Wildlife Dynamics and Management Optimization via Statistics and Mathematics

by

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Moose

Wolf

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Optimal Solutions
Sweden

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This presentation is based on the following three open access articles. More details can be obtained from the links found below.

[1] Lohmander, P., 2017. Optimal Stochastic Control in Continuous Time with Wiener Processes: General Results and Applications to Optimal Wildlife Management, *Iranian Journal of Operations Research*, Vol. 8, No. 2, 58-67. http://iors.ir/journal/article-1-541-en.pdf

[2] Lohmander, P., 2021. Dynamics, stability and sustainable optimal control in wolf-moose systems, *International Robotics & Automation Journal*, Volume 7, Issue 1, 2021, 24-33. https://medcraveonline.com/IRATJ/IRATJ-07-00223.pdf

[3] Lohmander, P., 2021. Optimal Adaptive Integer Pulse Control of Stochastic Nonlinear Systems: Application to the Wolf-Moose Predator Prey System, *Asian Journal of Statistical Sciences*, 1(1), 23-38. http://arfjournals.com/abstract/41816_article_no_3.pdf

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Abstract

Wildlife Dynamics and Management Optimization via Statistics and Mathematics

By Peter Lohmander

Three alternative approaches to dynamic optimization of the management of moose populations are presented. First, a Wiener process and stochastic optimal control theory are used to determine an explicit function for the optimal adaptive hunting level, according to the methods described in Lohmander [1]. Then, the dynamical predator- prey system with wolf and moose populations is defined along the lines presented in Lohmander [2]. The dynamics is analyzed via simulation and the analytical solution to the linearized system, close to the system equilibrium. The system is found to be stable, but it converges very slowly. A deterministic steady state control optimization approach is applied to this system. However, the stochastic variations in the moose and wolf populations are considerable. For this reason, the final approach is the following, based on Lohmander [3]. A numerical approach to optimal adaptive integer pulse control of stochastic nonlinear systems is presented. For most stochastic nonlinear systems, optimal adaptive control rules cannot be derived with analytical methods. A robust optimization algorithm is created. The complete nonlinear adaptively controlled stochastic system is simulated during 100 years, for 100 alternative sequences of stochastic disturbances, for every feasible integer combination of adaptive control rules. The optimal adaptive control rules that maximize the expected value of the objective function are selected as the optimal adaptive control rules. The method is very general and can easily be applied to most adaptive nonlinear stochastic control problems, from technology, management or other fields. The method is tested and applied to the wolf-moose predator prey system. The parameters of this stochastic nonlinear dynamical system have recently been estimated from empirical data from Isle Royale in Lake Superior, USA. The objective function is the expected total present value of all hunting net revenues and the environmental value of preserving the wolf population. The value of the wolf population is a strictly increasing and strictly concave function of the population level. Periodically, the region is visited and the population levels are determined. If the population levels, one for each species, exceed the optimal control limits, then the populations are reduced to the control limits, via hunting. Then, the system is left to develop until the next period. Optimal population control limits and objective function values are determined for alternative levels of the wolf population value function. The average optimal moose hunting level is a decreasing function of the wolf population value parameter and an increasing function of the level of risk in the predator prey system. The average optimal wolf population level is an increasing function of the wolf population value parameter and a decreasing function of the level of risk in the predator prey system.

Three alternative approaches to dynamic optimization of the management of moose populations are presented.

First, a Wiener process and stochastic optimal control theory are used to determine an explicit function for the optimal adaptive hunting level, according to the methods described in Lohmander [1].

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We maximize the objective function:

$$E\left[\int_0^T F(X_t,U_t,t)dt + S(X_t,T)\right] \tag{1}$$

where X_t is the state variable and U_t is the closed loop control variable. Time is represented by t and T is the time horizon. $F(.)$ is the instantaneous profit rate and $S(.)$ is the salvage value. $E[.]$ denotes expected value. z_t is a standard Wiener process.

$$dX_t = f(X_t,U_t,t)dt + G(X_t,U_t,t)dz_t, X_0 = x_0 \tag{2}$$

According to the Bellman principle of optimality, we may determine the value function $V(x,t)$ as the maximum of the sum of the net reward during the first short time interval, $F(.)dt$, and the value function directly after that time interval:

$$V(x,t) = \max_{u} E\left[F(x,u,t)dt + V(x + dX_t, t + dt)\right]. \tag{3}$$

A Taylor approximation gives

$$V(x + dX_t, t + dt) = V(x,t) + V_x dX_t + V_t dt + \frac{V_{xx}(dX_t)^2}{2} + \frac{V_{tt}(dt)^2}{2} + V_{xt}(dX_t)(dt) + o(.) \tag{4}$$

Hence, the value function is approximately given by

$$V = \max_{u} E\left[Fdt + V + V_x f dt + V_t dt + \frac{V_{xx} G^2}{2} dt + o(.) \right], \tag{11}$$

Since V_t is not a function of u, we obtain the *"Hamilton-Jacobi-Bellman equation"* as follows:

$$-V_t = \max_{u} E\left[F + V_x f + \frac{V_{xx} G^2}{2} \right], \tag{14}$$

with the boundary condition

$$V\left(x, T\right) = S\left(x, T\right). \tag{15}$$

3. The Particular Stochastic Optimization Problem

We want to maximize the expected present value of wildlife management. We need the following notations: $u = u(t)$ is the control variable, the level of hunting at time t, $x = x(t)$ is the size of the wildlife population, (k, p, f) are objective function parameters. The net revenue of the hunting and meat values, $ku - pu^2$, is a strictly concave function of the hunting level, fx, which is proportional to the population level, x, is the cost of destroyed forest plantations and cost of traffic accidents caused by the wildlife population. The population growth increases with the size of the population and decreases with the hunting level. The magnitude of the stochastic population changes depend on the standard Wiener process, z, the size of the population, and the risk parameter s. With r as the rate of interest in the capital market, we then have the following problem:

$$\max E\left(\int_0^{\infty} e^{-rt}(ku - pu^2 - fx)dt\right)$$
$$s.t. \qquad dx = (gx - u)\,dt + sx\,dz \tag{16}$$
$$k > 0, p > 0, f > 0, s > 0.$$

$$R(u, x) = (ku - pu^2 - fx)\,. \tag{17}$$

The *"Hamilton-Jacobi-Bellman equation"* becomes

$$-J_t(x,t) = e^{-rt} R(u(t), x(t)) + \frac{s^2 x^2 J_{xx}(x,t)}{2} + J_x(x,t)(gx(t) - u(t)). \tag{18}$$

Now, the problem is to determine the value function and the control function so that the Hamilton-Jacobi-Bellman equation (HJBE) is satisfied. Let us assume that the value function can be expressed as follows:

$$V(x) = a + bx + cx^2, \tag{19}$$

$$J(x(t),t) = e^{-rt} V(x) = e^{-rt}(a + bx + cx^2). \tag{20}$$

We need to optimize the control, u:

$$\max_{u} Z(u) = (ku - pu^2 - fx) + \frac{1}{2}s^2x^2 2c + (b + 2cx)(gx - u). \qquad (26)$$

Using the optimal values of the control, via the optimized control function, we get

$$Z^* = (k - b - 2cx)\left(\frac{k - b - 2cx}{2p}\right) - p\left(\frac{k - b - 2cx}{2p}\right)^2 - fx + \frac{1}{2}s^2x^2 2c + bgx + 2cgx^2. \qquad (32)$$

Now, we have obtained a quadratic function, that always has to be zero. If the function is notzero, then the HJBE is violated. Since the function must hold for all possible values of x, the size of the population, it is clear that we have three equations that can be used to determine the parameters (a, b, c):

$$0=\left(\frac{k^2+b^2-2bk}{4p}-ra\right)+\left(\frac{c(b-k)}{p}+bg-rb-f\right)x+\left(\frac{c^2}{p}+cs^2+2cg-rc\right)x^2. \qquad (41)$$

= 0   = 0   = 0

$$c = p(r - 2g - s^2)\ .$$

$$b = \frac{k(2g - r + s^2) - f}{g + s^2}$$

$$a = \frac{\left(f - k(g - r)\right)^2}{4pr(g + s^2)^2}$$

$$V(x) = a + bx + cx^2, \tag{47}$$

$$V(x) = \frac{\left(f - k(g - r)\right)^2}{4pr(g + s^2)^2} + \left(\frac{k(2g - r + s^2) - f}{g + s^2}\right)x + p(r - 2g - s^2)x^2. \tag{48}$$

Next, we will determine the optimal control function. We know

$$u^* = \left(\frac{k - b - 2cx}{2p} \right).$$

$$u^* = \frac{k(r-g)+f}{2p(g+s^2)} + (2g - r + s^2)x.$$

The Numerically Specified Case

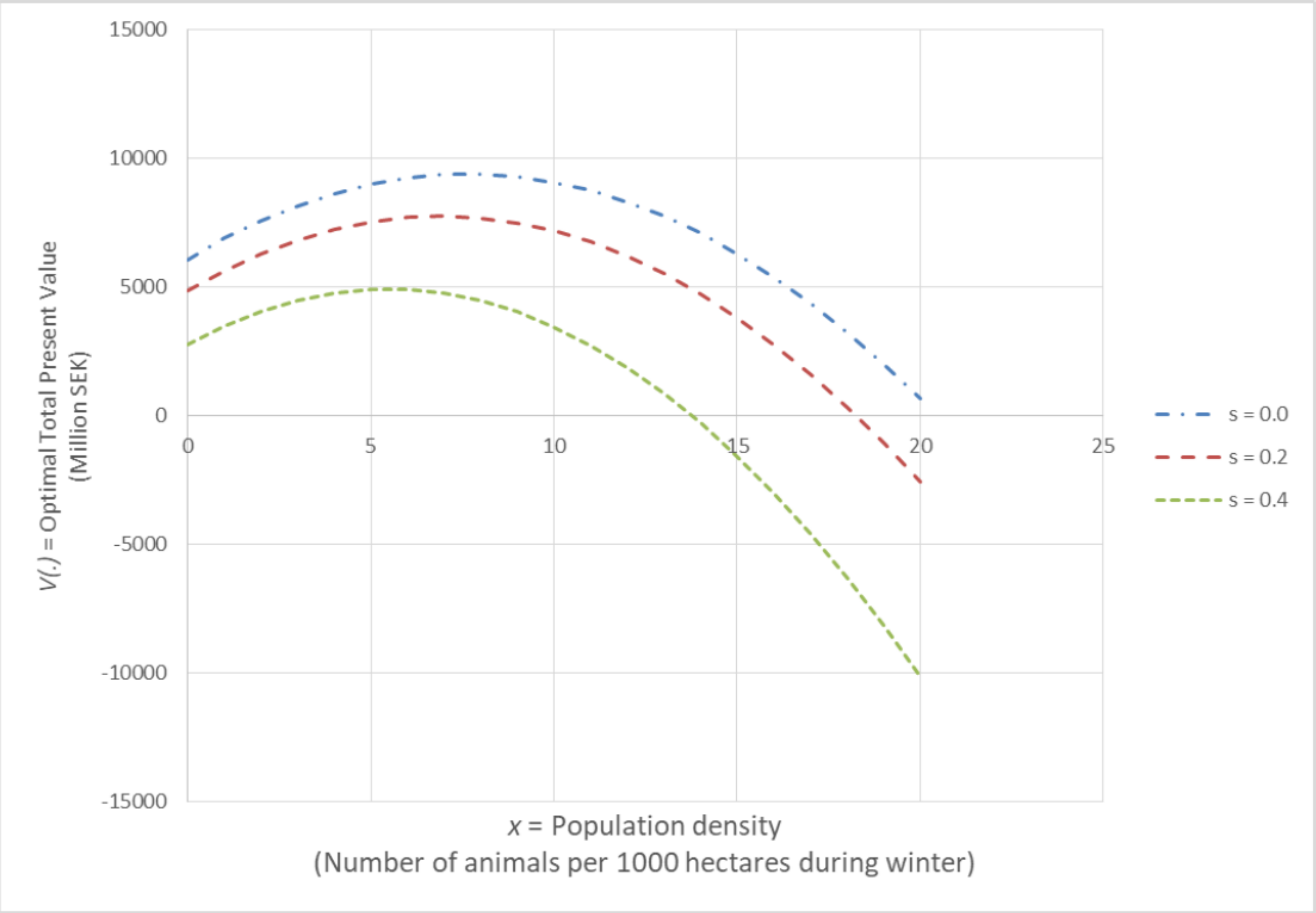

Figure 1. The optimal total present value function, $V(.)$ as a function of the population density, x, and the stochastic parameter s.

