Comparative statics analysis has been used to determine the directions of change of the optimal capacity levels under the influence of alternative parameter changes.

The expected fire sizes can also be numerically approximated via random numbers with relevant correlations, based on Cholesky factorization.

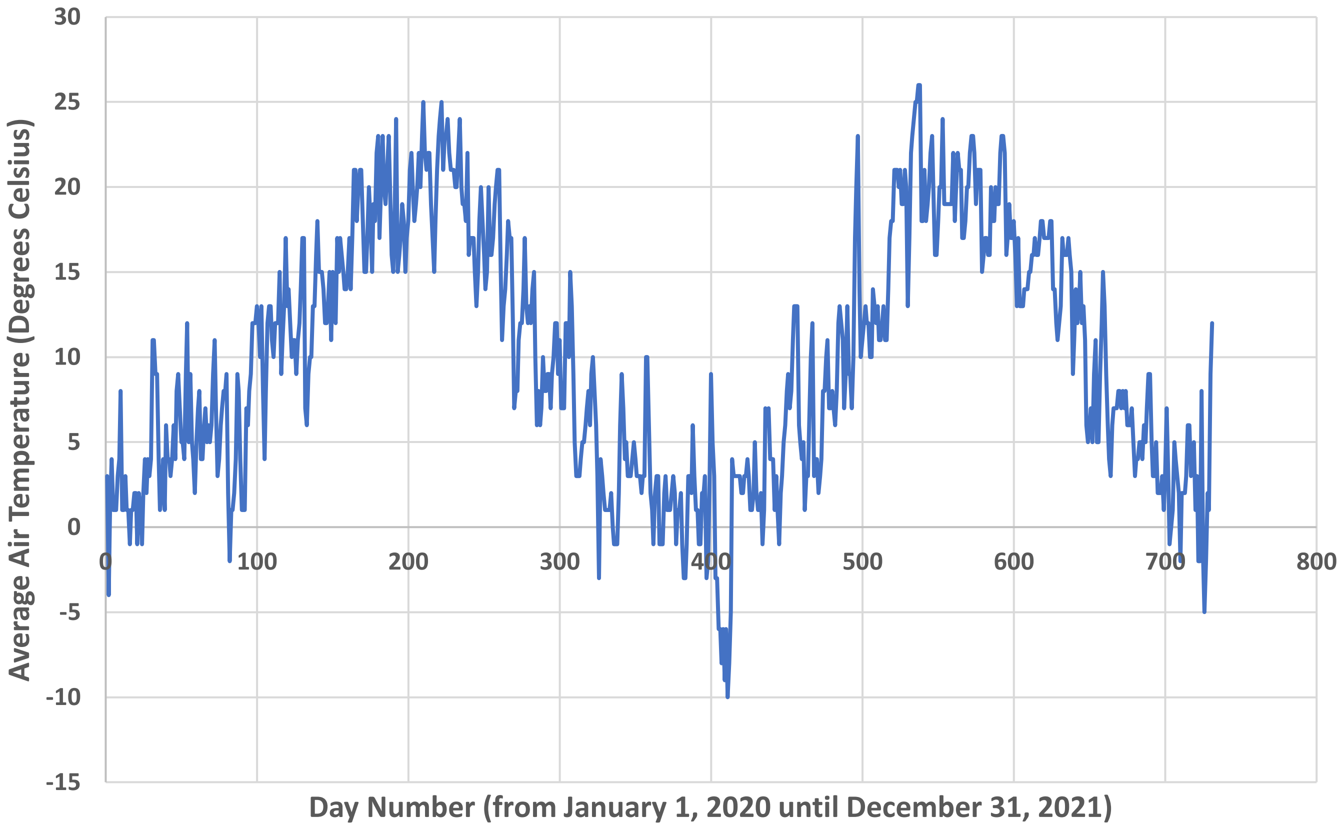
A stochastic dynamic programming version of the dynamic investment decision problem is also possible to present, based on a very flexible labor market, where the number of fire fighting units rapidly can be adapted to the sequentially revealed weather situation. However, since time is limited and since such labour conditions are not relevant in many countries, this will not be presented here.

