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A Quantitative Adaptive Optimization Model for Resource Harvesting in a Stochastic Environment

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This paper contains a flexible adaptive optimal control (harvesting or extraction) model for the personal computer. The profit function is an arbitrary third order polynomial where stochastic shifts in the demand function are represented by a bounded first order autoregressive process with arbitrary deterministic trend. The growth process is the sum of a second order polynomial and an exponential function. The value of the land released after harvest is considered in the objective function. Resources without growth, such as minerals and oil, are of course special cases. The present analysis restricts the attention to discrete time and a discrete state space. This makes sure that the obtained solution always is a global maximum. The computer program is included and used in the illustrating applications. The results are:

a. Problems from stochastic optimal stopping theory and from stochastic optimal "*continuous*" control theory have been solved through one model.

b. Some important properties of the optimal control of stochastic systems have been discovered through the numerical model. These properties are difficult to find via analytical optimization models because the optimal solution may be an interior solution in some periods and a boundary solution in other periods. In most analytical models, interior **or** boundary solutions are generally assumed in all periods.

1. Introduction

1.1. Earlier Work in the Field

The optimal intertemporal control (harvest or extraction) problem which arises in resource management has earlier been analysed in continuous and discrete time by the use of deterministic and stochastic optimal control theory. Some references in this field are:

a. Probability and stochastic processes (ITO & MCKEAN [8], GRIMMET & STIRZAKER [7])
b. General deterministic and stochastic optimal control theory (BERKOVITZ [1], FLEMING & RISHEL [5], CHOW [2])
c. Dynamic programming and general problems in management and economics (WAGNER [19], MALLIARIS & BROCK [14])
d. Developments of optimal control theory for resource management optimization (LOHMANDER [10, 11, 12, 13])
e. Resource management applications of deterministic optimal control (CLARK [3], JOHANSSON and LÖFGREN [9])

f. Resource management applications of stochastic optimal control (NORSTROM [15], RISVAND [17], GLEIT [6], PINDYCK [16], LOHMANDER [11, 12]).

Of course, the list is far from complete. However, it gives some insight to the area of optimization methods and the classes of resource management applications. Since the applications are different, the optimization methods used have also been different. In forestry, optimal stopping theory is frequently used, since "pulse extraction" generally is more profitable than "continuous thinning" in the pure forest producing enterprise. This is often the case if prices are exogenous to the firm and there are set up costs in production. The properties of the growth function must also be taken into consideration. Compare CLARK ([3]), RISVAND ([17]) and LOHMANDER ([10]). If the firm is sufficiently large, it will affect the market prices through the extraction and the wood sales. Then, the profit function is generally strictly concave in the harvest volume and it is not optimal to harvest all of the resource during one period. A continuous harvesting strategy is more profitable. This approach is used by CLARK [3], GLEIT [6], PINDYCK [16] and LOHMANDER [10, 11, 13].

NORSTROM [15], RISVAND [17], PINDYCK [16] and LOHMANDER [12] assume that price is a stochastic process and that growth is deterministic or nonexistent. These assumptions have been quite relevant because the treated resources have been forests and oil. The physical development of such resources is much easier to predict than future prices. GLEIT [6] assumes price to be deterministic and growth stochastic, which may be more relevant when the resource is a fish population.

The model presented in this paper serves as a bridge between two areas. The solution becomes an optimal stopping strategy **or** an optimal continuous control strategy, depending on the choice of parameters.

1.2. The Purpose of the Paper

The reader of different articles in the field of optimal harvesting will find several contradicting results and assumptions. A survey is presented in LOHMANDER [10]. Proper investigation shows that the obtained, almost always qualitative, results are extremely sensitive to the assumptions. The great importance of these assumptions concerning details in the stochastic specifications of the processes, the details of the profit function etc. is generally not clearly specified in the articles. In fact, the author of this paper sometimes gets the impression that new models are designed just in order to support a particular empirical result. The model (theory) is designed in a particular way in order to give a strong result (an unambiguous sign of a partial derivative) which is consistent with the empirical data or the common opinion in a normative problem. One attempt to demonstrate how sensitive the results generally are was made by LOHMANDER [11] via an analytical model. However, even if analytical models are important, this paper contains a flexible numerical model. There are several reasons for this.

a. Problems from optimal stopping theory and from optimal continuous control can be solved through the same model.
b. Thanks to the discrete state space and the solution algorithm, it is known that the obtained solution always is a global maximum.
c. The qualitative results presented in the literature can easily be checked.
d. The effects of new and more relevant assumptions are easily investigated.
e. Most of the qualitative results from the analytical models can be discussed as special cases of one model. (Of course, a numerically specified model is a special case of an identical, but not numerically specified, model. However, the analytical models generally have a simple structure in order to give strong results. Since the numerical

models can be given a more complicated structure, the analytical model is a special case of the more complex numerical model in the sence of structure.)

f. Results not yet discovered via the existing analytical models may be found via the numerical model. This may show us new and fruitful directions of analytical model building.

g. The quantitative results are almost always more important than the qualitative. The magnitude of an effect is however often completely neglected in the economics literature.

2. Analysis

2.1. General Model Properties and Qualitative Observations

In this paper we will deal with a stochastic price parameter process, as opposed to most earlier work in the field where the stochastic price has been regarded and defined as an exogenous process. See for instance PINDYCK [16]. Fig. 1 shows a possible sample path of the stochastic price parameter. It is defined through a finite state space in each time period. Every time period contains 9 possible states. There may be a deterministic trend in the price parameter process. In this version of the model, the only possible time trend is linear (More general functional forms of the time trend could easily be included.). The stochastic part of the price parameter process is defined as a first order autoregressive process (AR 1). The sample path shown in Fig. 1 is typical when the AR 1 process is stationary (around the deterministic trend).

The probability distribution of the price parameter state in period $t + 1$ is shown in Fig. 2 conditional on the price parameter state in period t. The graph is constructed

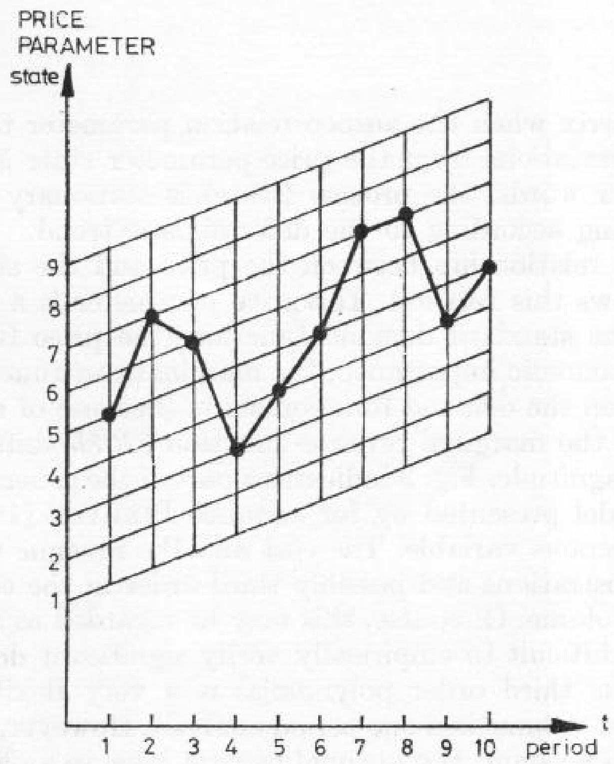

Fig. 1. A possible sample path of the stochastic price parameter process. There are 9 possible states in each period. The price parameter is a function of state and time