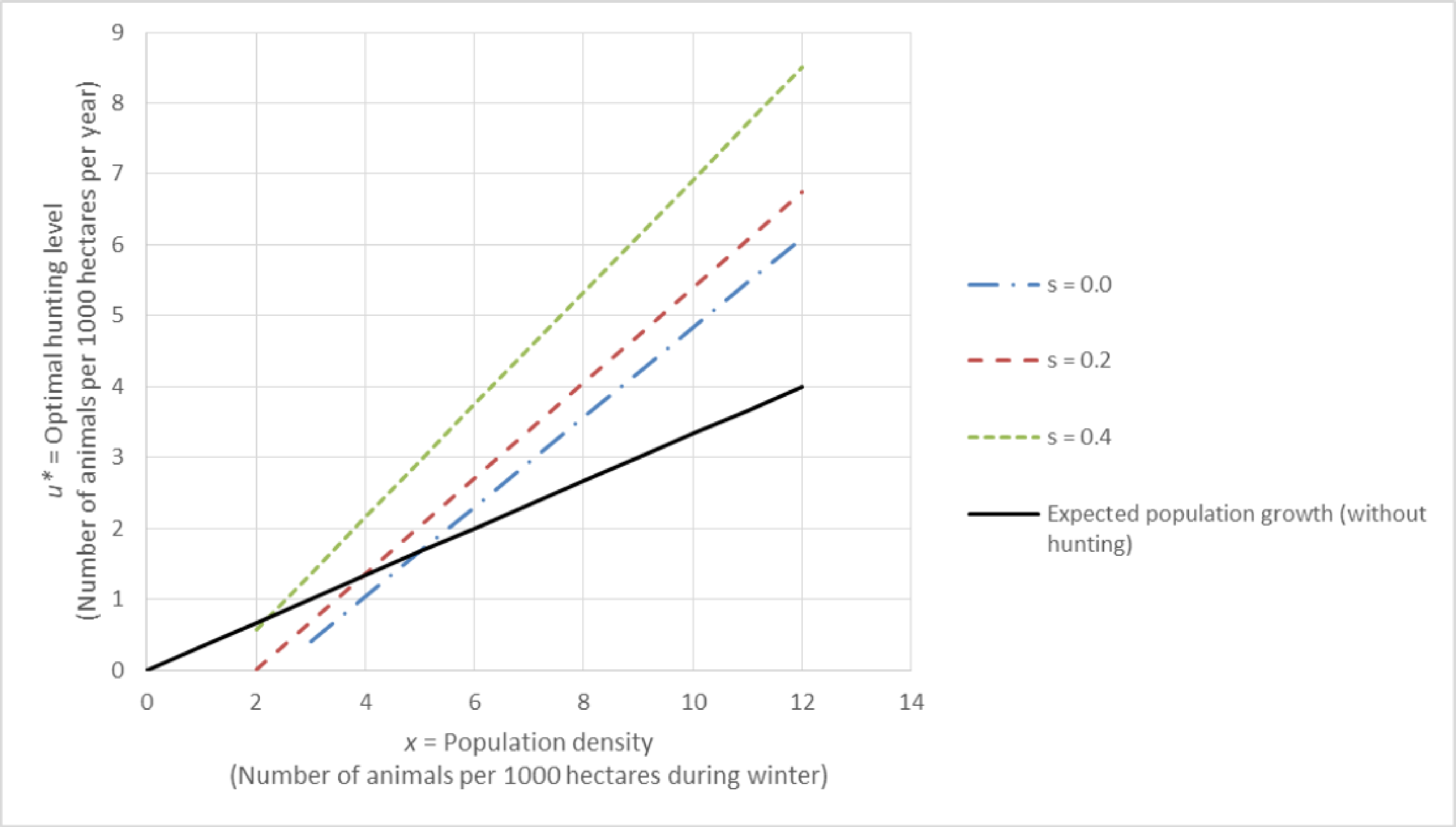

Figure 2. The optimal control, the hunting level, u^*, as a function of the population density, x, and the stochastic parameter s.

Then, the dynamical predator- prey system with wolf and moose populations is defined along the lines presented in Lohmander [2].

The dynamics is analyzed via simulation and the analytical solution to the linearized system, close to the system equilibrium.

The system is found to be stable, but it converges very slowly.

A deterministic steady state control optimization approach is applied to this system.

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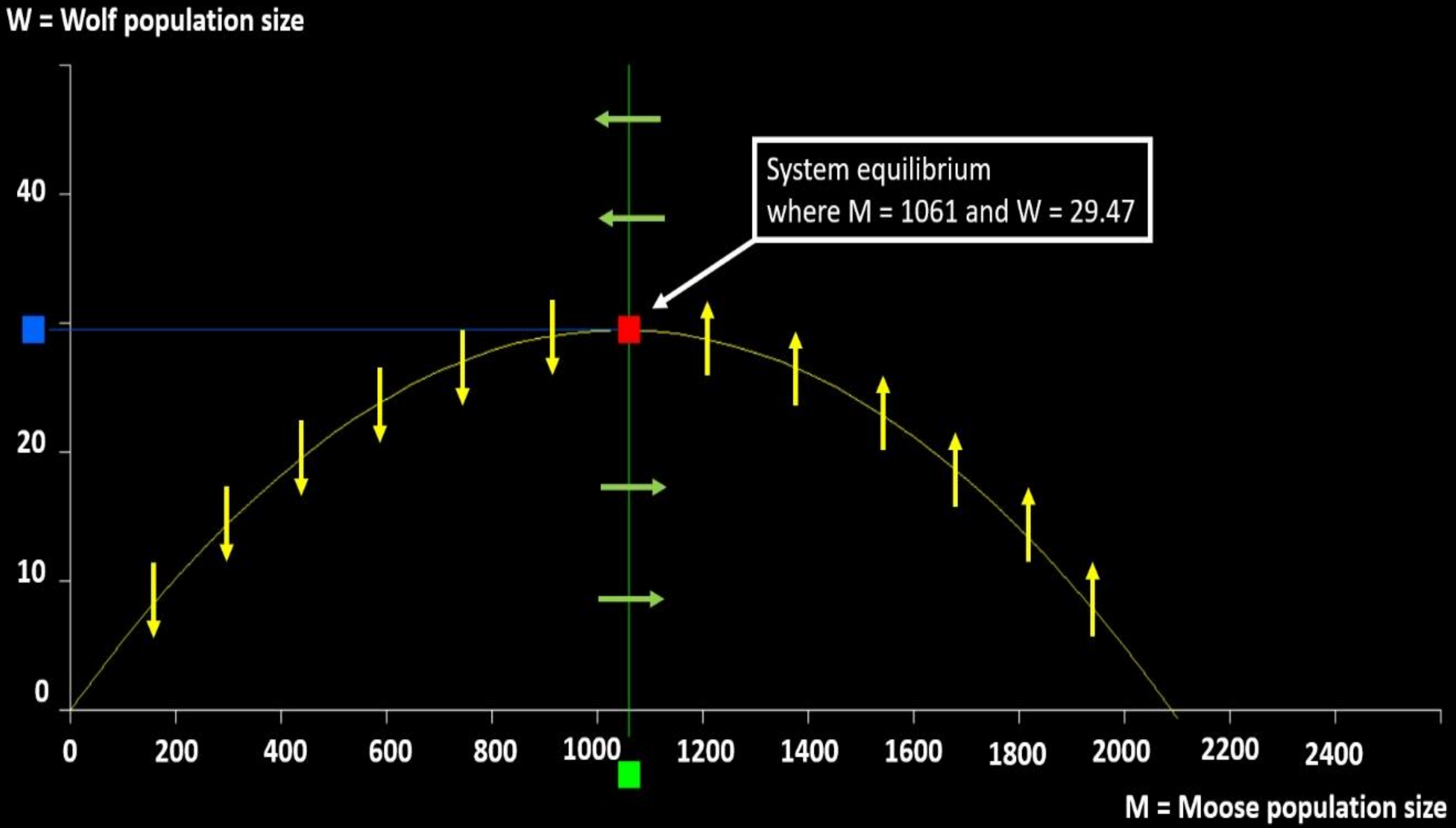
$$\begin{cases} \overset{\bullet}{M} \approx \dfrac{\Delta M}{\Delta t} = aM - bM^2 - cW \\ \overset{\bullet}{W} \approx \dfrac{\Delta W}{\Delta t} = -gW + hMW \end{cases} \quad (6)$$

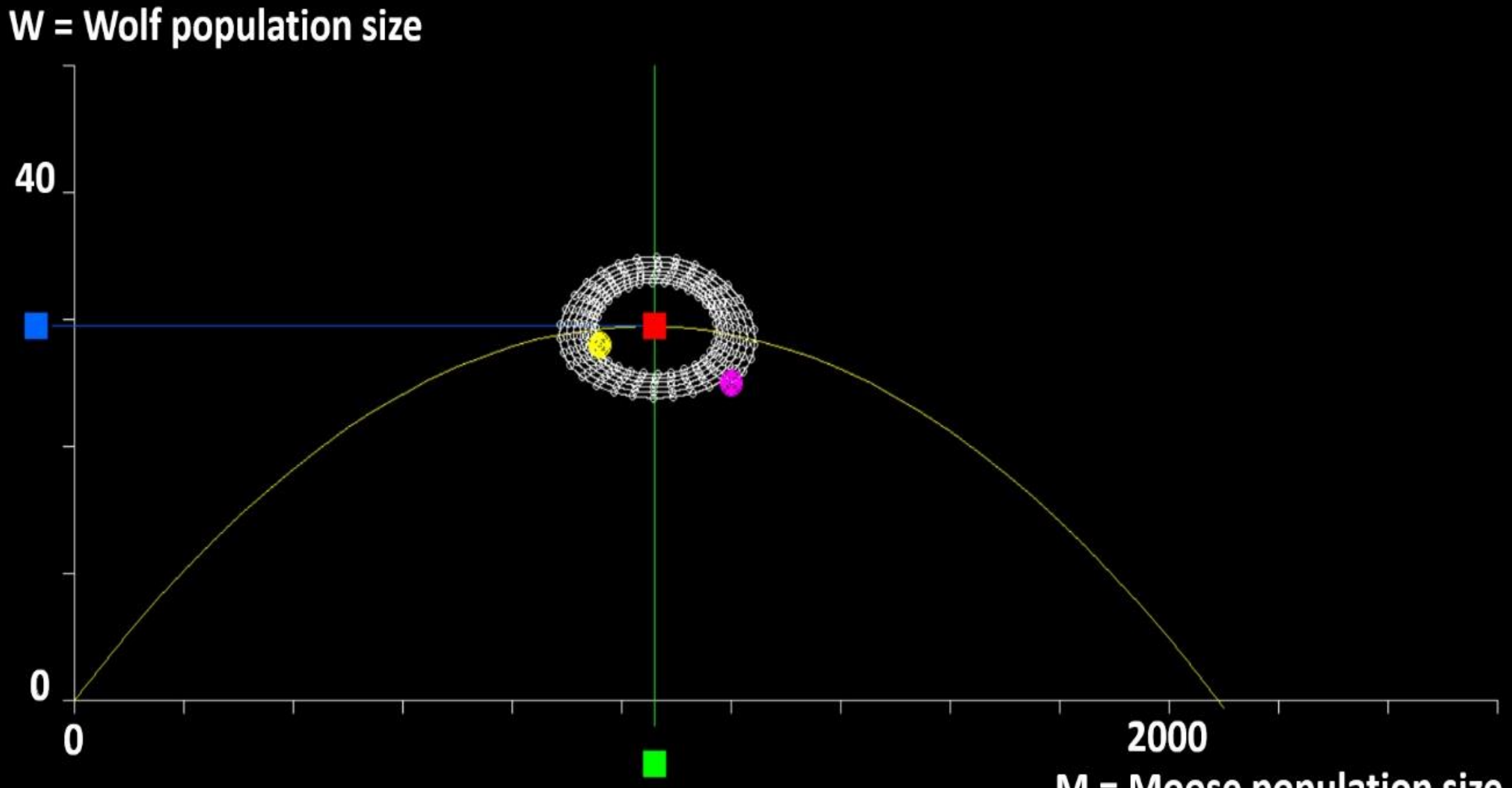
Then, two multiple regression analyses were performed. The residuals $(\varepsilon_M, \varepsilon_W)$ are assumed to be Normally distributed with zero means and correlations.

$$\begin{cases} \dfrac{\left(\dfrac{\Delta M}{\Delta t}\right)}{M} = a - bM - c\dfrac{W}{M} + \varepsilon_M \\ \dfrac{\left(\dfrac{\Delta W}{\Delta t}\right)}{W} = -g + hM + \varepsilon_W \end{cases} \quad (7)$$

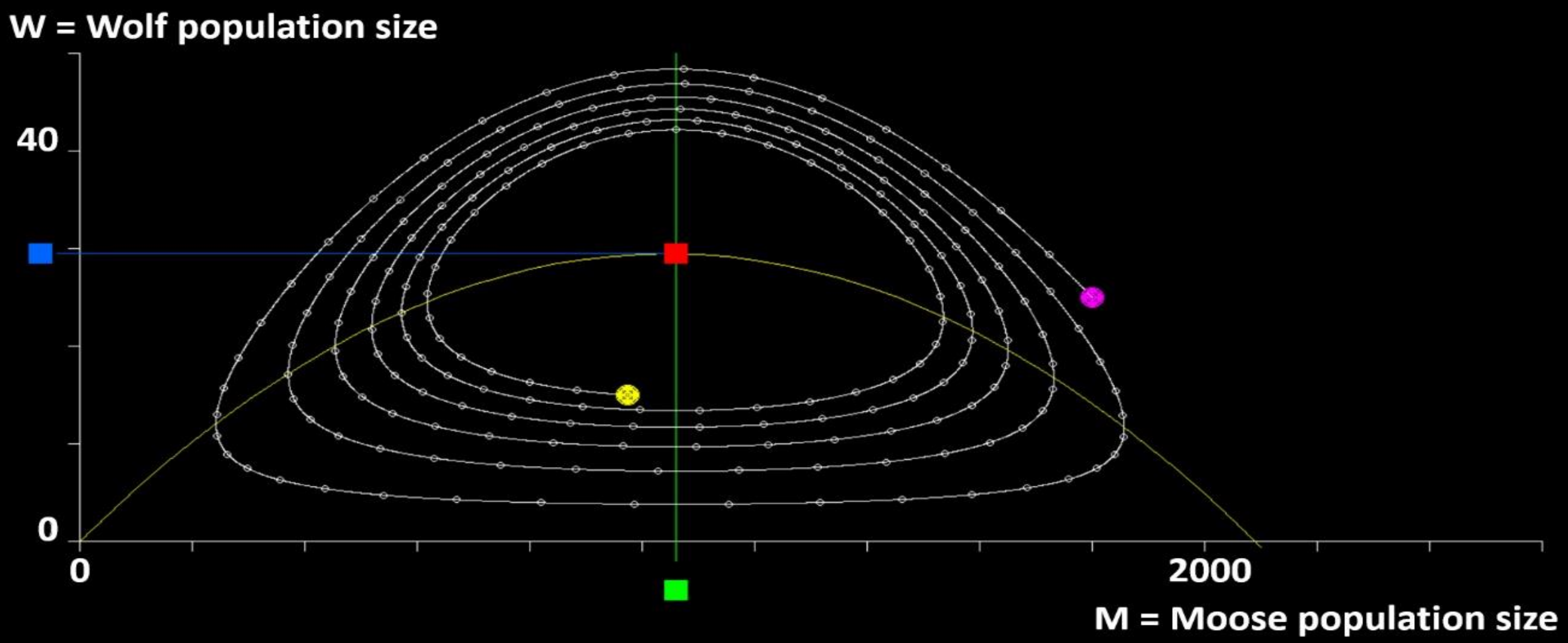
The simplified estimated differential equation system is:

$$\begin{cases} \dot{M} = 0.372M - 0.178 \times 10^{-3} M^2 - 6.593W \\ \dot{W} = -0.244W + 0.230 \times 10^{-3} MW \end{cases} \qquad (17)$$