Background to empirical weather and fire data

Dynamic and stochastic properties of air temperature, relative humidity and wind speed

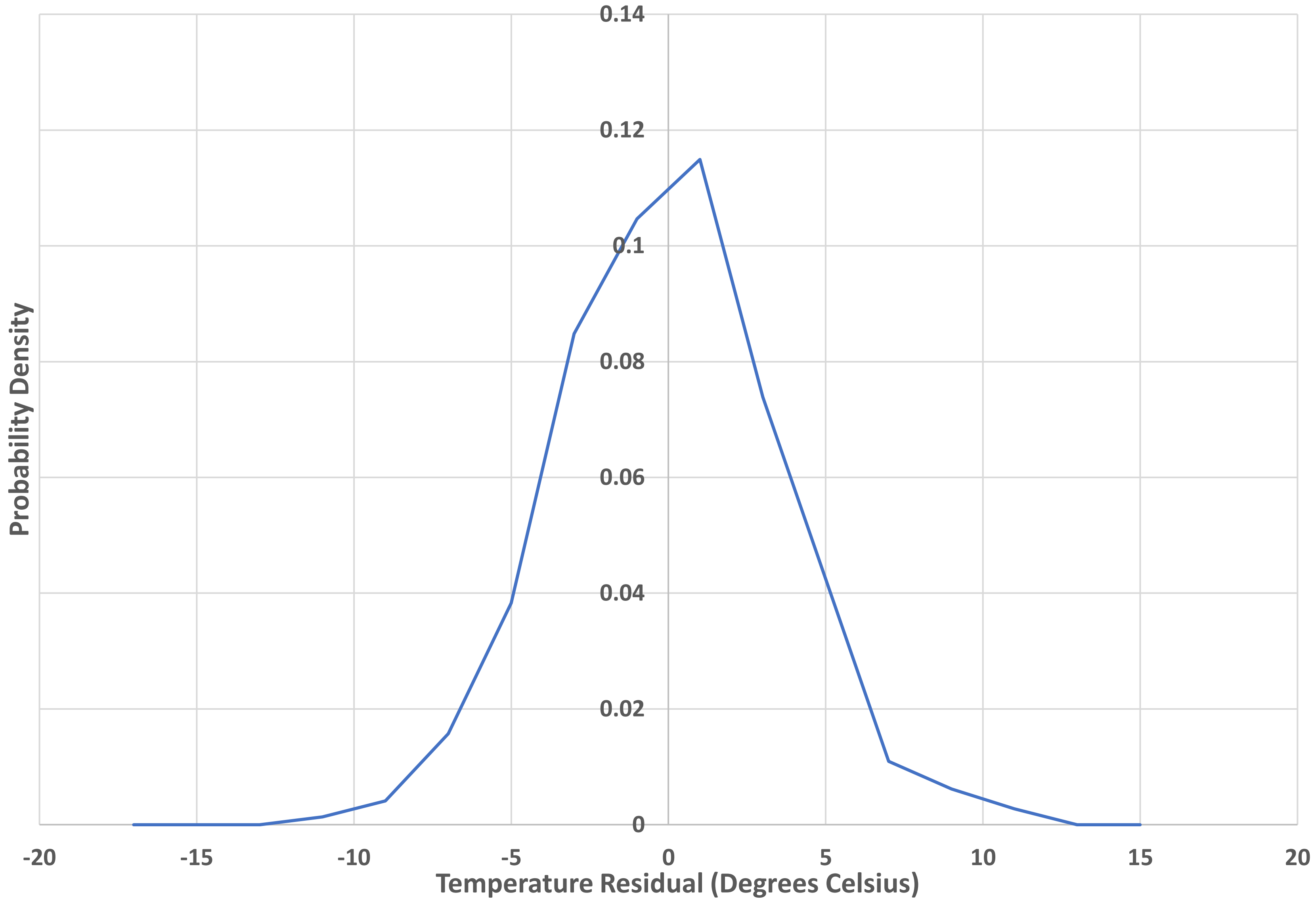
The following parameter estimations related to dynamic and stochastic properties of air temperature, relative humidity and wind speed, can be used in the fire fighting capacity optimization problem.