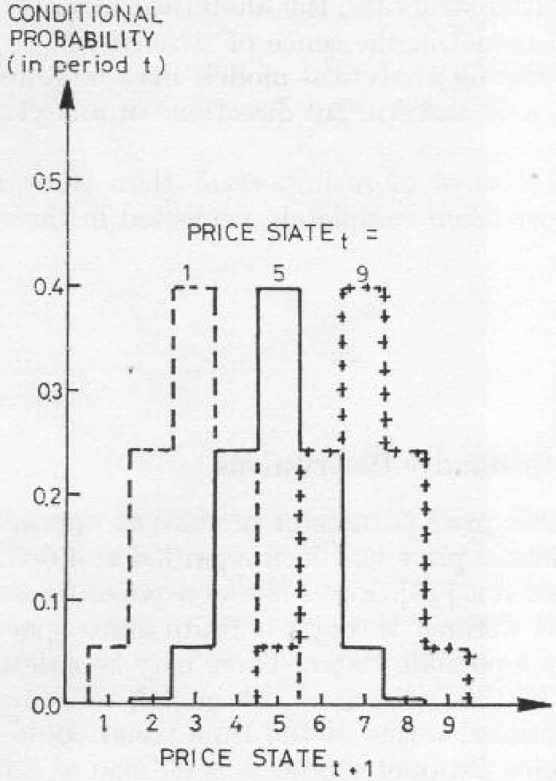

Fig. 2. The probability distribution of the price parameter state in period $t + 1$ (expectation formed in period t) conditional on the price parameter state in period t. The graph is constructed from the transition probability matrix when the autocorrelation parameter takes the value 0.5.

from the transition probability matrix when the autocorrelation parameter takes the value 0.5. The graph shows that deviations from the price parameter state 5 tend to dissapear in the long run. In other words, the process (state) is stationary but the equilibrium point (state 5) is moving according to the deterministic trend.

So far, we have not defined the relationship between the price and the stochastic price parameter process. Fig. 3 shows this relation. The price parameter is a measure of the stochastic deviation from the standard demand function (the price function). Fig. 3 also shows other entities of economic importance; the marginal cost function and the marginal revenue function. When the demand function shifts (because of a change in the price parameter state), then the marginal revenue function $\partial R/\partial h$ shifts in the same direction and with the same magnitude. Fig. 3 indicates a part of the generalization in this model compared to the model presented by for instance Pindyck [16] where price is not a function but an exogenous variable. The cost and the revenue are both polynomial (second order in the illustrations and possibly third order in the computer program) functions of the harvest volume. Of course, this may be regarded as a limitation. However, in most cases it is difficult to empirically verify significant deviations from the polynomial functions. The third order polynomial is a very flexible tool. $h^*(PAR)$ denotes the optimal harvest volume in a one period analysis. However, because we have many periods to take into account, the optimal harvest level in an arbitrary period t is generally lower than $h^*(PAR)$ in Fig. 3. Fig. 4 shows the profit function π in a particular period as a function of the price parameter state. The graph is constructed

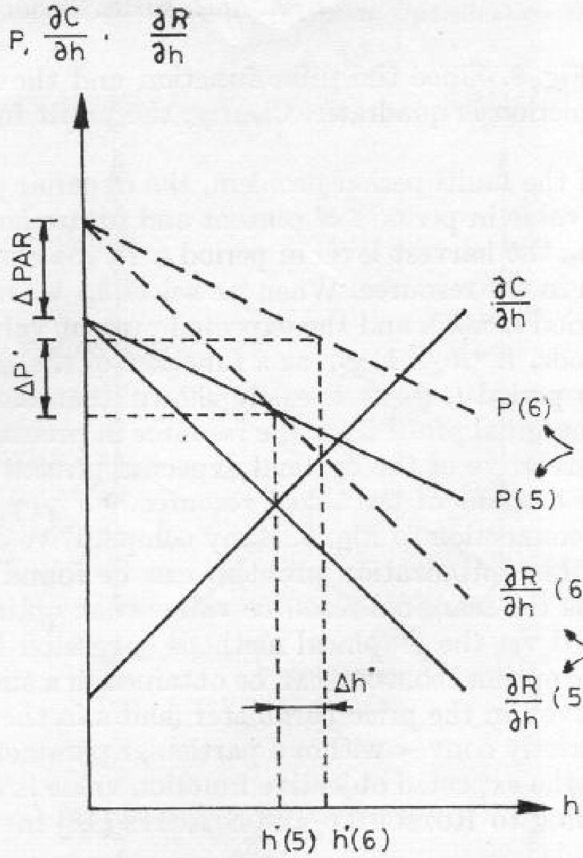

Fig. 3. Price, marginal cost, marginal revenue and the influence of the price parameter state. C, h, P and R denote harvest (extraction) cost, harvest (extraction) level, price (demand) function and revenue (P^*h). When the price parameter (PAR) increases, then the price (demand) function P and the marginal revenue function $\partial R/\partial h$ shift upwards. In the graph, P and $\partial R/\partial h$ are expressed as functions of PAR ($=5$ and 6). The optimal harvest level (in a one period analysis) is expressed by $h^*(PAR)$. P^* denotes optimal price.

Fig. 4. The profit function in a particular time period, $\pi = (P(h)h - C(h))$, and the price parameter state. The graph is constructed from the assumptions presented in Fig. 3. $h^*(.)$ is the optimal harvest level in a one period analysis.

from the assumptions presented in Fig. 3. Since the price function and the marginal cost function are linear, the profit function is quadratic. Clearly, the profit function is strictly concave in most cases.

Fig. 5 shows a simplified version of the multi period problem, the dynamic programming problem. The expected present value in period t of present and future harvesting, φ_t, is maximized via the selection of h_t, the harvest level in period t. In the example illustrated in Fig. 5, there is no growth in the resource. When we select h_t, we must consider the profit from harvesting in period t, $\pi(h_t)$, and the expected present value of the profits from harvesting in future periods, $W^*(t+1, \psi_t)$, as a function of the size of the resource saved for future purposes in period t, ψ_t. It is easily shown that the optimal rule in period t is the following; The marginal profit from the resource in present extraction $\partial\pi/\partial h_t$ should be equal to the derivative of the optimal expected present value of future harvesting with respect to the amount of the saved resource, $\partial W^*/\partial\psi_t$. This is shown in the text and calculations in connection to Fig. 5. Many comparative dynamics results from generalized versions of this optimization problem can be found in LOHMANDER [10] and [11]. We define λ^* as the marginal resource value when optimal decisions are made. λ^* is easily determined via the graphical methods suggested in Fig. 5. Furthermore, the graph shows that the optimal solution can be obtained in a simple way and that the objective function is convex in the price parameter (shifts in the demand function). The objective function is strictly convex within a particular parameter interval. Hence, it is possible to show that the expected objective function value is a strictly increasing function of the risk according to ROTSCHILD and STIGLITZ [18] in the price parameter. Compare LOHMANDER [10, 11].

Fig. 6 illustrates that the dual variable, the marginal resource value, λ^* is a kinked convex function of the price parameter if the second order derivatives $\pi_{h_t h_t}$ and $W^*_{\psi_t\psi_t}$ are constants less than zero. Thus, if the profit in period t is a strictly concave quadratic function and the optimal expected present value of future harvesting is a strictly concave quadratic function, then λ^* is a kinked convex function of the price parameter. For values of λ^* greater than zero, it is clear that λ^* is linear in the price parameter. This means, which is consistent with PINDYCK [16] and LOHMANDER [10, 11], that the expected marginal resource value will not change if the risk in the price parameter changes. But, Fig. 6 indicates that the earlier results have limited value in applied problems. λ^* is bounded from below. Hence, increasing risk in the price parameter in period t will generally increase the expected marginal resource value in period t. The optimal harvest level in earlier periods will then decrease.