W = Wolf population size
40
20
10
0
System equilibrium
where M = 1061 and W = 29.47
0
200
400
600
800
1000
1200
1400
1600
1800
2000
2200
2400
M = Moose population size



W = Wolf population size
40
0
0
2000
M = Moose population size


W = Wolf population size
40
0
0
2000
M = Moose population size

In equilibrium, we have:

$$\begin{cases} \dot{M} = aM_1 - bM_1^2 - cW_1 = 0 \\ \dot{W} = -gW_1 + hM_1W_1 = 0 \end{cases} \tag{33}$$

Now, we will let u and v represent small deviations from the equilibrium.

$$M = M_1 + u \tag{34}$$

$$W = W_1 + v \tag{35}$$

The differential equation system may now be rewritten as (36).

$$\begin{cases} \dot{M} = a\left(M_1 + u\right) - b\left(M_1 + u\right)^2 - c\left(W_1 + v\right) \\ \dot{W} = -g\left(W_1 + v\right) + h\left(M_1 + u\right)\left(W_1 + v\right) \end{cases} \quad (36)$$

The system (37) follows:

$$\begin{cases} \dot{M} = aM_1 + au - bM_1^2 - 2bM_1u - bu^2 - cW_1 - cv \\ \dot{W} = -gW_1 - gv + hM_1W_1 + hM_1v + hW_1u + huv \end{cases} \quad (37)$$

$$\begin{cases} \dot{u} = \dot{M} = au - 2bM_1 u - bu^2{}_1 - cv \\ \dot{v} = \dot{W} = -gv + hM_1 v + hW_1 u + huv \end{cases} \quad (39)$$

$$\begin{cases} \dot{u} = (a - 2bM_1)u + (-c)v - bu^2 \\ \dot{v} = (hW_1)u + (-g + hM_1)v + huv \end{cases} \quad (40)$$

As we approach an equilibrium, the second order terms go to zero much faster than the first order terms.

$$\left(u \to 0 \wedge v \to 0\right) \Rightarrow \left(\frac{u^2}{u} \to 0 \wedge \frac{uv}{u} \to 0 \wedge \frac{uv}{v} \to 0\right) \qquad (41)$$

Hence, close to the equilibrium, $u^2 \approx 0 \wedge uv \approx 0$.

We may express the linear differential equation system as (43).

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} (a - 2bM_1) & -c \\ hW_1 & (-g + hM_1) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{43}$$

We remember that the wolf population is in equilibrium. This gives (44).

$$\left(\dot{W} = 0 \right) \Rightarrow (-g + hM_1) = 0 \tag{44}$$

Hence, the differential equation system is simplified.

$$\begin{bmatrix} \overset{\bullet}{u} \\ \overset{\bullet}{v} \end{bmatrix} = \begin{bmatrix} (a - 2bM_1) & -c \\ hW_1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{45}$$

The parameter values that we now need are these:

$$a = 0.372,\ b = 0.178 \times 10^{-3},\ c = 6.593,\ g = 0.244,\ h = 0.230 \times 10^{-3},\ M_1 \approx 1061,\ W_1 \approx 29.47 \tag{46}$$

With the parameter values, we get:

$$\begin{bmatrix} \overset{\bullet}{u} \\ \overset{\bullet}{v} \end{bmatrix} = \begin{bmatrix} -5.716 \times 10^{-3} & -6.593 \\ 6.778 \times 10^{-3} & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{47}$$

We consider exponential functional forms, as in (48).

$$u(t) = u_0 e^{\lambda t} \wedge v(t) = v_0 e^{\lambda t} \tag{48}$$

Then, the time derivatives can be obtained and included in the equations, as in (49).

$$\begin{bmatrix} \lambda u \\ \lambda v \end{bmatrix} = \begin{bmatrix} -5.716 \times 10^{-3} & -6.593 \\ 6.778 \times 10^{-3} & 0 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} \tag{49}$$

Clearly, it is necessary that (50) is satisfied.

$$\begin{bmatrix} \left(-5.716\times10^{-3}-\lambda\right) & -6.593 \\ 6.778\times10^{-3} & \left(0-\lambda\right) \end{bmatrix}\begin{bmatrix} u \\ v \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (50)$$

We want to be sure that nontrivial solutions can be obtained. This implies the condition found in (51).

$$\left(u\neq0\wedge v\neq0\right)\Rightarrow\left(|D|=\begin{vmatrix} \left(-5.716\times10^{-3}-\lambda\right) & -6.593 \\ 6.778\times10^{-3} & \left(0-\lambda\right) \end{vmatrix}=0\right) \qquad (51)$$

Hence, a quadratic equation has to be solved. This is found in (52).

$$|D|=\lambda^{2}+5.716\times10^{-3}\lambda+44.687\times10^{-3}=0 \qquad (52)$$

$$|D| = \lambda^2 + 5.716 \times 10^{-3} \lambda + 44.687 \times 10^{-3} = 0 \qquad (52)$$

$$|D| = \lambda^2 + p\lambda + q = 0 \qquad (53)$$

There are, in general, two solutions.

$$\lambda = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q} \qquad (54)$$

$$\lambda = -0.002858 \pm \sqrt{-0.0446788} \qquad (55)$$

Obviously, we have two complex roots. The real part is negative. This means that the solutions will be converging spirals. Since the real part is close to zero, the solution converges very slowly. In fact, it takes approximately 243 years for the spiral to get 50% closer to the equilibrium. This seems reasonable if we compare the spirals in the Figures 2 & 3.

$$\lambda = -0.002858 \pm 0.21137i \quad (56)$$

$$\phi = -0.002858, \psi = 0.21137 \quad (57)$$

$$\begin{cases} u(t) = & e^{\phi t}\left(A\cos\left(\psi t\right) + B\sin\left(\psi t\right)\right) \\ v(t) = & \dfrac{1}{\Omega} e^{\phi t}\left(\left(\left(\phi-\gamma\right)A + \psi B\right)\cos\left(\psi t\right) + \left(\left(\phi-\gamma\right)B - \psi A\right)\sin\left(\psi t\right)\right) \end{cases} \quad (66)$$

Equilibrium 1 is:

$$(M_1, W_1) = \left(\left(\frac{g}{h} \right), \frac{\left(aM_1 - bM_1^{\ 2} \right)}{c} \right) \tag{80}$$

This can also be stated as:

$$(M_1, W_1) = \left(\left(\frac{g}{h} \right), \frac{\left(a\left(\frac{g}{h} \right) - b\left(\frac{g}{h} \right)^2 \right)}{c} \right) \tag{81}$$

In order to determine if the system is stable or unstable, it turns out that it is necessary to investigate the sign of the first part of the element in the first row and first column of the determinant (84). In order to do this, we first investigate the solution to an alternative problem, found in equation (85).

$$\max_{M}\left(aM - bM^{2}\right) = M^{*} \tag{85}$$

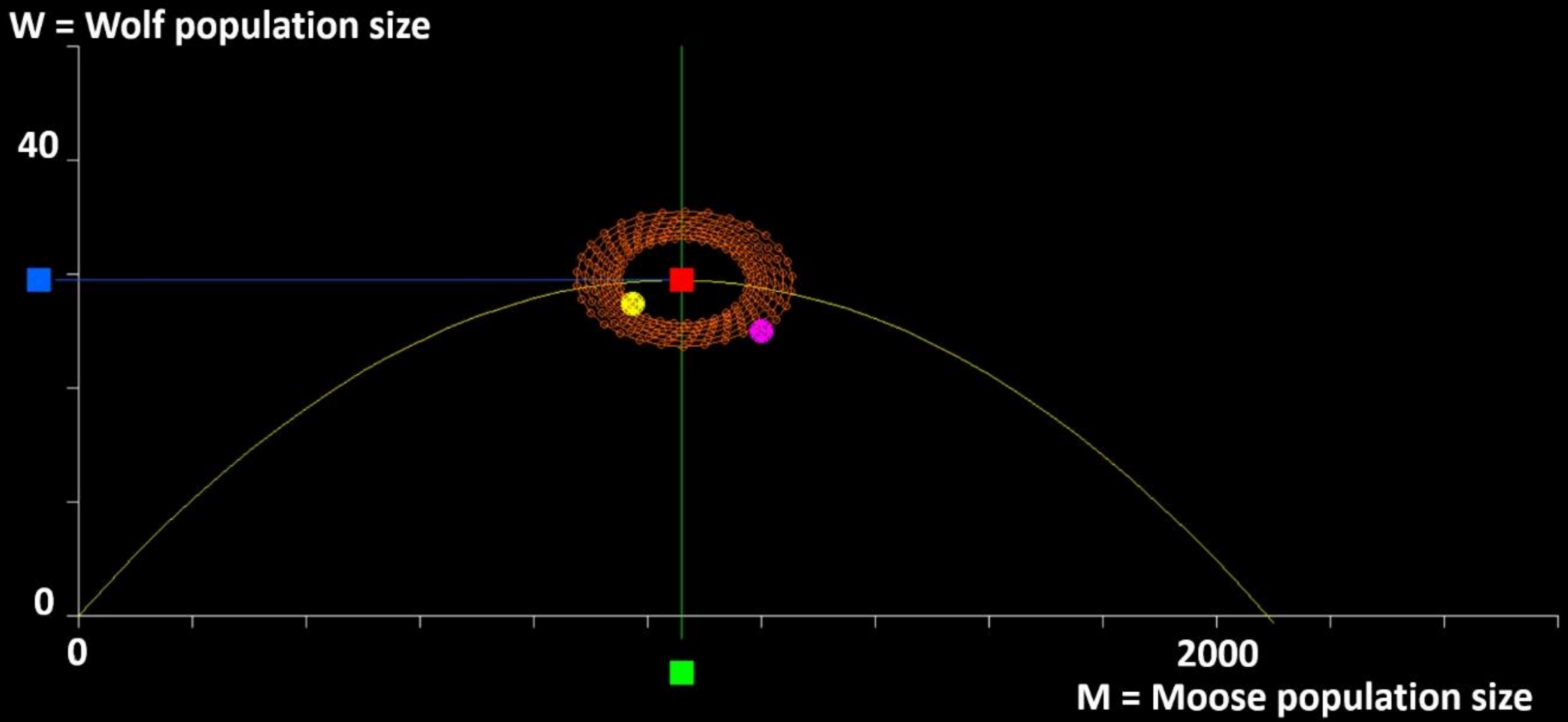
The size of the moose population that maximizes the net moose production per year.