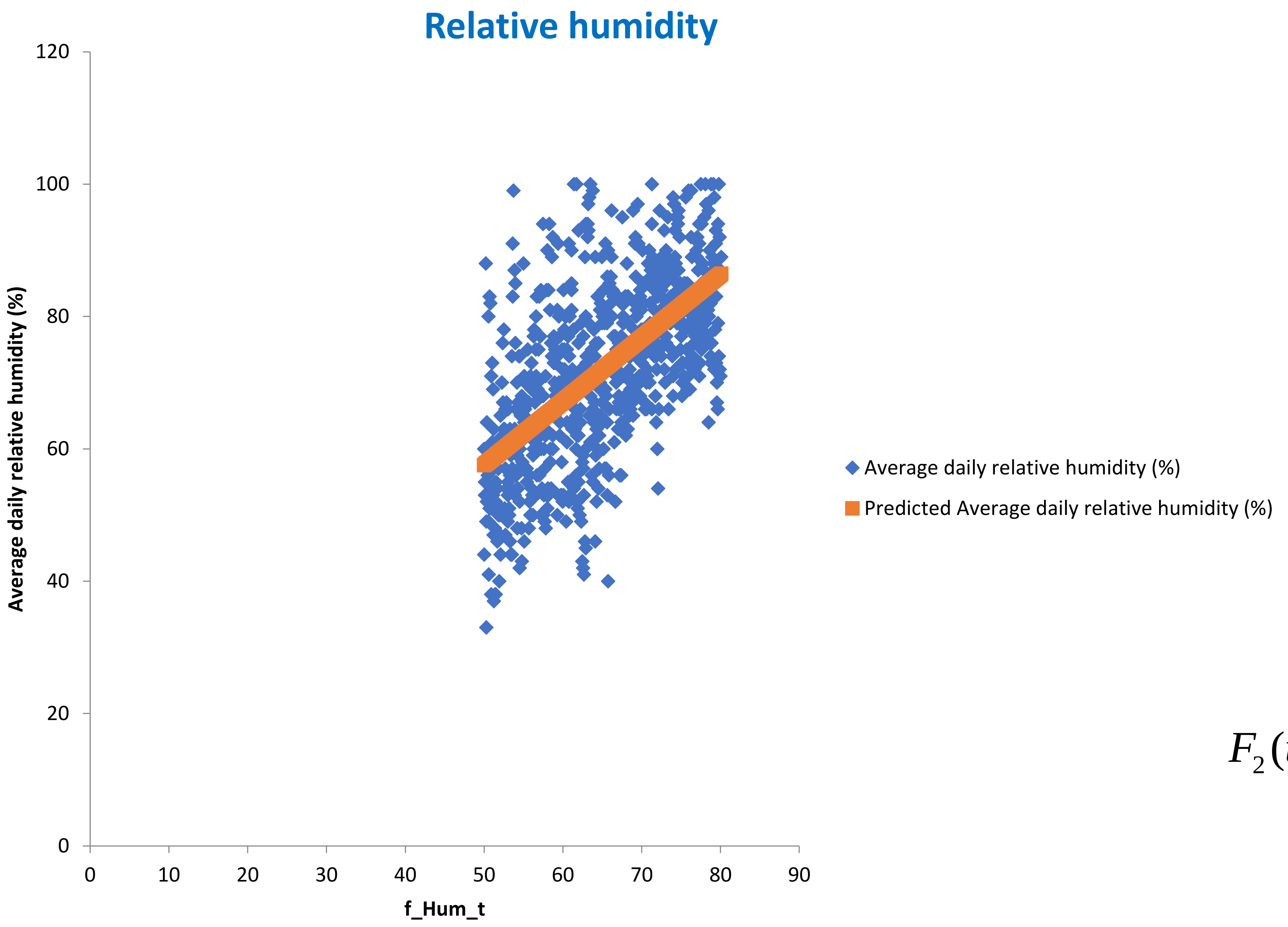
$$A(t) = a_0 + a_1 F_1(t) + \varepsilon_A(t)$$



$$s.t. \begin{cases} E(a_0) \approx 10.068 & \sigma_{a_0} \approx 0.131 \\ E(a_1) \approx \ 9.153 & \sigma_{a_1} \approx 0.185 \\ E(\varepsilon_A(t)) = 0 & \sigma_A \approx 3.531 \end{cases}$$