Fig. 7 shows a possible shape of the function λ^* when π and W^* are not quadratic functions. Now it is clear that λ^* is strictly concave within a particular price parameter interval. Since λ^* is a kinked function, the expected value of λ^* may increase, be unchanged or decrease when the risk in the price parameter increases according to ROTSCHILD and STIGLITZ [18]. Then, the optimal present harvest level may change in any direction in the presence of risk in the future demand functions. There are three important observations:

- Stochastic processes (with normally distributed (unbounded) stochastic components) are not consistent with real world prices, since prices are bounded from below.
- In most resource economics applications of stochastic optimal control, it is assumed that the optimal solution is an "*interior*" solution (the intersection of π_{h_t} and $W^*_{\psi_t}$ in Fig. 6). This would however imply that $\lambda^* < 0$ in some cases, which is not consistent with the Kuhn-Tucker conditions of nonlinear programming. As shown in connection to Fig. 6, this has important (normative) implications.
- Wiener processes are not relevant in the description of bounded physical processes. Physical quantities are positive in most applied control problems.

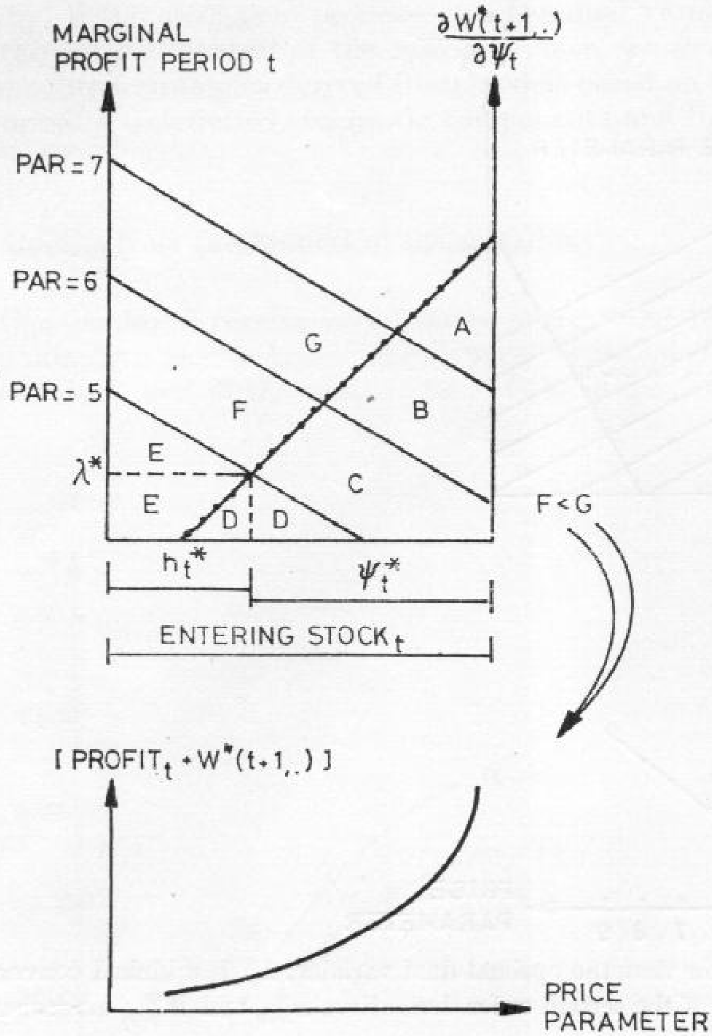

Fig. 5. Illustration of the simplified dynamic programming problem. The expected present value in period t, φ_t, is maximized via the selection of h_t, the harvest level in period t. The price parameter state PAR in period t has been observed and is known.

$$\max_{h_t, \psi_t} \varphi_t = \pi(h_t, PAR) + W^*(t+1, \psi_t)$$

$$\text{s.t.} \quad h_t + \psi_t \overset{(\leq)}{=} \psi_{t-1}$$

π is the profit in period t and W^* is the expected present value of future harvesting if optimal decisions are taken then (the Bellman principle of optimality). ψ_t denotes the available resource in period t after harvest (= the entering stock in period $t+1$). We assume that $\pi_{h_t h_t}$ and $W^*_{\psi_t \psi_t}$ are strictly negative. The Lagrange function is:

$L = \pi(h_t, PAR) + W^*(t+1, \psi_t) + \lambda(\psi_{t-1} - h_t - \psi_t)$. The first order optimum conditions are:

$$L_\lambda = \psi_{t-1} - h_t - \psi_t = 0$$

$$L_{h_t} = \pi_{h_t} - \lambda = 0$$

$$L_{\psi_t} = W_{\psi_t} - \lambda = 0.$$

The graphs show the optimal solution and that the objective function is convex in the price parameter PAR (strictly convex for PAR such that $5 < PAR < 7$).

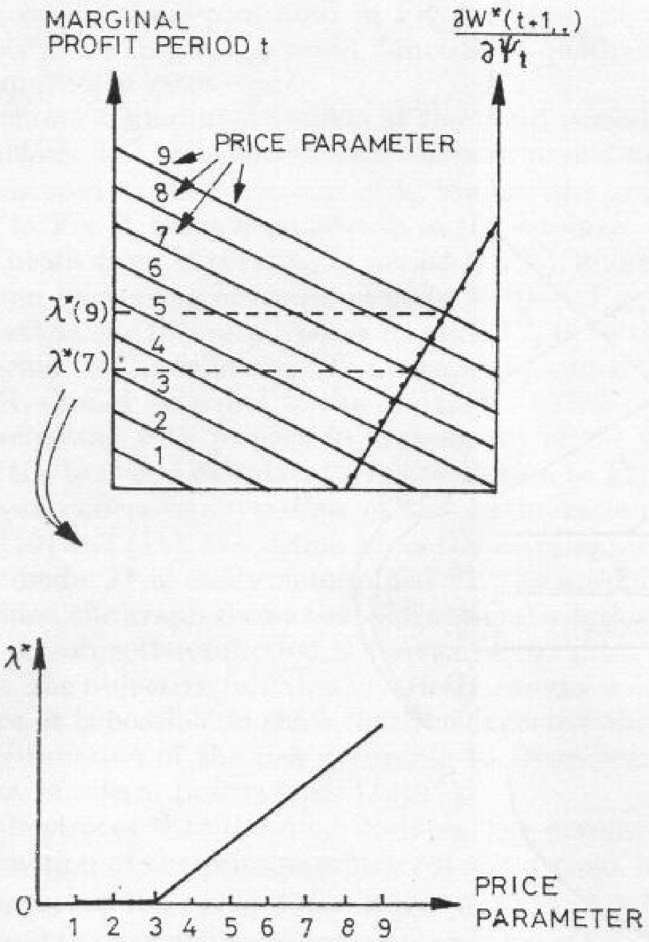

Fig. 6. The graphs show that the optimal dual variable, λ^*, is a kinked convex function of the price parameter if the second order derivatives $\pi_{h_t h_t}$ and $W^*_{\psi_t \psi_t}$ are constants less than zero. λ^* may be interpreted as the marginal resource value. Because of convexity, increasing risk according to ROTSCHILD and STIGLITZ [18] in the price parameter implies that the expected value of λ^* (before the price parameter is known) increases nonstrictly.