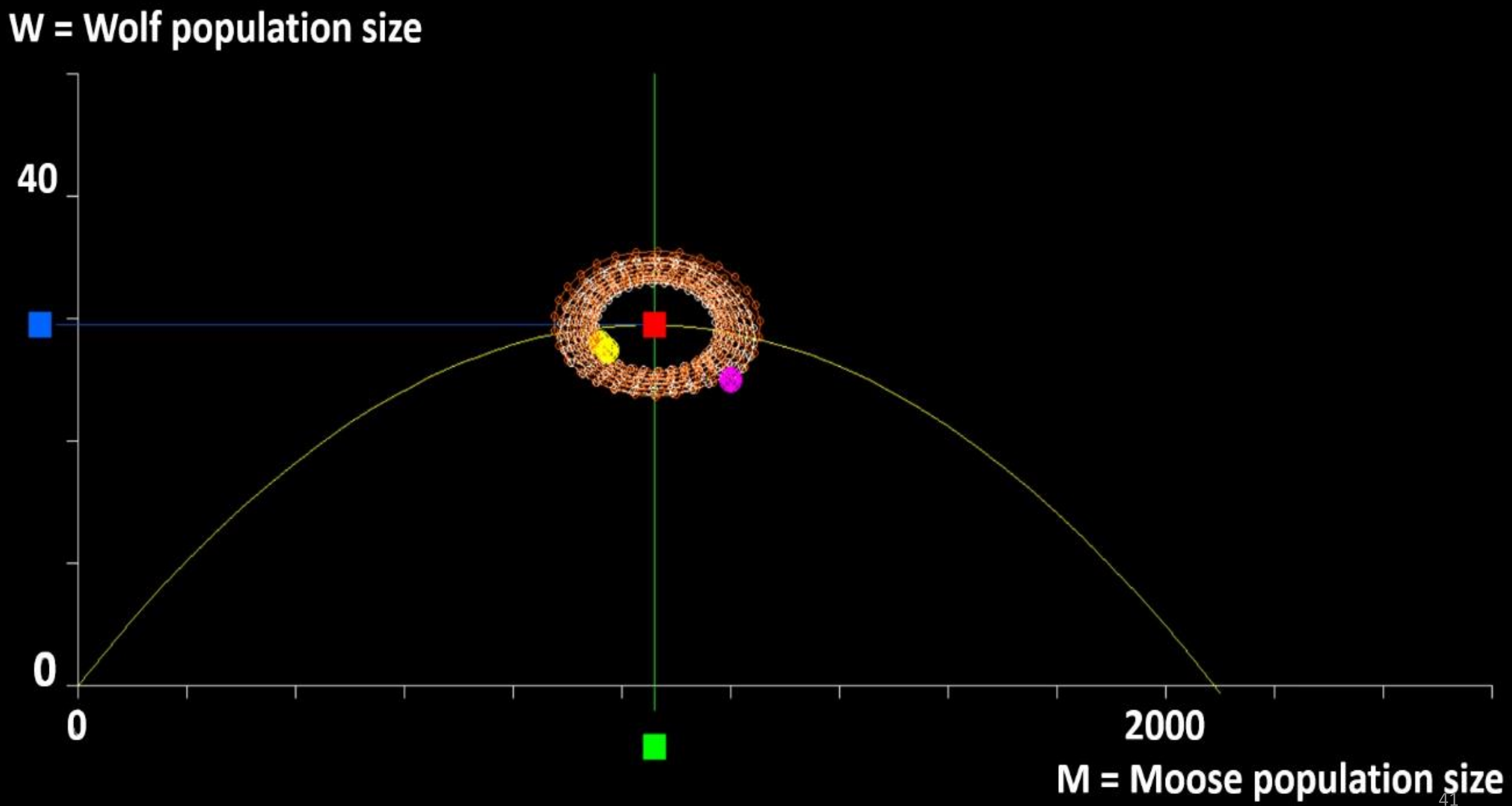
General results concerning the nature of Equilibrium 1:

$$\left(M_1 < M^*\right) \Rightarrow (n > 0) \Rightarrow \underline{Equilibrium : Unstable\ diverging\ spiral}. \qquad (95)$$

$$\left(M_1 = M^*\right) \Rightarrow (n = 0) \Rightarrow \underline{Equilibrium : Center}. \qquad (96)$$

$$\left(M_1 > M^*\right) \Rightarrow (n < 0) \Rightarrow \underline{Equilibrium : Stable\ converging\ spiral}. \qquad (97)$$


W = Wolf population size
40
0
0
2000
M = Moose population size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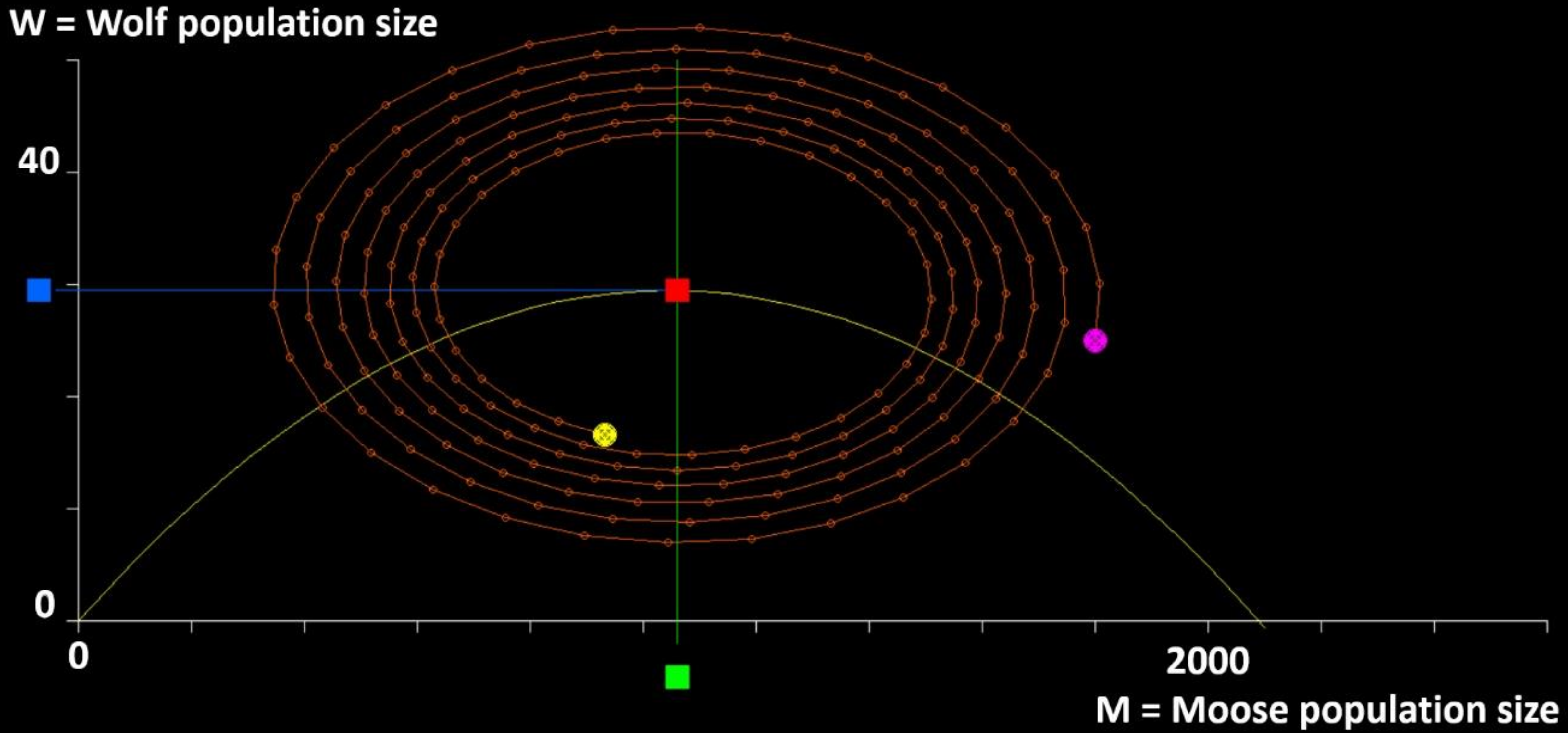
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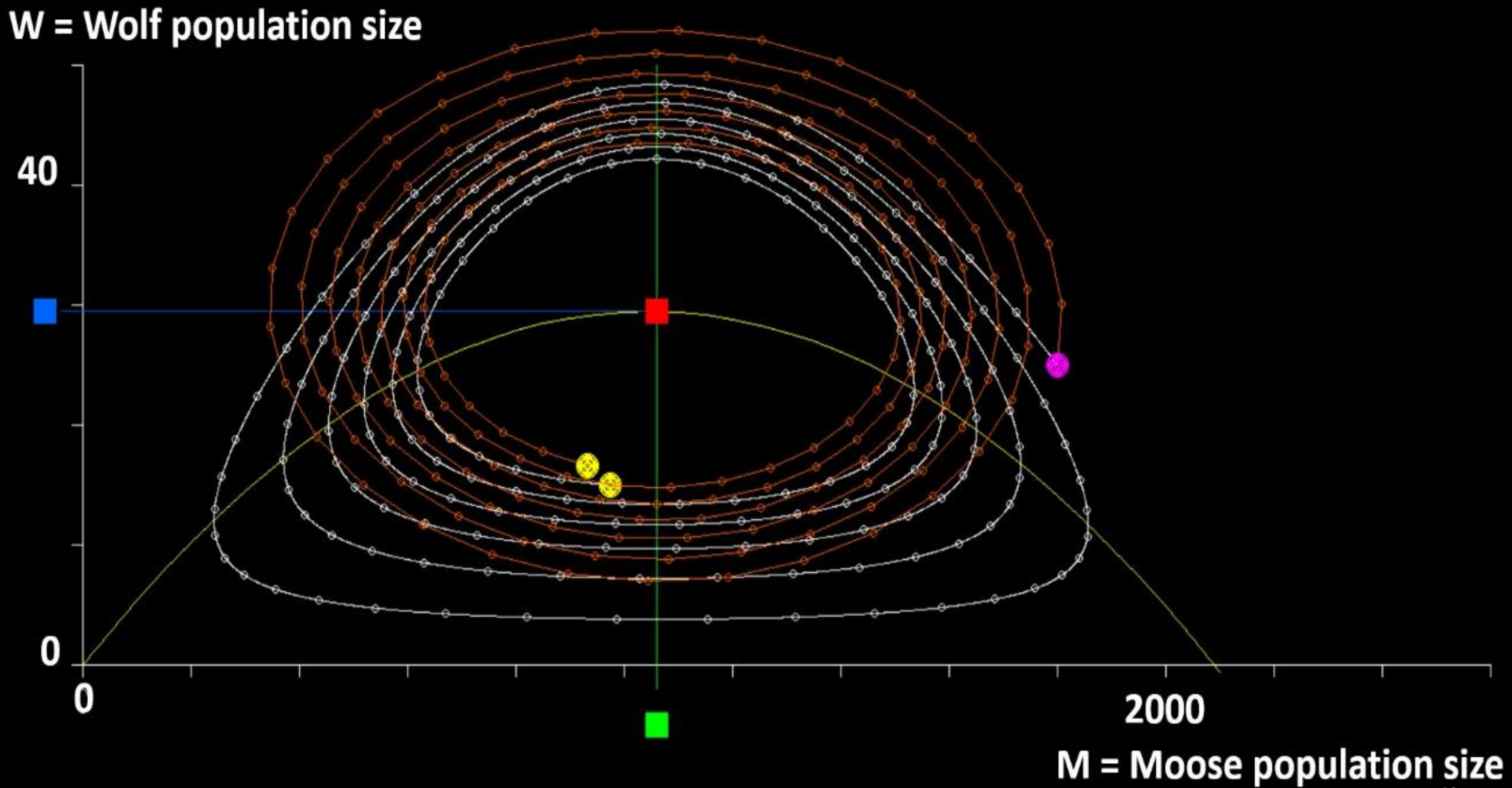
W = Wolf population size
40
0
0
2000
M = Moose population size

Sustainable optimal system control

$$\begin{cases} \dot{M} = aM - bM^2 - cW - R \\ \dot{W} = -gW + hMW - SW \end{cases} \qquad (98)$$

$$\pi = P_R R + P_S SW + P_W LN(W) \qquad (100)$$

The two optimal values of the moose population size are:

$$M_{opt1} = -\frac{p}{2} + \sqrt{\frac{p^2}{4} - q} \tag{121}$$

$$M_{opt2} = -\frac{p}{2} - \sqrt{\frac{p^2}{4} - q} \tag{122}$$

One of the two optima gives a maximum. (The other optimum is a minimum, which is mostly also an unfeasible solution with one or two negative population(s).) Inspection of the second order maximum conditions and the signs of the derived populations, reveals the feasible maximum. Since there is only one feasible solution that satisfies the local maximum conditions, we accept this as the unique global maximum. When we know the value of the moose population, we can also determine the values of the wolf population, the moose harvest level and the wolf adjustment level, that all, in combination, lead to the maximum of the objective function. This is done in (123), (124) and (125).

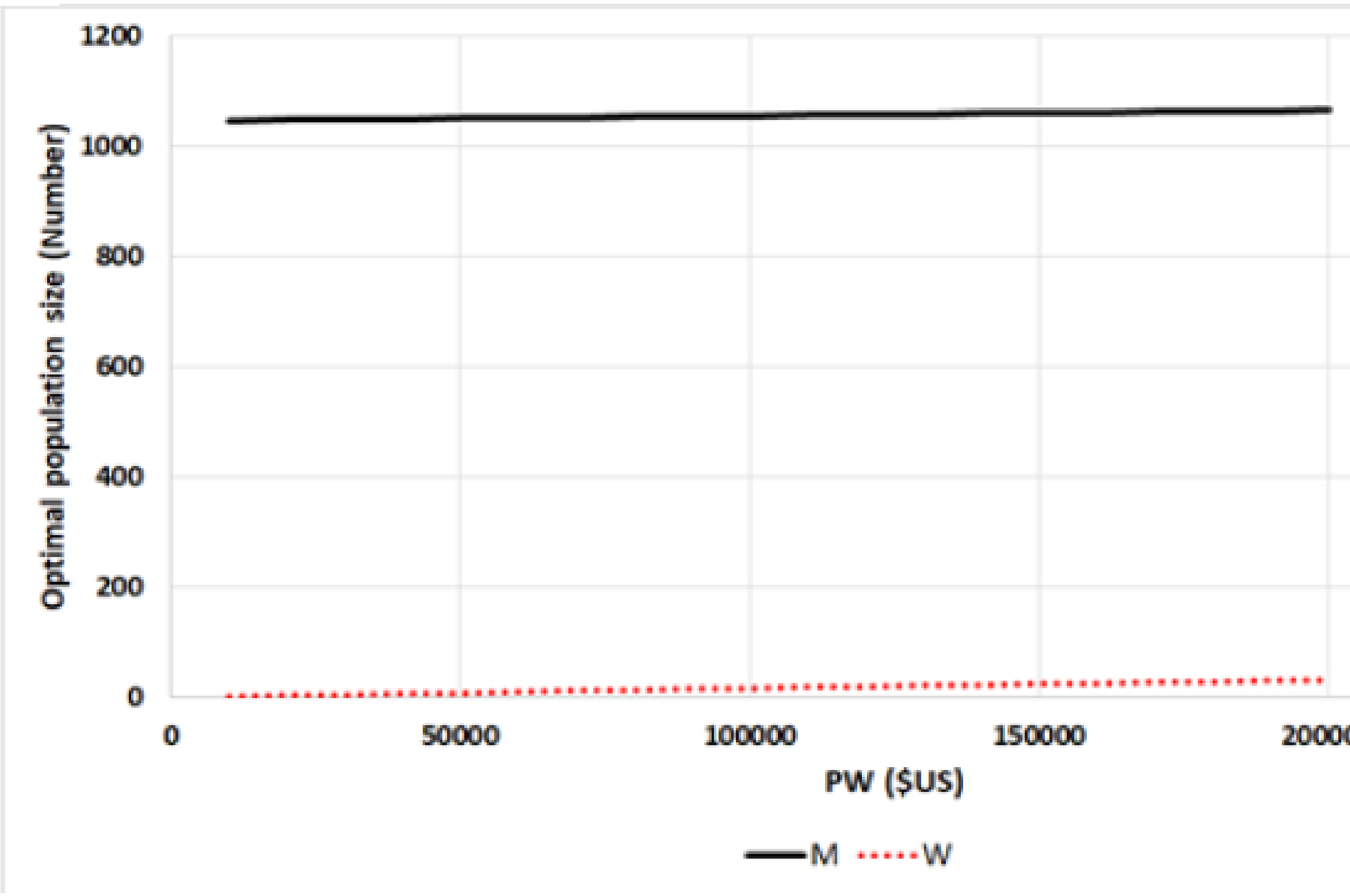

Figure 8 Optimal populations of moose (M) and wolf (W) for different levels of the wolf population value parameter P_W .