$$F_1(t) = \sin\left(\frac{4}{3} + 2\left(\frac{t}{365}\right)\pi\right)$$

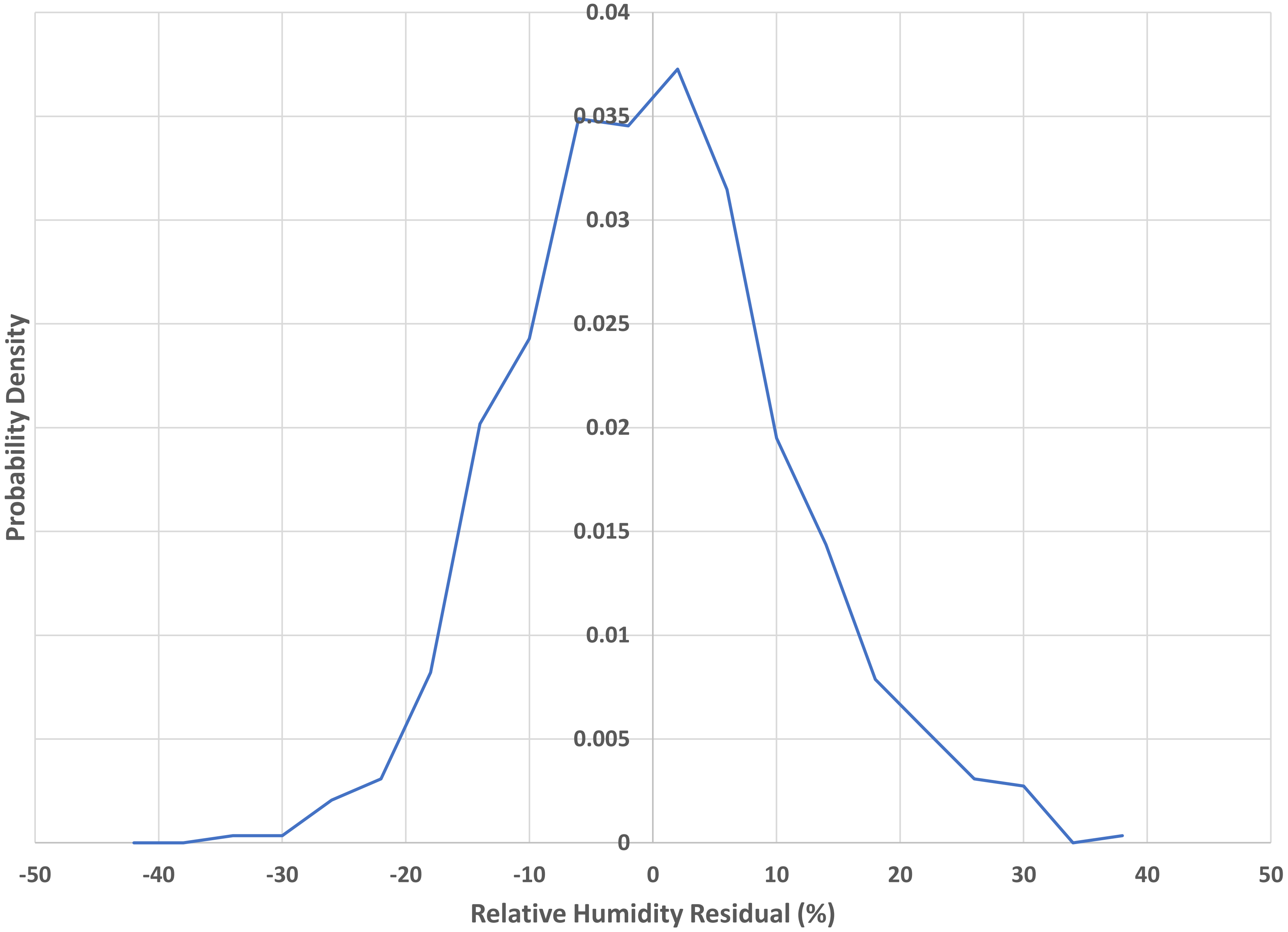

0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
-20
-15
-10
-5
0
5
10
15
20
Probability Density
Temperature Residual (Degrees Celsius)



$$\bar{H}(t) = \bar{h}_0 + \bar{h}_1 F_2(t) + \bar{\varepsilon}_H(t)$$

$$s.t. \begin{cases} E(\bar{h}_0) \approx 9.494 & \sigma_{\bar{h}_0} \approx 3.062 \\ E(\bar{h}_1) \approx 0.961 & \sigma_{\bar{h}_1} \approx 0.0467 \\ E(\bar{\varepsilon}_H(t)) = 0 & \sigma_{\bar{H}} \approx 10.945 \end{cases}$$