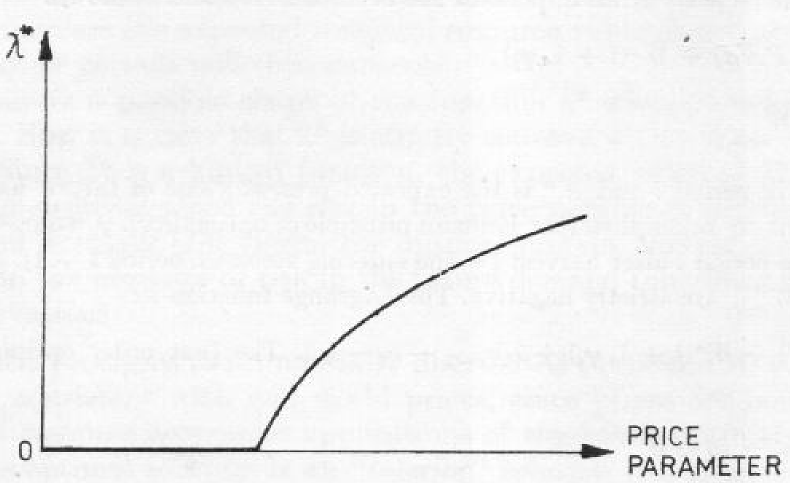

Fig. 7. The graph shows a possible shape of the function λ^*, the marginal resource value. when the second order derivatives $\pi_{h_t h_t}$ and $W^*_{\psi_t \psi_t}$ are **not** constants. (Compare Fig. 6.) In this case, λ^* is a strictly concave function of the price parameter for $\lambda^* > 0$. However, since the function is kinked, the expected value of λ^* may increase, be unchanged or decrease, when the risk in the price parameter increases.

- Generally, if the stochastic processes (or the dual variables) are bounded, this will affect the optimal control of the system. Hence, we should generally not believe in optimal control strategies derived from models based on the assumptions of processes with normally distributed stochastic components and "*interior*" solutions.

2.2. Results from the Numerical Optimizations

This section contains results and associated remarks from optimizations with the numerical adaptive model ADAPTEXT, which is included in the appendix. There, the parameter values used in the construction of the graphs can be found in a table.

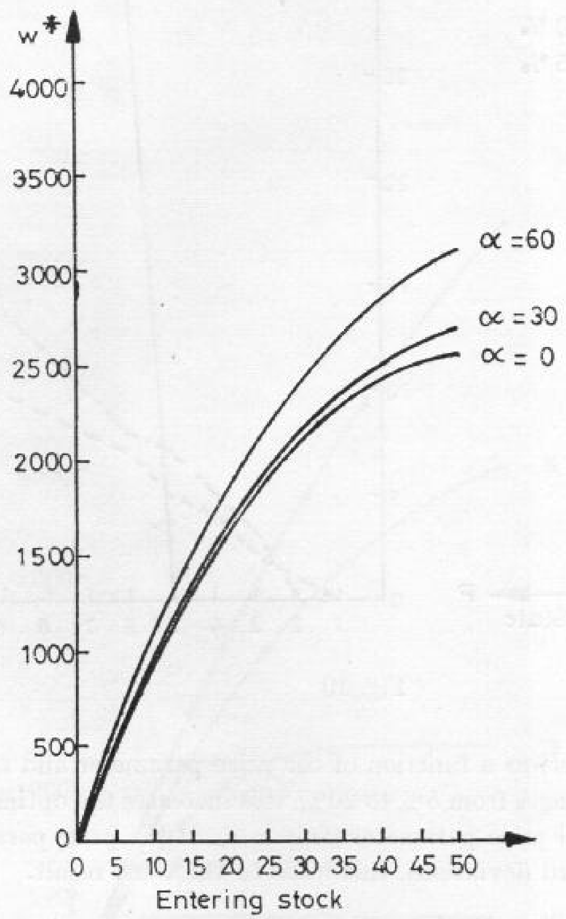

Fig. 8. The graph shows the expected present value of future extraction (harvesting) as a function of the entering stock and the standard deviation of the stochastic component in the price parameter process (adjusted because of a bounded integer state space). Fig. 5 shows that the objective function is convex in the price parameter. This is the reason why increasing risk in the price parameter process (as shown in this figure) leads to an increasing expected present value of future harvesting. Of course, Fig. 5 shows the analysis of a very special and restricted case. A more rigorous derivation of the proofs is found in LOHMANDER [10] and [11]. The logic behind the result is: When there are price parameter fluctuations, we may increase the harvest during good years and decrease it during the low price years. Since we adapt to these changes, we benefit from the price variability and risk is valuable.

Fig. 8 shows that the expected present value of future harvesting is a strictly increasing function of the risk in the price parameter process. This is not really surprising since one has the option to increase harvesting during the good price years and to reduce harvesting during low price years.

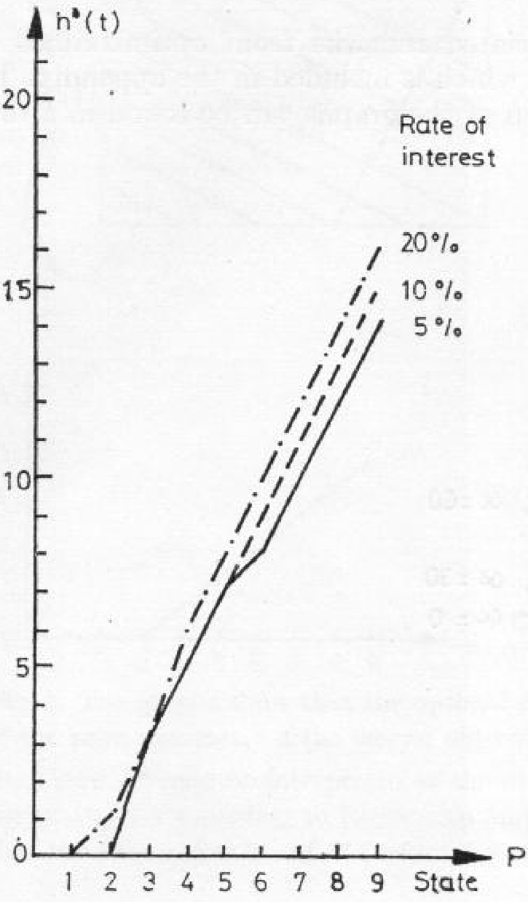

Fig. 9

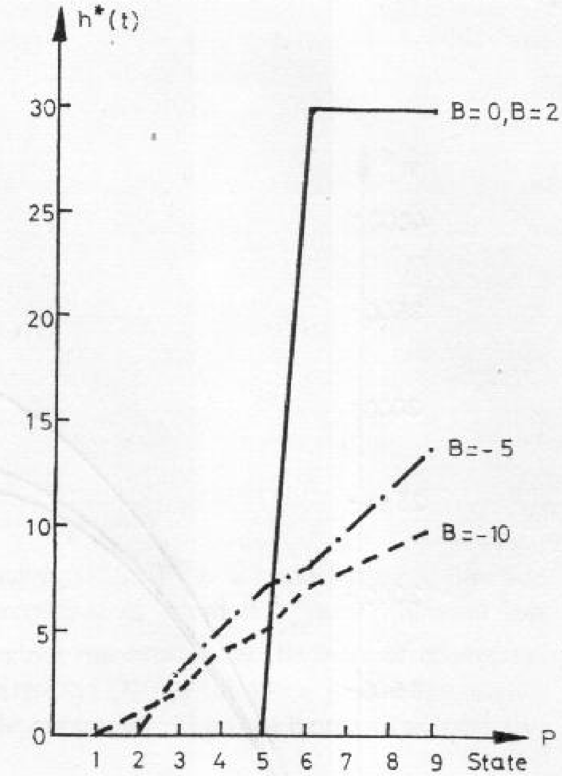

Fig. 10

Fig. 9. The optimal extraction level as a function of the price parameter and the rate of interest. If the rate of interest changes from 5% to 20%, this increases the optimal extraction with 2 units (when the initial price parameter state is 6). If the price parameter increases with one unit (one standard deviation), this leads to the same result.

Fig. 10. The optimal extraction level as a function of the price parameter and the second order derivative of the profit function π, $\pi = 100h + Bh^2$. When the profit function is strictly concave ($B < 0$), it is optimal to distribute the harvest quantity between different years even if there are price differences. The optimal harvest level is a slowly increasing function of the price parameter. This is called continuous extraction. However, if the profit function is linear or even strictly convex because of economies of scale ($B \geq 0$), then it is optimal to harvest everything during one period. This strategy is called pulse extraction and the relevant approach is optimal stopping theory. Then, if the price parameter belongs to the continuation region (≤ 5), it is optimal to save all of the resource at least one more period. If the price parameter belongs to the stopping region (≥ 6), all of the resource should instantly be harvested.