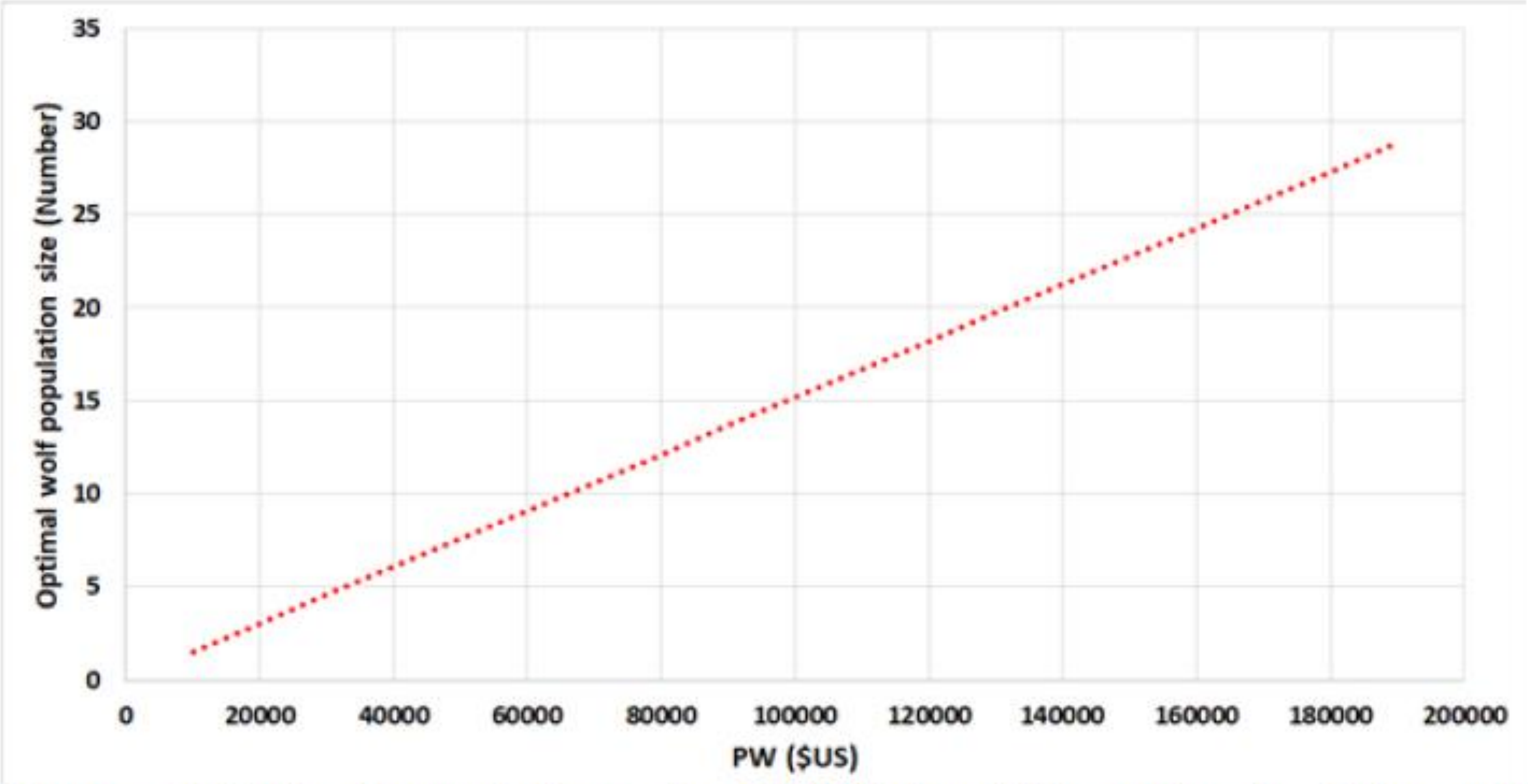

Figure 9 Optimal populations of wolf (W) for different levels of the wolf population value parameter P_W .

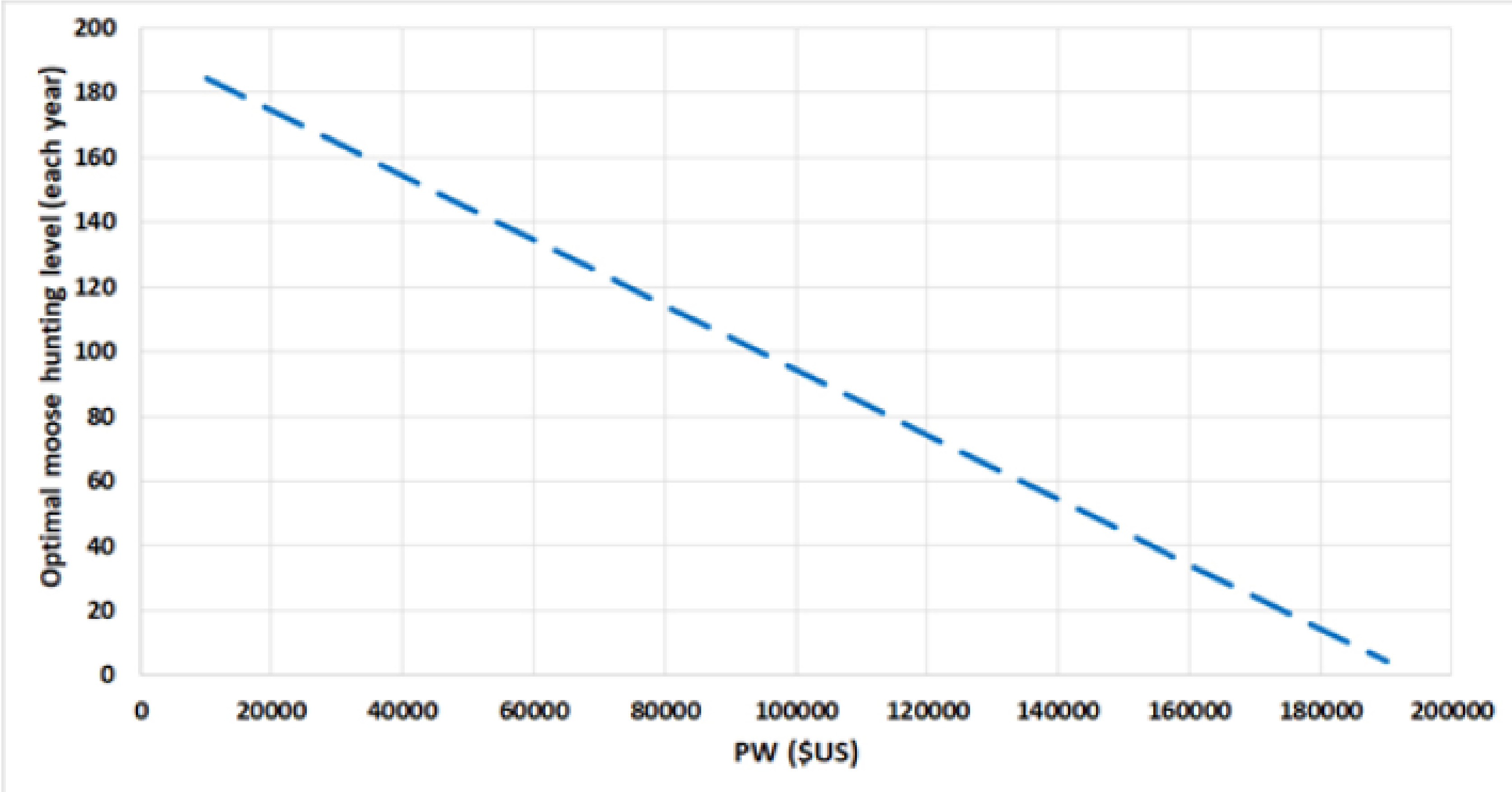

Figure 10 Optimal moose hunting for different levels of the wolf population value parameter P_W .

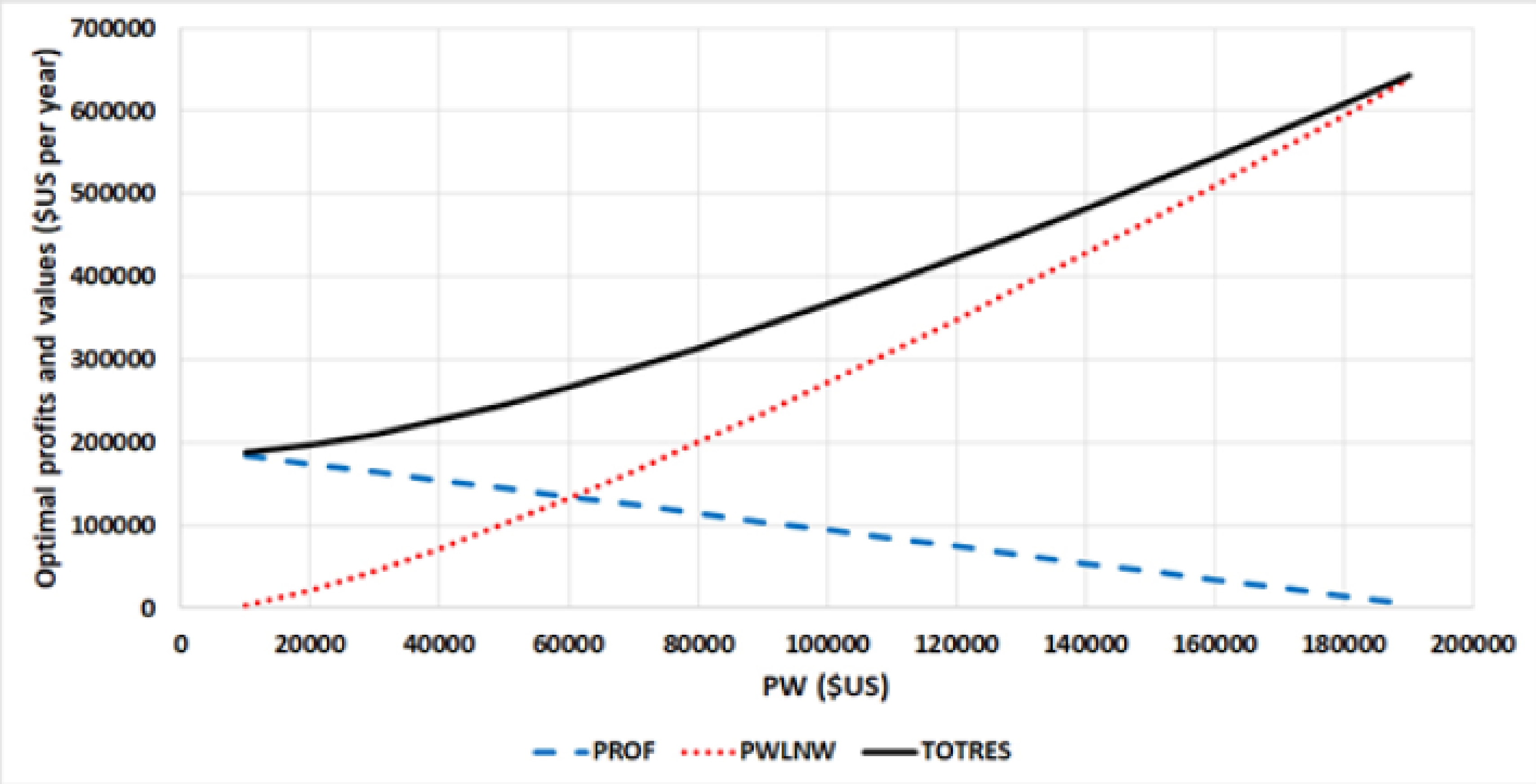

Figure 11 Optimal profits and values for different levels of the wolf population value parameter P_W . PROF is the sum of the profit from moose hunting and wolf hunting. PWLNW is the value of the wolf population. TOTRES is the sum of PROF and PWLNW.

The stochastic variations in the moose and wolf populations are considerable. For this reason, the final approach is the following, based on Lohmander [3].