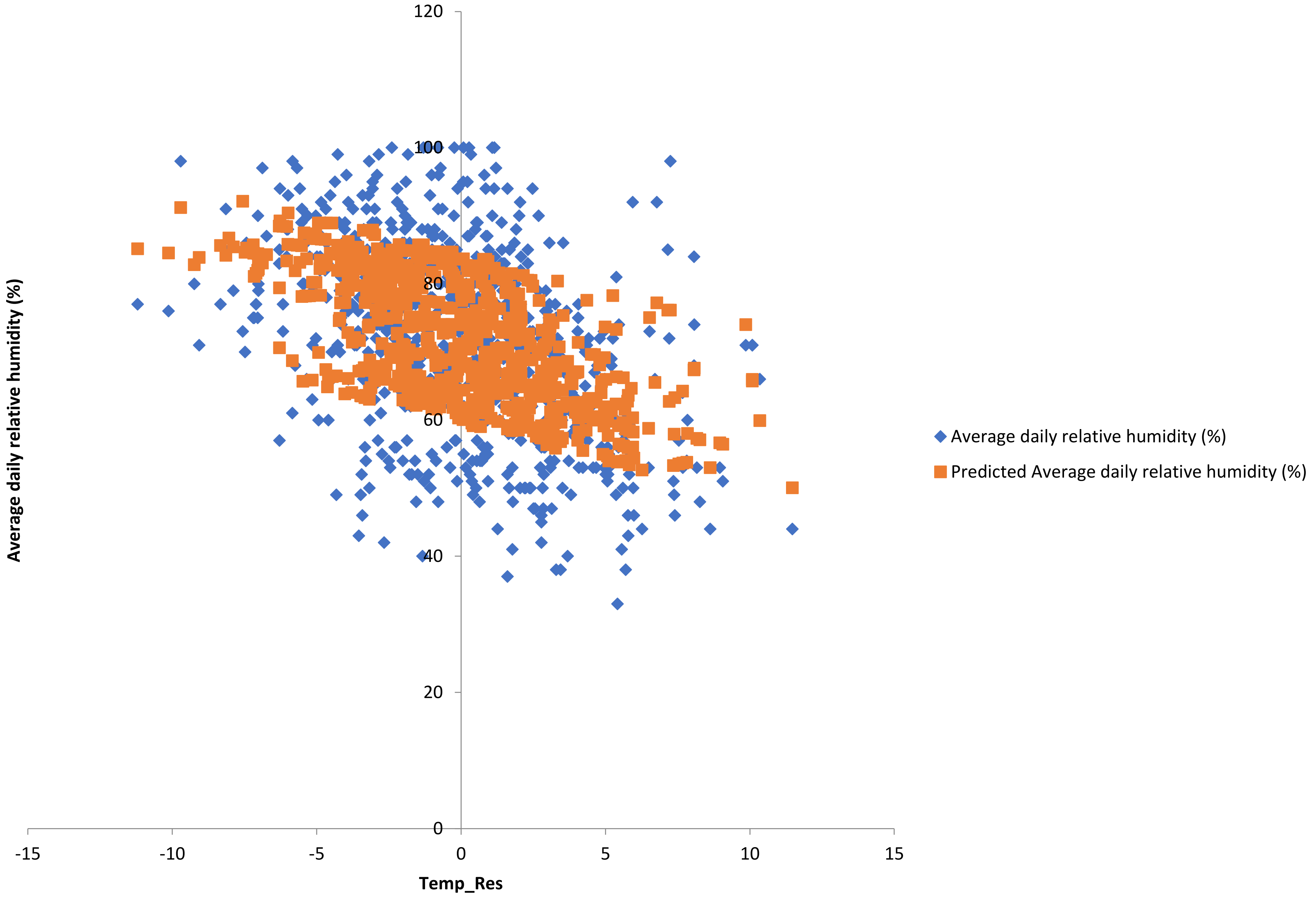
$$F_2(t) = \begin{cases} 80 - 0.3t & 0 \le t \le 100 \\ 50 + \frac{30}{265}(t - 100) & 100 < t \le 366 \end{cases}$$