Fig. 9 illustrates the sensitivity of the optimal supply function to the rate of interest in the capital market. It is clear in the example that normal changes in the rate of interest do not necessarily affect optimal supply more than normal price changes. In deterministic models where stochastic price changes are completely ignored, the effects of changes in the rate of interest are the main object of analysis. This model shows us that small shifts in the demand function may have a greater impact on optimal harvesting. From this standpoint, the relevance of the deterministic resource management optimization models is questioned.

Fig. 10 is interesting because it demonstrates the relations between the two control approaches optimal stopping theory and continuous optimal control. If the profit function in each period is strictly concave, there is an incentive to keep a smooth harvest path. In other words, the resource quantity should be more or less evenly distributed over the years irrespective of small price changes. Deviations from the "*optimal*" harvest

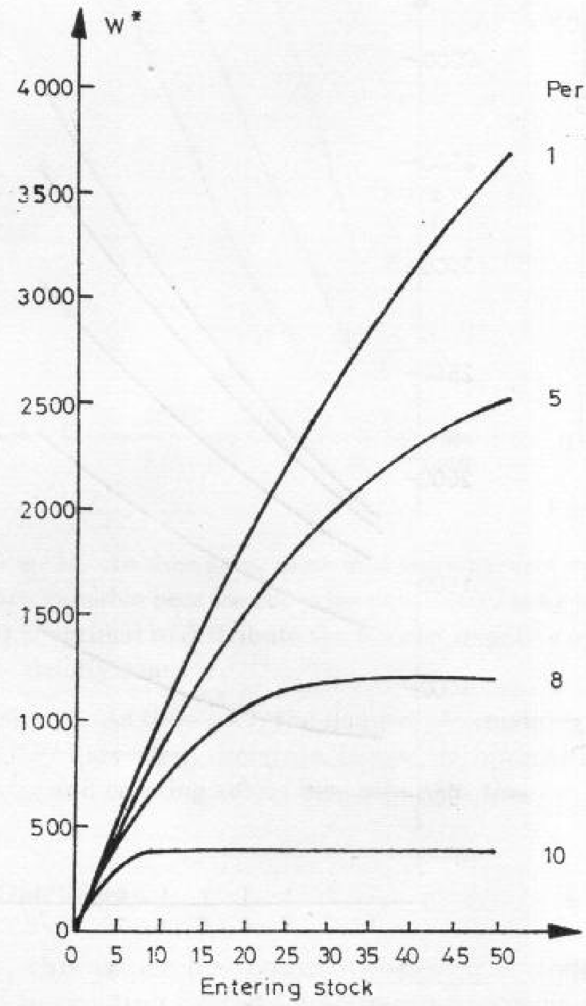

Fig. 11. The expected present value is an increasing function of the size of the entering stock and a decreasing function of time. As time goes, less periods remain, which implies less options to get good price parameter states. Furthermore, since the profit function is strictly concave, it is favourable if the harvest quantity can be distributed over many remaining periods. This way, the marginal contribution of each harvest unit to the objective function increases.

level are expensive. However, in some cases, the profit function may be linear or even convex. If there are economies of scale in harvesting operations, which is frequently the case in forestry, then it is generally optimal to harvest all of the resource during one period. Fig. 10 illustrates this phenomenon. If the profit function is strictly concave, then the harvest level is not very sensitive to the price during a particular period. If the profit function is linear or strictly convex, an optimal stopping strategy can be defined as follows. If the price exceeds the optimal stopping boundary, the price belongs to the optimal stopping region and all of the resource should instantly be harvested. If the price is below the optimal stopping boundary, then it belongs to the continuation region. All of the resource should be saved at least one more period. A so called bang-bang policy is optimal. Harvest all or nothing.

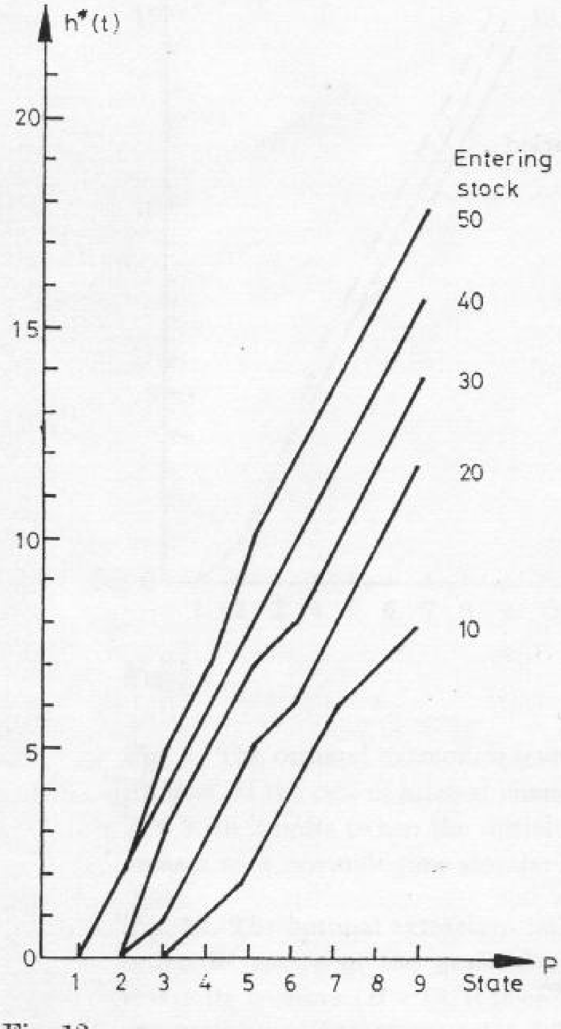

Fig. 12

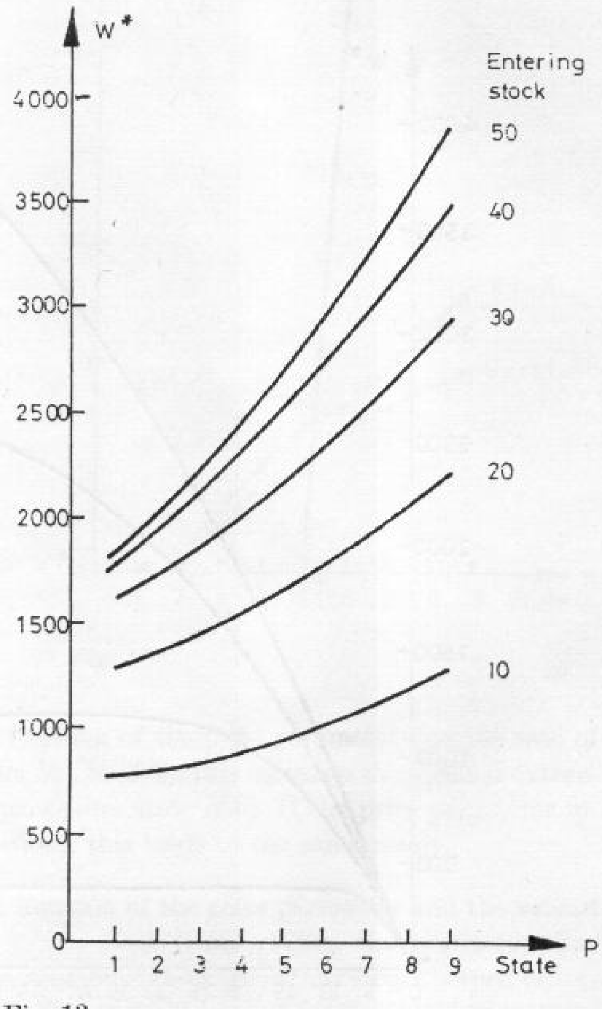

Fig. 13

Fig. 12. The optimal extraction level is an increasing functions of the price parameter and the size of the entering stock

Fig. 13. The optimal expected present value W^* is a (strictly) convex function of the price parameter. The logic behind this result is shown in Fig. 5.
Furthermore, W^* is an increasing function of the entering stock.