A numerical approach to optimal adaptive integer pulse control of stochastic nonlinear systems is presented.

For most stochastic nonlinear systems, optimal adaptive control rules cannot be derived with analytical methods.

A robust optimization algorithm is created. The complete nonlinear adaptively controlled stochastic system is simulated during 100 years, for 100 alternative sequences of stochastic disturbances, for every feasible integer combination of adaptive control rules. The optimal adaptive control rules that maximize the expected value of the objective function are selected as the optimal adaptive control rules.

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The method is very general and can easily be applied to most adaptive nonlinear stochastic control problems, from technology, management or other fields.

The method is tested and applied to the wolf-moose predator prey system. The parameters of this stochastic nonlinear dynamical system have recently been estimated from empirical data from Isle Royale in Lake Superior, USA.

The objective function is the expected total present value of all hunting net revenues and the environmental value of preserving the wolf population.

The value of the wolf population is a strictly increasing and strictly concave function of the population level.

Periodically, the region is visited and the population levels are determined.

If the population levels, one for each species, exceed the optimal control limits, then the populations are reduced to the control limits, via hunting.

Then, the system is left to develop until the next period.

$$\pi = E\left(\sum_t e^{-rt}\left(P_M u(t,\bullet) + P_W v(t,\bullet) + P_{ARW}\Psi(W(t,\bullet))\right)\right) \qquad (1)$$

$$\begin{cases} \Delta M & = \phi\left(M, W, u, \varepsilon_M\right) \quad ,\forall t \\ \Delta W & = \varphi\left(M, W, v, \varepsilon_W\right) \quad ,\forall t \end{cases} \qquad (2)$$

In equation (3) we see how the expected value of the objective function is estimated from N complete stochastic scenarios. In every stochastic scenario, n, the adaptive control functions are the same. However, the random number sequences, $\epsilon_M(t, n)$ and $\epsilon_W(t, n)$, are different for different n.

$$\pi = N^{-1} \sum_{t} \sum_{n=1}^{N} e^{-rt} \left(P_M u(t, n) + P_W v(t, n) + P_{ARW} \Psi(W(t, n)) \right) \tag{3}$$

Furthermore, all scenarios have the same initial conditions. These are found in (4).

$$(M(0, n), W(0, n)) = (M_0, W_0) = (1200, 25) \quad , \forall n \tag{4}$$

$$\Delta M = \underset{[0.135]}{0.372}M - \underset{[0.0845\times10^{-3}]}{0.178\times10^{-3}}M^2 - \underset{[2.215]}{6.593}W + M\varepsilon_M \tag{6}$$

$$\varepsilon_M \sim N\left(\mu_M, \sigma_M^2\right), \quad \mu_M = 0, \sigma_M \approx 0.174 \tag{7}$$

$$\Delta W = -\underset{[0.117]}{0.244}W + \underset{[0.107\times10^{-3}]}{0.230\times10^{-3}}MW + W\varepsilon_W \tag{8}$$

$$\varepsilon_W \sim N\left(\mu_W, \sigma_W^2\right), \quad \mu_W = 0, \sigma_W \approx 0.334 \tag{9}$$

The stochastic nonlinear system, without controls, is (10):

$$\begin{cases} \Delta M & = 0.372M - 0.178 \times 10^{-3}M^2 - 6.593W + M\varepsilon_M \\ \Delta W & = -0.244W + 0.230 \times 10^{-3}MW + W\varepsilon_W \end{cases} \tag{10}$$

When we introduce the controls, u and v, that are functions of different variables and parameters, we get (11):

$$\begin{cases} \Delta M & = 0.372M - 0.178 \times 10^{-3}M^2 - 6.593W - u(\bullet) + M\varepsilon_M \\ \Delta W & = -0.244W + 0.230 \times 10^{-3}MW - v(\bullet) + W\varepsilon_W \end{cases} \tag{11}$$

$$\begin{cases} \Delta M & = 0.372M - 0.178 \times 10^{-3}M^2 - 6.593W - u(\bullet) + M\varepsilon_M \underline{\times RISK} \\ \Delta W & = -0.244W + 0.230 \times 10^{-3}MW - v(\bullet) + W\varepsilon_W \underline{\times RISK} \end{cases} \tag{12}$$

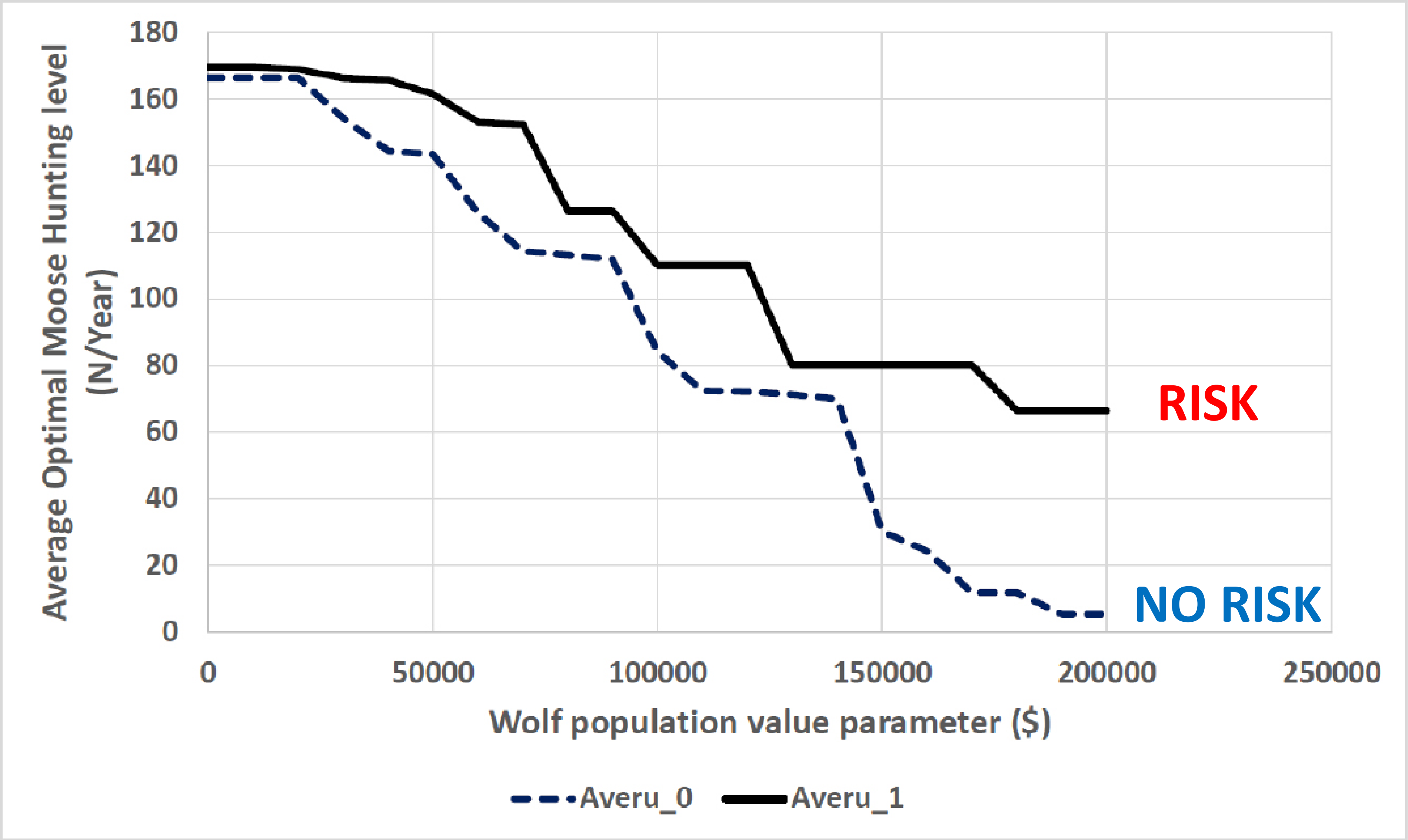

Average Optimal Moose Hunting level (N/Year)
180
160
140
120
100
80
60
40
20
0
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
RISK
NO RISK
Averu_0
Averu_1

Figure 1.