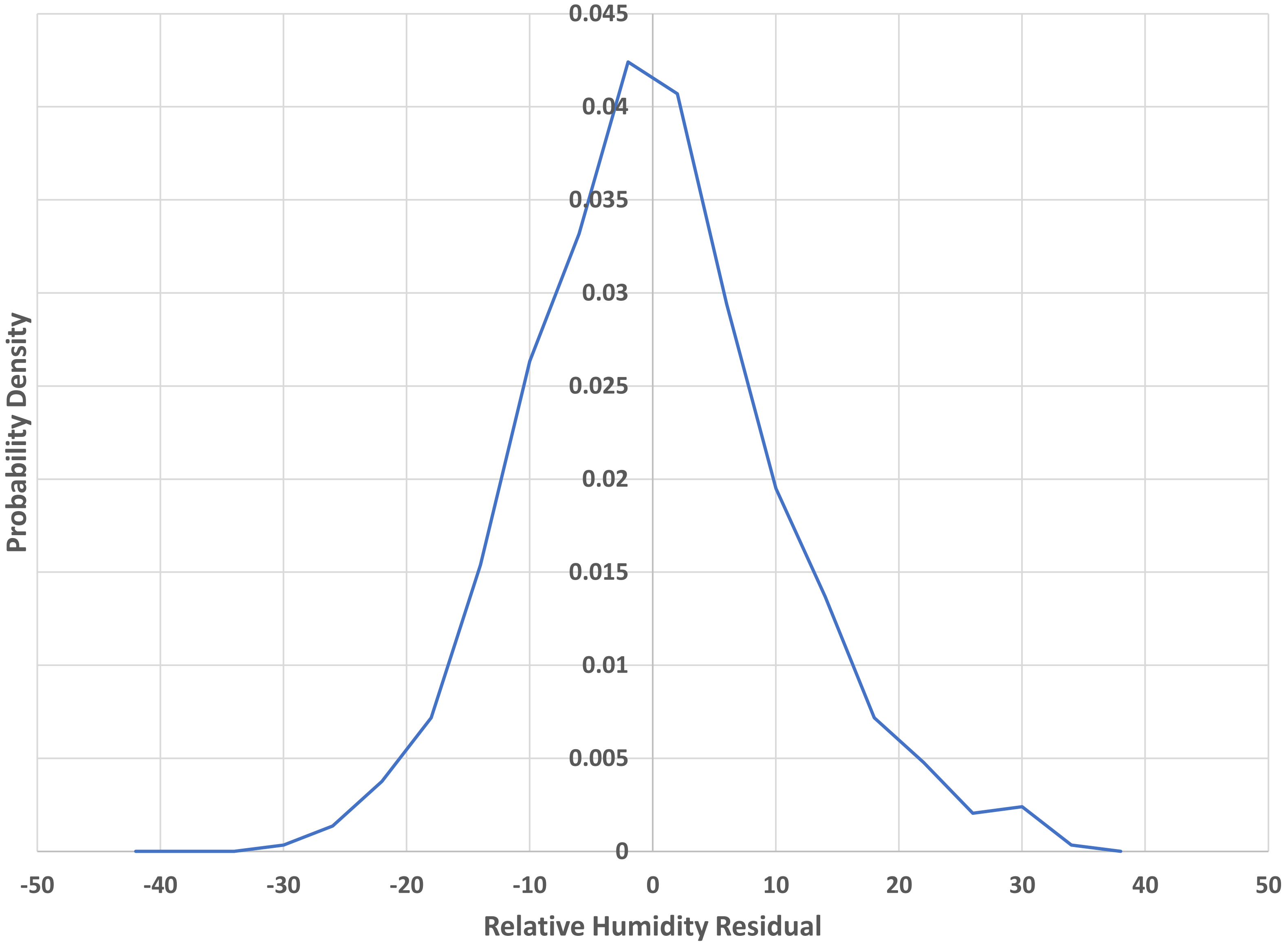
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
Probability Density
Relative Humidity Residual (%)