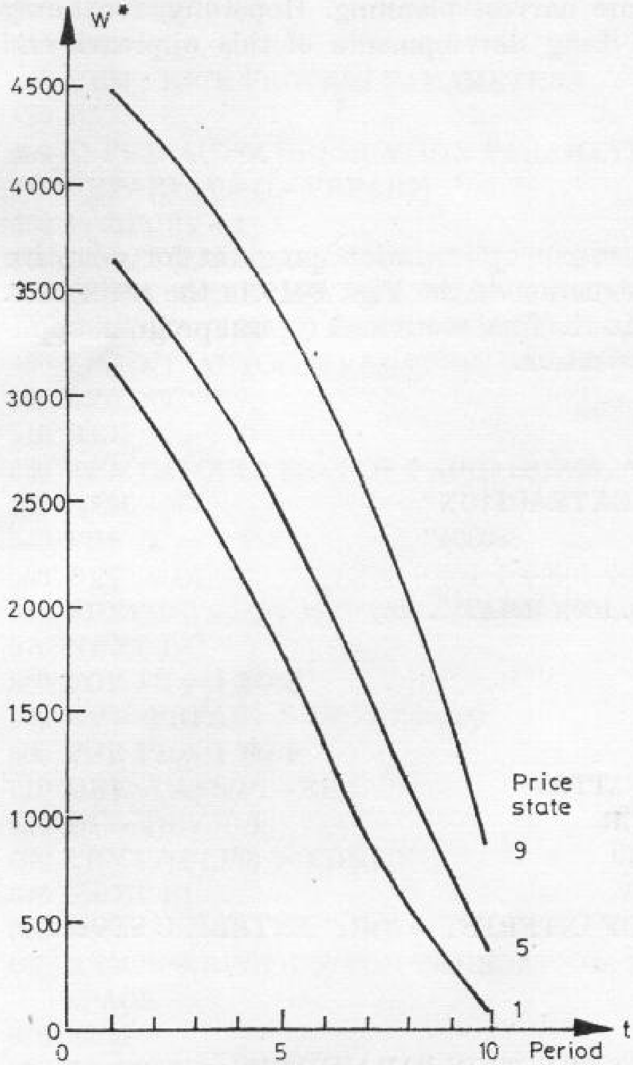

Fig. 14

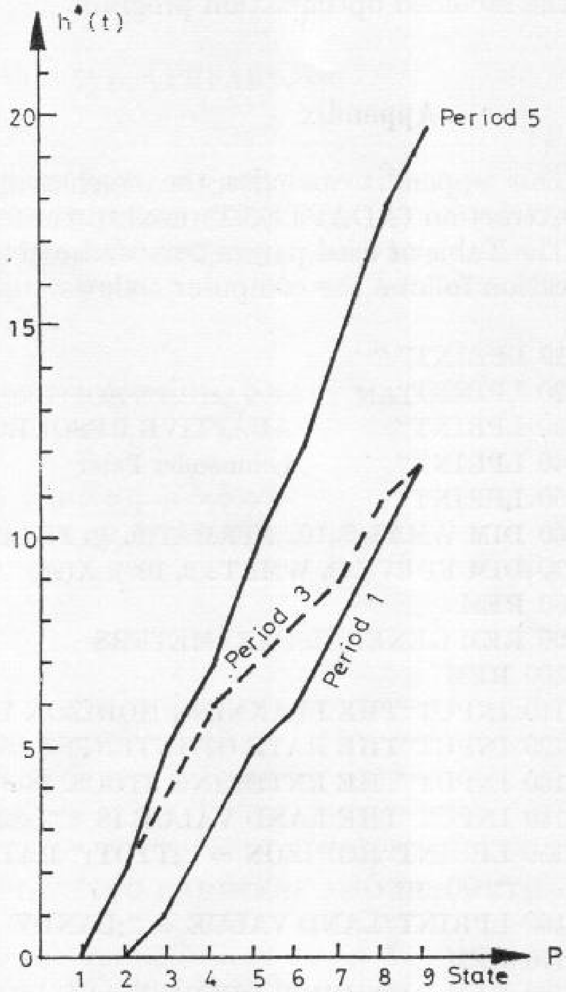

Fig. 15

Fig. 14. As time goes, more and more harvest options are lost. Remaining harvest options are valuable because the price parameters may be favourable in future periods and because it is optimal to distribute the harvest quantity over many years when the profit function π is strictly concave.

Fig. 15. As time goes, the number of remaining periods (when the resource may be profitably harvested) decreases. Hence, the optimal harvest level (for a particular price parameter and entering stock) increases with time.

3. Discussion

Hopefully, this paper has contributed to the understanding of the fundamental rules of optimal harvesting in a stochastic environment. It has been possible to describe and solve problems from applied optimal stopping theory and optimal control within the framework of one simple model. The author has the strong opinion that it is necessary to use numerical models as complements to analytical models, because, as discussed also in the introduction, normative problems are almost always quantitative problems. Furthermore, it is easy to investigate the importance of new and more relevant, model assumptions through numerical modelling. Hopefully, the results presented in this paper will have a direct influence on future harvest planning. Hopefully, remaining problems within this area can be solved using developments of this approach and the included optimization program.