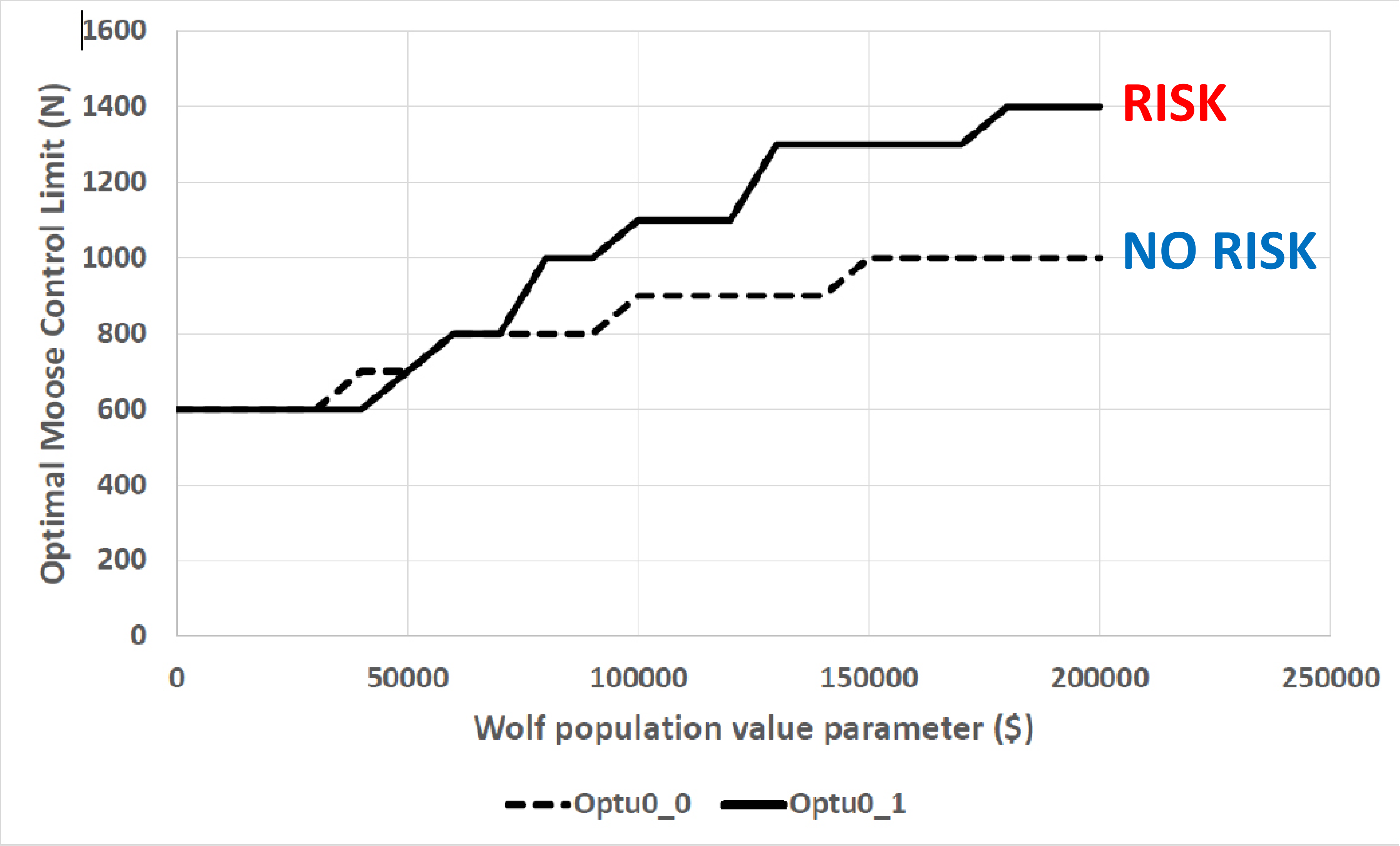
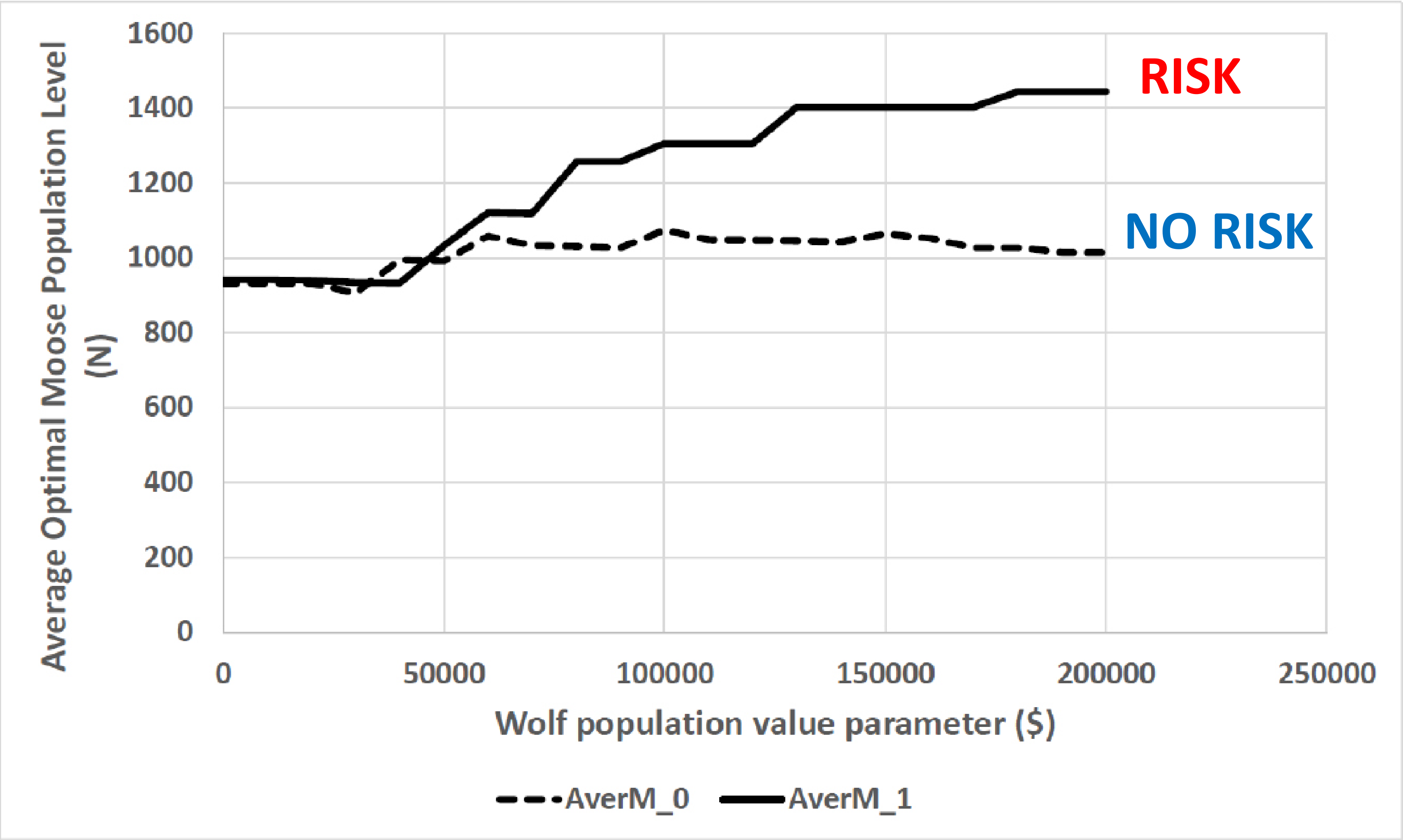
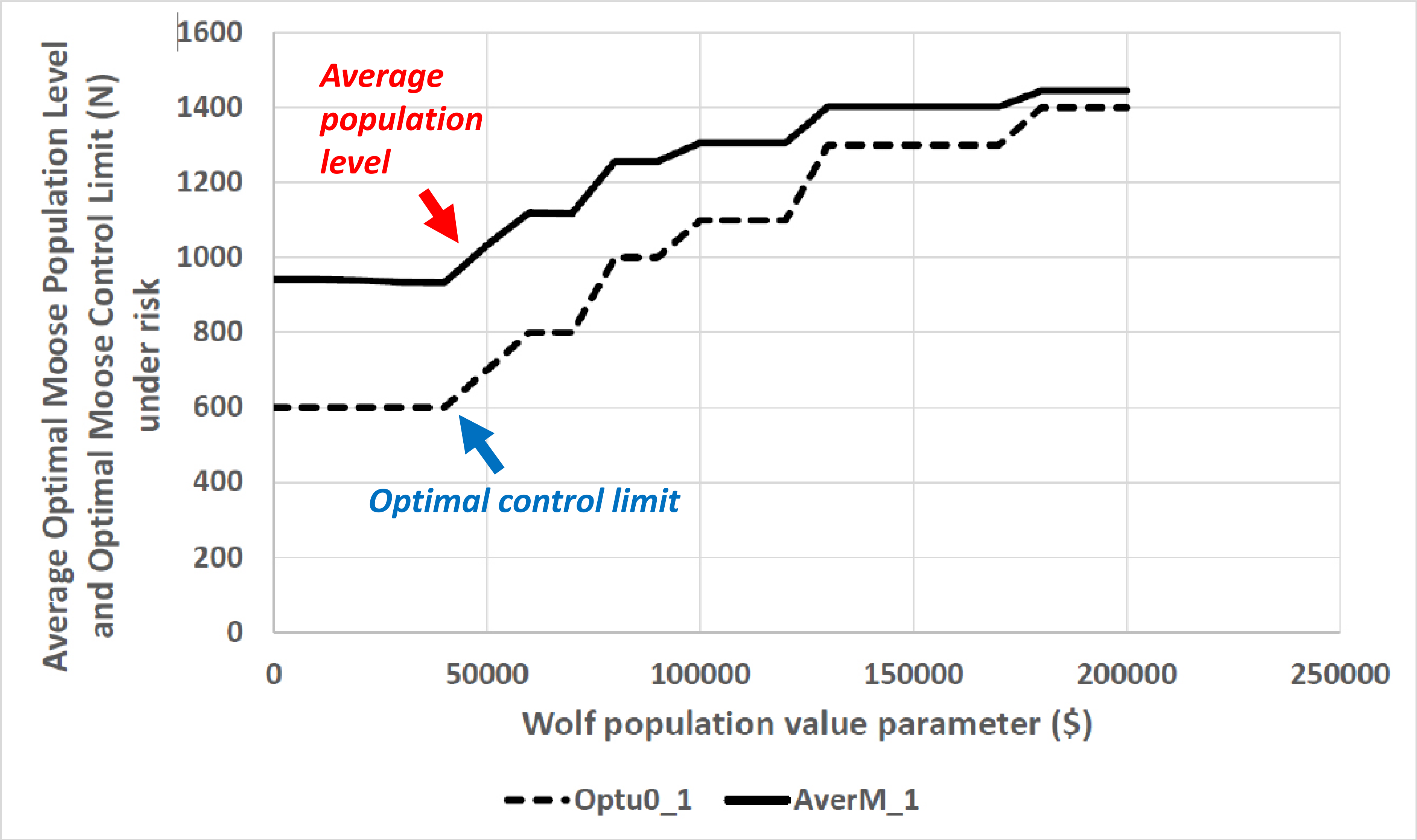

1600
1400
1200
1000
800
600
400
200
0
Optimal Moose Control Limit (N)
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
RISK
NO RISK
Optu0_0
Optu0_1

Figure 2.



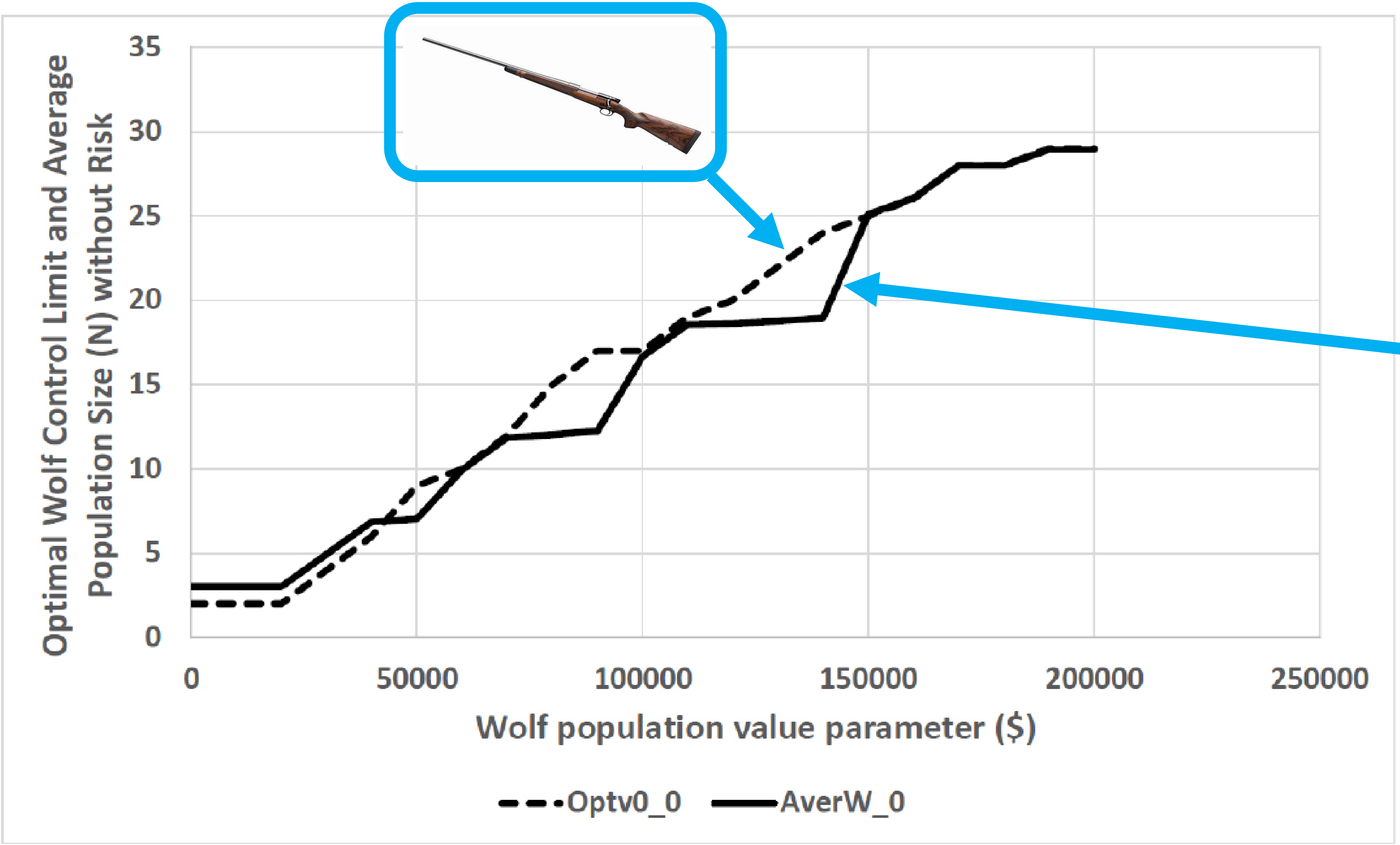
RISK
NO RISK
Average Optimal Moose Population Level (N)
1600
1400
1200
1000
800
600
400
200
0
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
AverM_0
AverM_1

Figure 3.



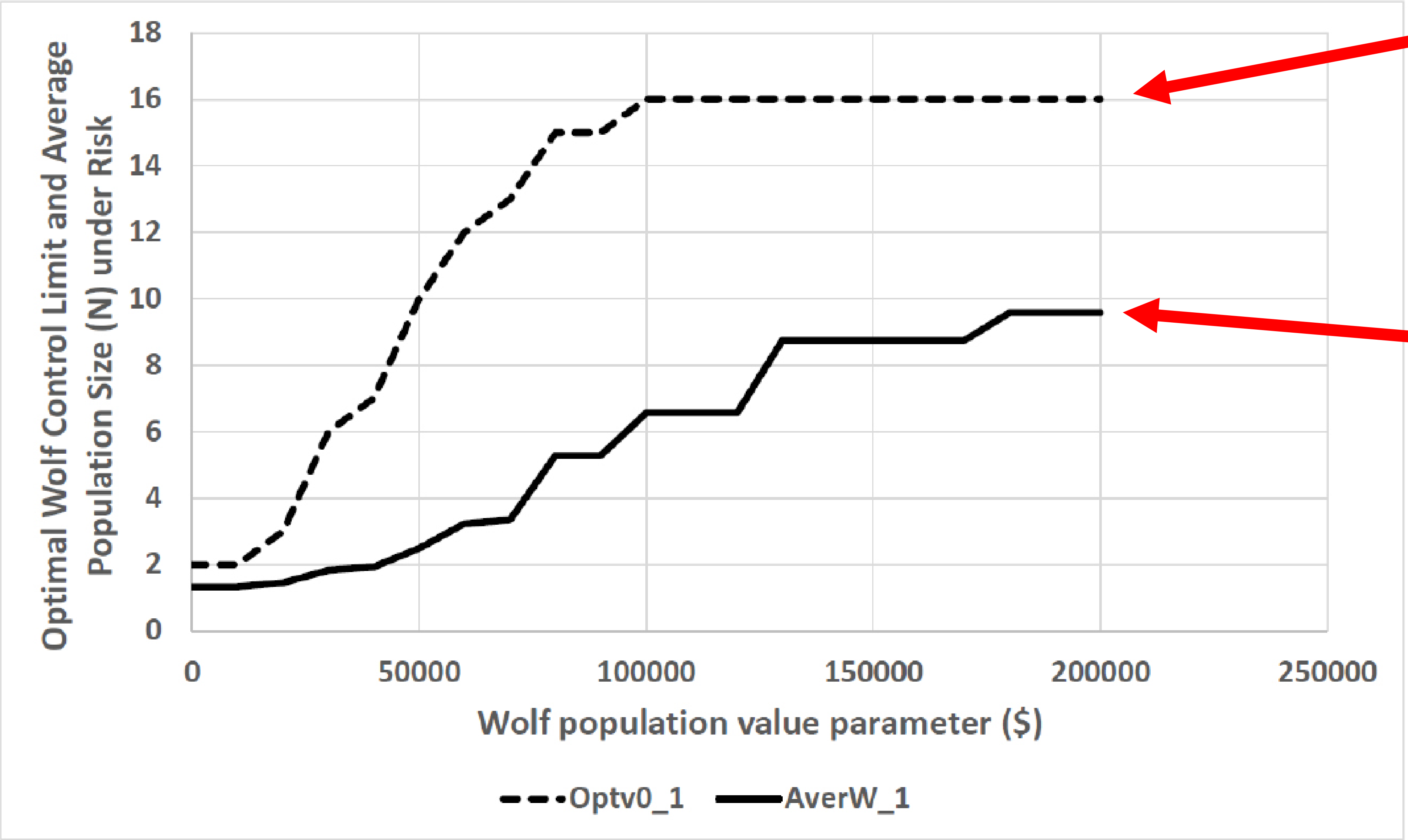
Average Optimal Moose Population Level and Optimal Moose Control Limit (N) under risk
1600
1400
1200
1000
800
600
400
200
0
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
Average population level
Optimal control limit
Optu0_1
AverM_1

Figure 4.