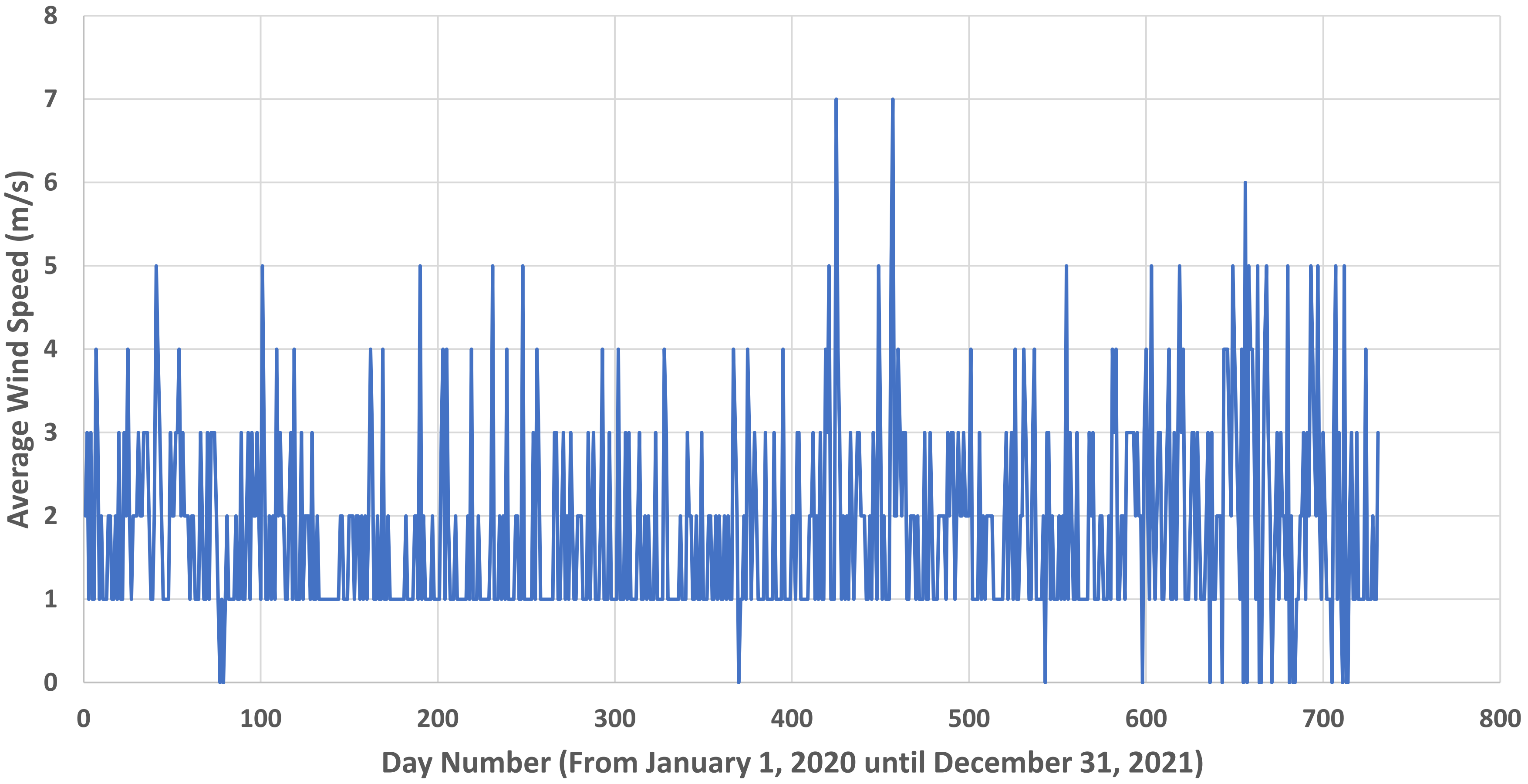
$$H(t) = h_0 + h_1 F_2(t) + h_2 \varepsilon_A(t) + \varepsilon_H(t)$$

$$s.t. \begin{cases} E(h_0) \approx 17.395 \quad \sigma_{h_0} \approx 3.010 \\ E(h_1) \approx 0.839 \quad \sigma_{h_1} \approx 0.0459 \\ E(h_2) \approx -1.067 \quad \sigma_{h_2} \approx 0.113 \\ E(\varepsilon_H(t)) = 0 \quad \sigma_H \approx 10.337 \end{cases}$$