Appendix

This appendix includes the stochastic dynamic optimization program for adaptive extraction (ADAPTEXT) used in the construction of the Figs. 8–15 in the main text. The Table of used parameters is also given in the final section. A typical program application follows the computer code as an illustration.

```
10 LPRINT" "
20 LPRINT"          ++++++++++++++++++++++++"
30 LPRINT"          ADAPTIVE RESOURCE EXTRACTION"
40 LPRINT"          Lohmander Peter                  880304"
50 LPRINT"          ++++++++++++++++++++++++"
60 DIM WMAT(9, 100), TRMAT(9, 9), FIMAT(9, 100), HMAT(9, 100)
70 DIM FDEV(11), WMAT2(9, 100), X(20)
80 REM
90 REM GENERAL PARAMETERS
100 REM
110 INPUT"THE PLANNING HORIZON IS ?",TTOT
120 INPUT"THE RATE OF INTEREST IS ?",R
130 INPUT"THE ENTERING STOCK IS ?",Q0
140 INPUT"THE LAND VALUE IS ?",LANDV
150 LPRINT"HORIZON = ";TTOT;" RATE OF INTEREST = ";R;" ENTERING STOCK =
    ";Q0
160 LPRINT"LAND VALUE = ";LANDV
170 REM
180 REM RESOURCE DENSITY AND PROFIT FUNCTION PARAMETERS
190 REM
200 INPUT"FIX PROFIT = ",PPAR0
210 INPUT"PPAR1       = ",PPAR1
220 INPUT"PPAR2       = ",PPAR2
230 INPUT"PPAR3       = ",PPAR3
240 LPRINT" PROFIT FUNCTION PARAMETERS = ";PPAR0;PPAR1;PPAR2;PPAR3
250 PRINT"++++++++++++++"
260 INPUT"                          FIX DENSITY     = ",GPAR0
270 INPUT" FIRST ORDER DENSITY PARAMETER   = ",GPAR1
280 INPUT"SECOND ORDER DENSITY PARAMETER = ",GPAR2
290 INPUT" EXPONENTIAL DENSITY PARAMETER  = ", GPAR3
300 LPRINT"DENSITY FUNCTION PARAMETERS = ";GPAR0;GPAR1;GPAR2;GPAR3
310 LPRINT" "
320 LPRINT"         THE PROFIT AS A FUNCTION OF THE EXTRACTION VOLUME"
330 LPRINT"         (THE LAND VALUE IS NOT INCLUDED, P IS IN EQUILIBRIUM
                    AND T = 0)"
340 LPRINT" "
350 FOR H = 0 TO 20
360 PROFIT=PPAR0 + PPAR1 *H + PPAR2 *H *H + PPAR3 *H *H *H
370 LPRINT"         EXTRACTION VOLUME = ";H;" PROFIT = ";PROFIT
380 NEXT H
```

```
390 LPRINT" "
400 REM
410 REM PRICE PROCESS PARAMETERS
420 REM
430 INPUT"AUTOCORRELATION PARAMETER (0 - 1) = ",PRPAR2
440 PRPAR1=5*(1-PRPAR2)
450 PRSTDEV=1
460 INPUT"STANDARD DEVIATION IN PRICE PROCESS = ",PRPAR4
470 INPUT"DETERMINISTIC TIME TREND IN PRICE IS = ",PRPAR5
480 LPRINT"STDV IN PRICE PROCESS = ";PRPAR4;" TIME TREND = ";PRPAR5
490 LPRINT"AUTOCORRELATION PARAMETER = ";PRPAR2
500 LPRINT" "
510 REM
520 REM CALCULATION OF PRICE INDEX TRANSITION PROBABILITY MATRIX
530 REM
540 FOR DI = 1 TO 11
550 DEV = DI-1
560 FDEV(DI) = 1/(2*3.141593*PRSTDEV^2)^.5*EXP(-DEV^2/2/PRSTDEV^2)
570 NEXT DI
580 FOR P0 = 1 TO 9
590 EP = PRPAR1 + PRPAR2*P0
600 FOR P1 = 1 TO 9
610 DEV = ABS(P1-EP)
620 DI = DEV + 1
630 TRMAT(P1,P0) = FDEV(DI)
640 NEXT P1
650 REM
660 REM CORRECTION FOR TRUNCATION, REFLECTING BARRIERS AND DISCRETE
    SPACE
670 REM
680 PROBTOT = 0
690 FOR P1 = 1 TO 9
700 PROBTOT = PROBTOT + TRMAT(P1,P0)
710 NEXT P1
720 FOR P1 = 1 TO 9
730 TRMAT(P1,P0) = TRMAT(P1,P0)/PROBTOT
740 NEXT P1
750 NEXT P0
760 LPRINT" PRICE STATE TRANSITION PROBABILITY MATRIX "
770 LPRINT"          row = P(t+1), column = P(t)"
780 LPRINT" "
790 FOR P1 = 1 TO 9
800 FOR P0 = 1 TO 9
810 X(P0) = TRMAT(P1,P0)
820 NEXT P0
830 LPRINT USING"###.###";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
840 NEXT P1
850 REM
860 REM DEFINITION OF WMAT2(P,V) (= 0 FOR ALL P, V) IN THE FINAL PERIOD
870 REM
880 FOR P = 1 TO 9
890 FOR V = 1 TO Q0
```

```
900 WMAT2(P,V)=0
910 NEXT V
920 NEXT P
930 REM
940 REM STOCHASTIC DYNAMIC PROGRAMMING VIA THE BACKWARD ALGORITHM
950 REM
960 FOR S = 1 TO TTOT
970 T = TTOT - S + 1
980 DENSITY = GPAR0 + GPAR1*T + GPAR2*T*T + GPAR3^T
990 LPRINT" "
1000 LPRINT"*********** THE PERIOD IS ";T;" **************
"
1010 LPRINT"                    DENSITY IS ";DENSITY
1020 REM
1030 REM SELECTION OF THE OPTIMAL EXTRACTION LEVEL FOR EACH (P,V)
1040 REM
1050 FOR PIND = 1 TO 9
1060 FOR VOLIN = 1 TO Q0
1070 PRMAX = 0
1080 HOPT = 0
1090 FOR H = 0 TO VOLIN
1100 VOLOUT = VOLIN - H
1110 G=H*DENSITY
1120 W = WMAT2(PIND,VOLOUT)
1130 PR = PPAR0 + (PPAR1 + PRPAR5+PRPAR4*(PIND-5))*G + PPAR2*G*G +
     PPAR3*G*G*G
1140 PR = EXP(-R*T)*(PR + H*LANDV) + W
1150 IF PR > PRMAX THEN HOPT = H
1160 IF PR > PRMAX THEN PRMAX = PR
1170 NEXT H
1180 FIMAT(PIND,VOLIN) = PRMAX
1190 HMAT(PIND,VOLIN) = HOPT
1200 NEXT VOLIN
1210 NEXT PIND
1220 LPRINT" "
1230 LPRINT"THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD
     ";T
1240 LPRINT"row = entering stock (1 - Q0), column = price state (1 - 9)"
1250 LPRINT" "
1260 FOR V = 1 TO Q0
1270 FOR P = 1 TO 9
1280 X(P) = HMAT(P,V)
1290 NEXT P
1300 LPRINT USING"###";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
1310 NEXT V
1320 REM
1330 REM DETERMINATION OF THE EXPECTED VALUE MATRIX. NAMELY WMAT
     (P,V)
1340 REM
1350 FOR P0 = 1 TO 9
1360 FOR VOLIN = 1 TO Q0
1370 WMAT(P0,VOLIN) = 0
```

```
1380 FOR P1 = 1 TO 9
1390 WMAT(P0,VOLIN) = WMAT(P0,VOLIN) + TRMAT(P1,P0)*FIMAT(P1,VOLIN)
1400 NEXT P1
1410 NEXT VOLIN
1420 NEXT P0
1430 REM
1440 REM PRINTOUT OF THE EXPECTED PRESENT VALUE MATRIX WMAT(P,V)
1450 REM
1460 LPRINT" "
1470 LPRINT"THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD ";T
1480 LPRINT"row = entering stock (1 — Q0), column = price state (1 — 9)"
1490 LPRINT" "
1500 FOR V = 1 TO Q0
1510 FOR P = 1 TO 9
1520 X(P) = WMAT(P,V)
1530 NEXT P
1540 LPRINT USING"#######.";X(1);X(2);X(3);X(4);X(5);X(6);X(7);X(8);X(9)
1550 NEXT V
1560 REM
1570 REM BEFORE A CHANGE IN PERIOD, WE LET WMAT(P,V) REPLACE WMAT2(P,V)
1580 REM
1590 FOR P = 1 TO 9
1600 FOR V = 1 TO Q0
1610 WMAT2(P,V) = WMAT(P,V)
1620 NEXT V
1630 NEXT P
1640 NEXT S
1650 STOP
```

```
        +++++++++++++++++++++
        ADAPTIVE RESOURCE EXTRACTION
        Lohmander Peter                    880304
        +++++++++++++++++++++
HORIZON = 2 RATE OF INTEREST = .05 ENTERING STOCK = 10
LAND VALUE = 10
 PROFIT FUNCTION PARAMETERS = 0 100 -10 0
DENSITY FUNCTION PARAMETERS = .9 .1 0 0

        THE PROFIT AS A FUNCTION OF THE EXTRACTION VOLUME
        (THE LAND VALUE IS NOT INCLUDED, P IS IN EQUILIBRIUM AND T = 0)
        EXTRACTION VOLUME = 0   PROFIT = 0
        EXTRACTION VOLUME = 1   PROFIT = 90
        EXTRACTION VOLUME = 2   PROFIT = 160
        EXTRACTION VOLUME = 3   PROFIT = 210
        EXTRACTION VOLUME = 4   PROFIT = 240
        EXTRACTION VOLUME = 5   PROFIT = 250
        EXTRACTION VOLUME = 6   PROFIT = 240
        EXTRACTION VOLUME = 7   PROFIT = 210
        EXTRACTION VOLUME = 8   PROFIT = 160
        EXTRACTION VOLUME = 9   PROFIT = 90
        EXTRACTION VOLUME = 10   PROFIT = 0
        EXTRACTION VOLUME = 11   PROFIT = -110
```