NO RISK

Figure 5.



RISK

Figure 6.

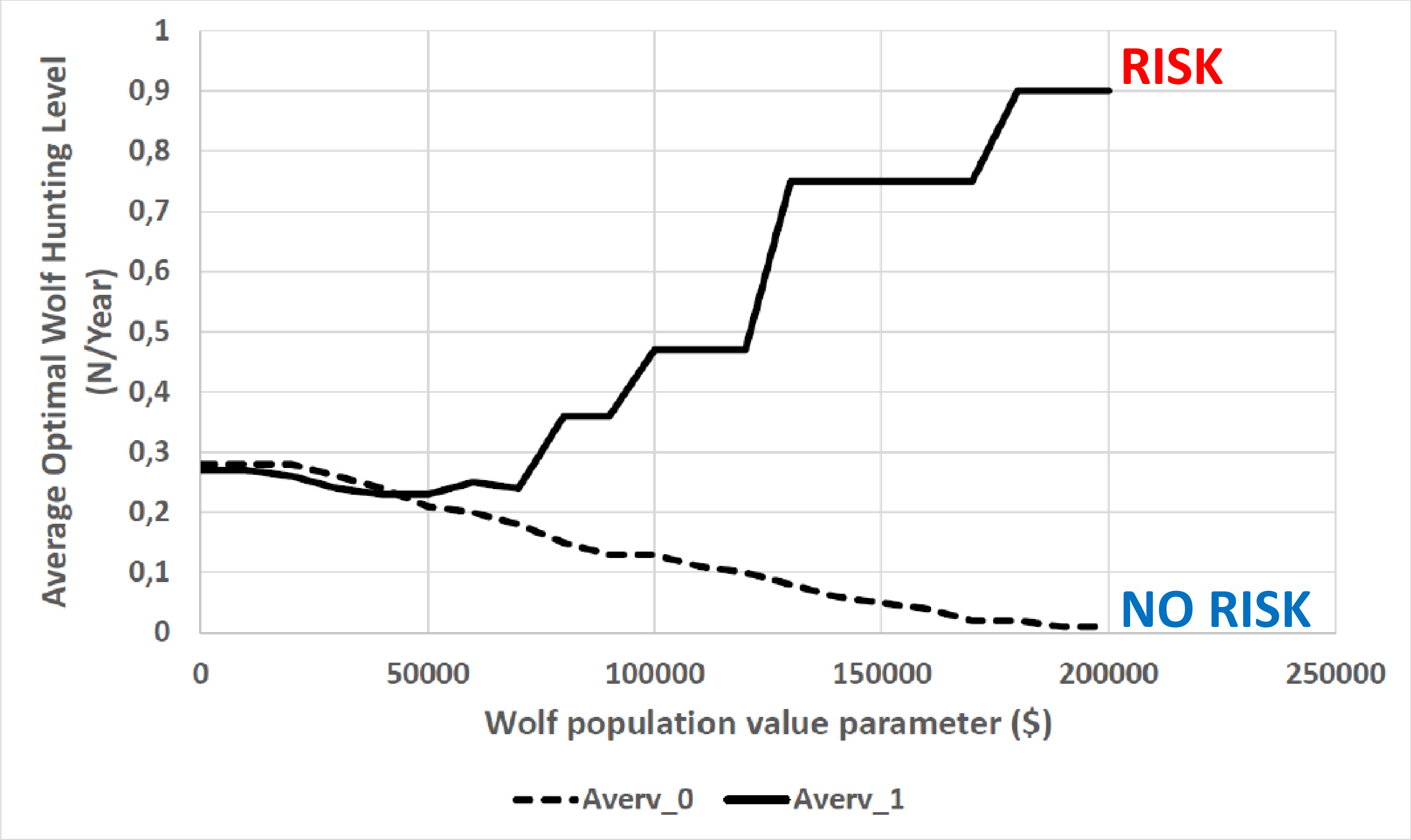

RISK
NO RISK
1
0,9
0,8
0,7
0,6
0,5
0,4
0,3
0,2
0,1
0
Average Optimal Wolf Hunting Level (N/Year)
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
Averv_0
Averv_1

Figure 7.

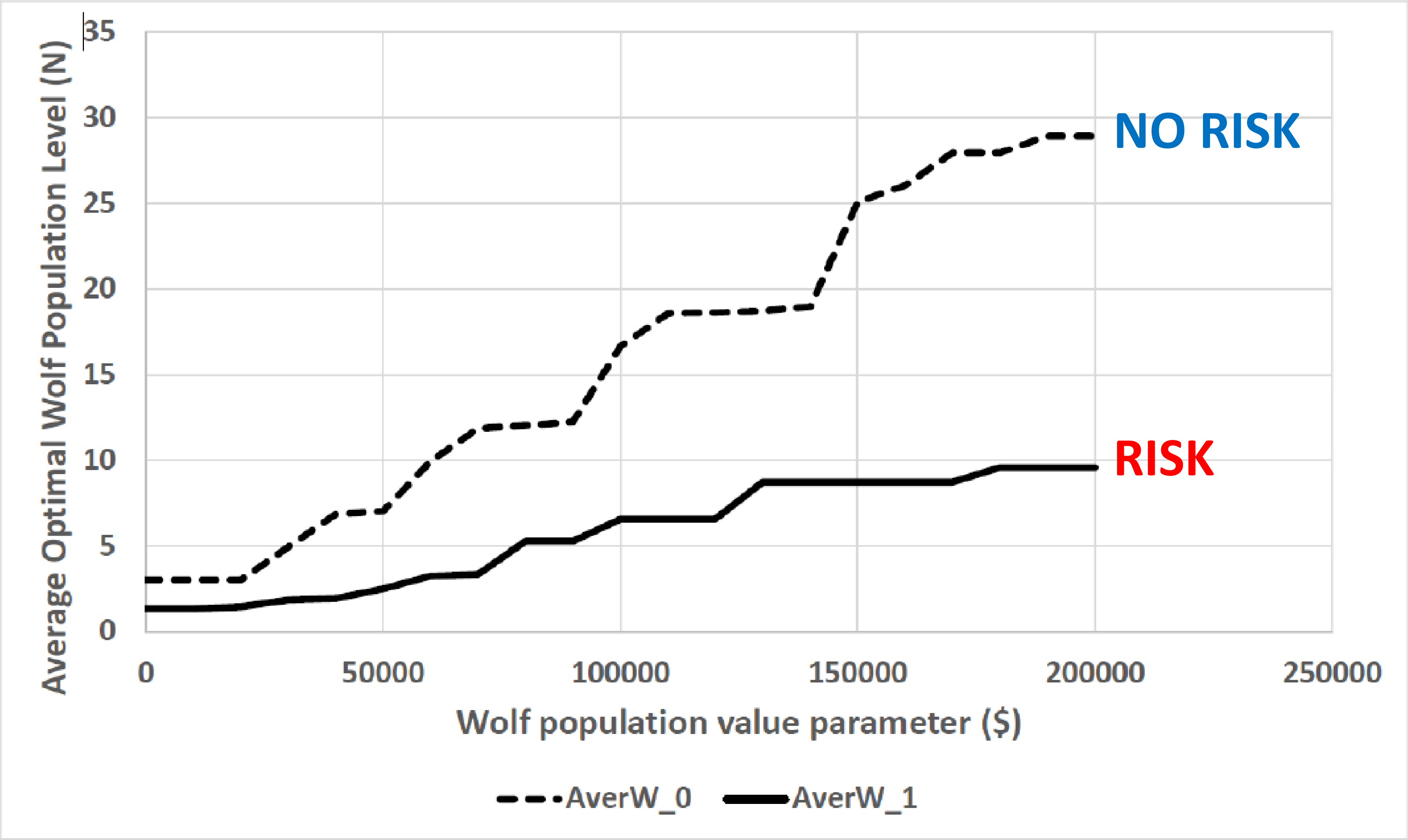

35
30
25
20
15
10
5
0
Average Optimal Wolf Population Level (N)
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
NO RISK
RISK
AverW_0
AverW_1

Figure 8.


100
THE UNITED STATES OF AMERICA
IN GOD WE TRUST
ONE HUNDRED DOLLARS

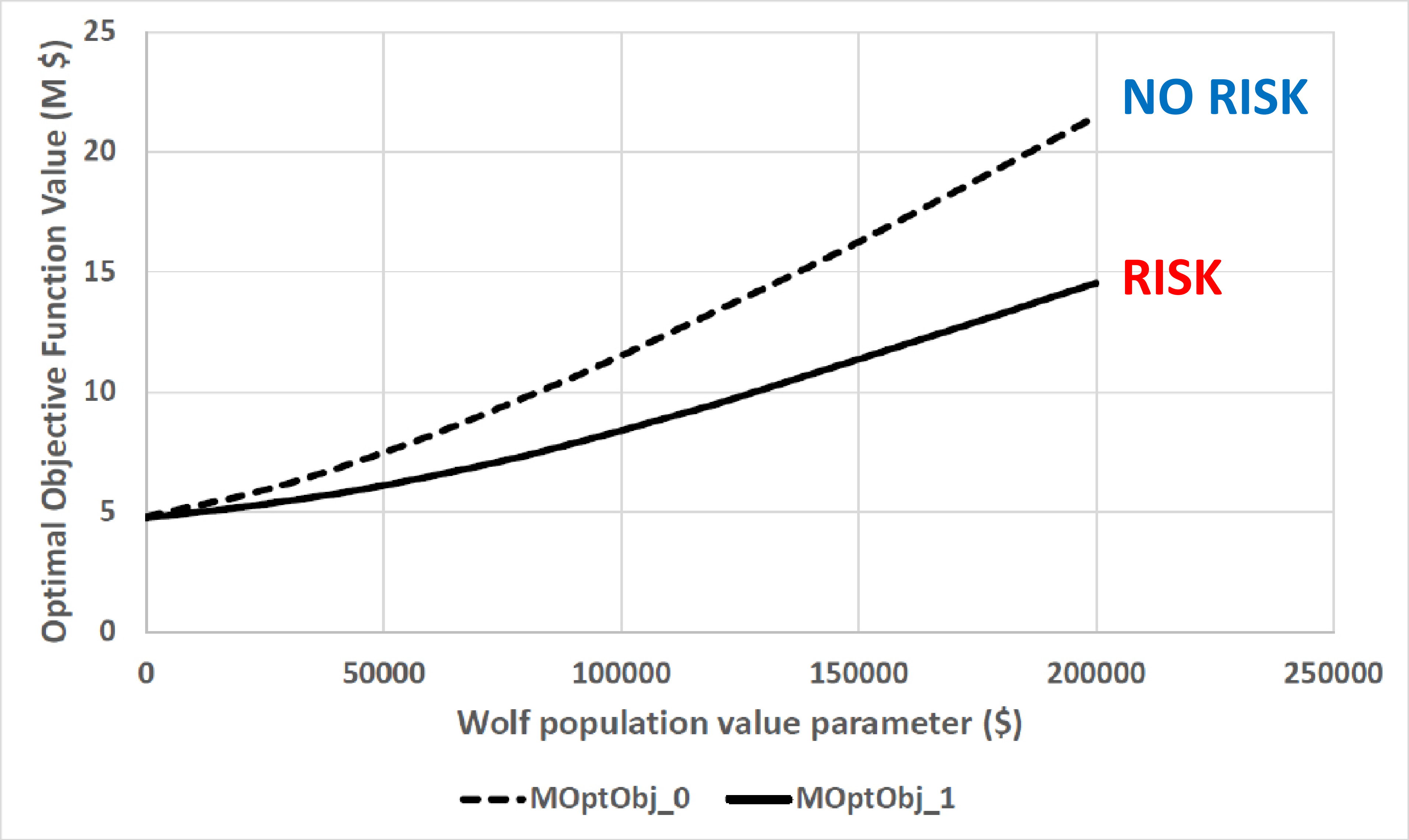

Optimal Objective Function Value (M $)
0
5
10
15
20
25
0
50000
100000
150000
200000
250000
Wolf population value parameter ($)
NO RISK
RISK
MOptObj_0
MOptObj_1

Figure 9.

What we have done:

Optimal population control limits and objective function values are determined for alternative levels of the wolf population value function.

Two, of many, central observations:

The average optimal moose hunting level is a decreasing function of the wolf population value parameter and an increasing function of the level of risk in the predator prey system.

The average optimal wolf population level is an increasing function of the wolf population value parameter and a decreasing function of the level of risk in the predator prey system.

Conclusions

- Many different methods can be used to study the dynamics and to optimize the management of wild life populations.
- This presentation includes three alternatives.
- Please continue the process and try to find the most relevant approach to your particular problems!

THANK YOU VERY MUCH FOR YOUR TIME AND A MOST INTERESTING CONFERENCE!

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http://www.lohmander.com/Information/Ref.htm

Wildlife Dynamics and Management Optimization via Statistics and Mathematics

by

Peter Lohmander

Moose

Wolf

Professor Peter Lohmander
Optimal Solutions
Sweden

Fifth International Webinar on

RECENT TRENDS IN STATISTICAL THEORY AND APPLICATIONS-2021 (WSTA-2021)

09:00 PM - 09:35 PM IST, June 30, 2021 (Day 2), TS 21

Organized by

Indian Society for Probability and Statistics (ISPS), Kerala Statistical Association (KSA) & Department of Statistics (School of Physical and Mathematical Sciences) University of Kerala, Trivandrum, India