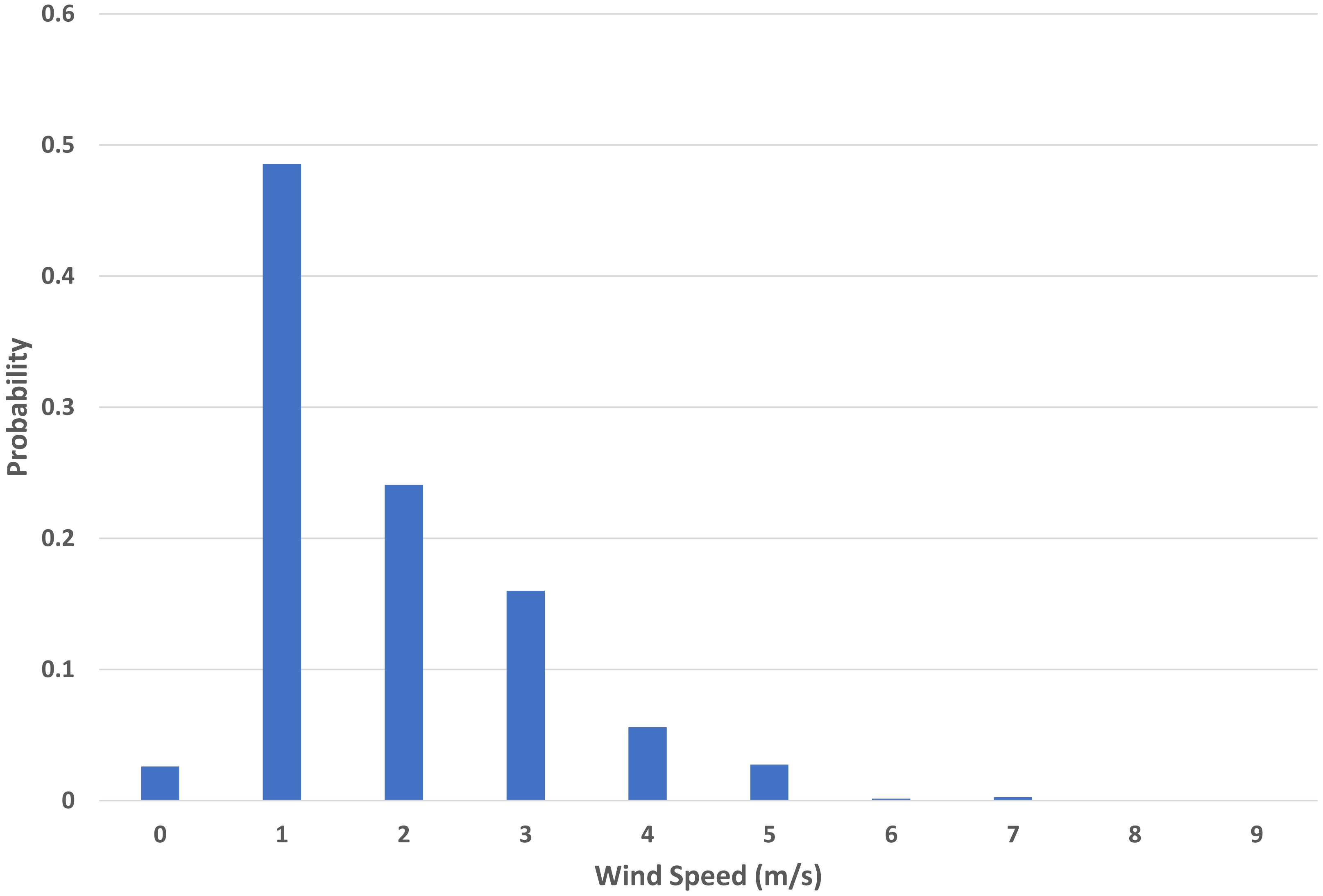


120
100
80
60
40
20
0
-15
-10
-5
0
5
10
15
Average daily relative humidity (%)
Temp_Res
Average daily relative humidity (%)
Predicted Average daily relative humidity (%)


0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Probability Density
-50
-40
-30
-20
-10
0
10
20
30
40
50
Relative Humidity Residual



Average Wind Speed (m/s)
0
1
2
3
4
5
6
7
8
0
100
200
300
400
500
600
700
800
Day Number (From January 1, 2020 until December 31, 2021)



0.6
0.5
0.4
0.3
0.2
0.1
0
Probability
0
1
2
3
4
5
6
7
8
9
Wind Speed (m/s)

Conclusions

The optimal fire fighting capacity is a dynamically changing function of several parameters. The optimal solution has been derived and presented in general form. Comparative statics analysis has been used to show how the optimal decisions are affected by parameter changes.

Since the optimal decisions are functions of many local cost parameters and since different regions have different and dynamically changing weather parameters, the optimal capacity solutions are not the same in different regions.

The general approach presented here should be possible to use in most countries and regions of our world.

THANK YOU VERY MUCH FOR YOUR TIME AND A MOST INTERESTING CONFERENCE!

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http://www.lohmander.com/Information/Ref.htm