EXTRACTION VOLUME = 12 PROFIT = −240
EXTRACTION VOLUME = 13 PROFIT = −390
EXTRACTION VOLUME = 14 PROFIT = −560
EXTRACTION VOLUME = 15 PROFIT = −750
EXTRACTION VOLUME = 16 PROFIT = −960
EXTRACTION VOLUME = 17 PROFIT = −1190
EXTRACTION VOLUME = 18 PROFIT = −1440
EXTRACTION VOLUME = 19 PROFIT = −1710
EXTRACTION VOLUME = 20 PROFIT = −2000

STDV IN PRICE PROCESS = 30 TIME TREND = 0
AUTOCORRELATION PARAMETER = .5

PRICE STATE TRANSITION PROBABILITY MATRIX
row = P(t+1), column = P(t)

0.054	0.007	0.004	0.000	0.000	0.000	0.000	0.000	0.000
0.243	0.090	0.054	0.007	0.004	0.000	0.000	0.000	0.000
0.401	0.403	0.242	0.090	0.054	0.007	0.004	0.000	0.000
0.243	0.403	0.399	0.403	0.242	0.090	0.054	0.007	0.004
0.054	0.090	0.242	0.403	0.399	0.403	0.242	0.090	0.054
0.004	0.007	0.054	0.090	0.242	0.403	0.399	0.403	0.243
0.000	0.000	0.004	0.007	0.054	0.090	0.242	0.403	0.401
0.000	0.000	0.000	0.000	0.004	0.007	0.054	0.090	0.243
0.000	0.000	0.000	0.000	0.000	0.000	0.004	0.007	0.054

**** THE PERIOD IS 2 ******************
DENSITY IS 1.1

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 2
row = entering stock (1 − Q0), column = price state (1 − 9)

0	1	1	1	1	1	1	1	1
0	1	2	2	2	2	2	2	2
0	1	2	3	3	3	3	3	3
0	1	2	4	4	4	4	4	4
0	1	2	4	5	5	5	5	5
0	1	2	4	5	6	6	6	6
0	1	2	4	5	6	7	7	7
0	1	2	4	5	6	8	8	8
0	1	2	4	5	6	8	9	9
0	1	2	4	5	6	8	9	10

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 2
row = entering stock (1 − Q0), column = price state (1 − 9)

40.	53.	68.	83.	98.	113.	127.	142.	157.
62.	86.	115.	144.	173.	203.	233.	263.	292.
71.	101.	142.	183.	228.	272.	317.	362.	405.
73.	105.	155.	203.	261.	319.	379.	438.	496.
74.	106.	160.	211.	279.	346.	420.	493.	566.
74.	106.	161.	213.	287.	358.	443.	527.	614.
74.	106.	161.	214.	288.	361.	453.	544.	643.
74.	106.	161.	214.	289.	362.	456.	549.	656.
74.	106.	161.	214.	289.	362.	457.	550.	661.
74.	106.	161.	214.	289.	362.	457.	550.	662.

**** THE PERIOD IS 1 ******************
DENSITY IS 1

THE OPTIMAL EXTRACTION STRATEGY MATRIX HMAT(.) IN PERIOD 1
row = entering stock (1 — Q0), column = price state (1 — 9)

0	0	0	0	0	1	1	1	1
0	0	0	1	1	1	2	2	2
0	0	1	1	2	2	2	3	3
0	1	1	2	2	2	3	3	3
0	1	2	2	3	3	3	4	4
0	1	2	3	3	3	4	4	4
0	1	2	3	4	4	4	5	5
0	1	2	4	4	5	5	5	6
0	1	2	4	5	5	5	6	6
0	1	2	4	5	6	6	6	7

THE EXPECTED PRESENT VALUE MATRIX WMAT(.) IN PERIOD 1
row = entering stock (1 — Q0), column = price state (1 — 9)

68.	75.	83.	91.	102.	112.	125.	138.	152.
118.	134.	153.	172.	194.	215.	239.	262.	288.
157.	182.	212.	241.	274.	306.	342.	377.	413.
186.	221.	262.	302.	346.	389.	435.	481.	528.
208.	251.	302.	352.	406.	461.	518.	575.	633.
224.	273.	333.	393.	458.	522.	590.	658.	727.
233.	287.	356.	423.	499.	575.	654.	733.	812.
238.	295.	371.	446.	531.	616.	706.	795.	886.
241.	300.	381.	461.	556.	649.	750.	849.	950.
243.	303.	388.	472.	574.	674.	784.	893.	1005.

Table. The numerical cases. This table shows the parameters used in the construction of some of the graphs (Figs. 8–15) in the main text. The parameters in the columns >1 are the same as those in column 1 if no other value is given in the Table

PLANNING HORIZON	5		10						
RATE OF INTEREST	.05			.10				.20	
ENTERING STOCK	50								
LAND VALUE	10								
FIX PROFIT	0								
PPAR1	100								
PPAR 2	-5				-10	0	2		
PPAR 3	0								
FIX DENSITY	.97								
1 ORDER DENSITY PAR	.03								
2 ORDER DENSITY PAR	0								
EXP DENSITY PAR	0								
AUTOCOR PAR	.5								
STDEV PRICE PROC RES	30	0							60
DETERM PRICE TREND	0								
Fig. 8	X	X							X
Fig. 9	X			X				X	
Fig. 10	X				X	X	X		
Fig. 11			X						
Fig. 12	X								
Fig. 13	X								
Fig. 14			X						
Fig. 15	X								

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Book Review

Banks, S. P.: **Mathematical Theories of Nonlinear Systems.** Prentice Hall International, New York–London–Toronto–Tokyo 1988, 345 pp.

In recent textbooks and books on system theory linear systems are described, only, for which the mathematical apparatus is well developed and prepared for engineering application. But the most practical systems are nonlinear and require nonlinear methods for their effective handling (analysis and synthesis). At present a systematic, general theory of nonlinear systems like the linear one does not exist: Either the nonlinear system is simply linearized or a developed nonlinear method only refers to quite a special system.

This book represents a first approach towards a systematic, general theory of nonlinear systems in a range similar to that of linear systems. Therefore it is of great interest although it is restricted to deterministic systems described by ordinary differential equations $\dot{z} = f(z, u)$, $y = s(z, u)$ in the state space (u: vector input including state depending inputs, z: state vector, y: output vector). The book requires some deeper mathematical knowledge summarized in chapter 1 (Differentiable manifolds, Lie Groups, Lie algebras, ...) which is to be completed (especially from the engineer) by additional mathematical textbooks (references are given).

The second chapter represents and proofs conditions for such fundamental properties of nonlinear systems as controllability, observability, invertibility, etc. known from the linear systemtheory. The third chapter represents and proofs general conditions for the global linearization by a diffeomorphic change of variables. The tensor representation of nonlinear systems is described more deeply. On this basis a wide class of nonlinear systems can be described by bilinear systems, which are described in chapter four. The used here Volterra expansion series allows the extension of the application of the well known from the linear system theory Laplace- an Fourier transforms to nonlinear systems.

The last chapter 5 presents an introduction to nonlinear distributed parameter systems.

The book gives a good and systematic overview of the literature up to 1986. in this field. The new quality is the systematization. The given simple examples are helpfully for better understanding. The concentration on bilinear systems (with possible difficulties with the infinite dimensions) is restrictive but it demonstrates the state of the art of this field.

The book is more directed to mathematicians with interest in technical applications. The required mathematical level seems to be a bit to high for engineers understanding.

Nevertheless, the book is a very good first step to a general, systematic theory on nonlinear systems and should have its place in libraries of institutes looking for engineering solutions of control problems concerning nonlinear plants.

N. Ahlbehrendt