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DECISION OPTIMIZATION WITH STOCHASTIC SIMULATION SUBROUTINES
Relation to Analytical Optimization of Capacity Investment and Production

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In several multi period decision optimization problems where the decision environment can be modelled as stochastic processes, one can show that adaptive optimization is appropriate. The decisions should be based on the latest available information.

This paper starts with an analytical treatment of the economic two stage industrial capacity investment and production problem. Several new explicit results are derived with uniform and triangular net price distributions. Some of these are: The optimal capacity level and the expected optimal present value are increasing functions of the expected net price and of the net price risk.

When there are several stochastic processes and adaptive control options in the problem, continuous state and decision variables, and detailed answers are needed, not many methods can be used. The adaptive control rule parameters can however be optimized with an iterative stochastic quasi gradient method in which the objective function sequentially is approximated by large numbers of stochastic full system simulations.

This paper contains a discussion of this approach and a detailed forest industry enterprise example.

KEY WORDS Decision optimization, stochastic simulation, industrial capacity investment.

1. INTRODUCTION

In the discussion of a particular qualitative property of reality, a complete analytical model should, when possible, be designed and the qualitative properties of this derived. Often, however, this is almost impossible. In several multiperiod decision optimization problems where the decision environment can be modelled as stochastic processes, one can show that adaptive optimization is appropriate. The decisions should be based on the latest available information. This kind of decision problem may often be handled numerically by discrete time discrete state stochastic dynamic programming. Qualitative properties of solutions may sometimes be found via continuous analytical versions of these models. The recursive character of stochastic dynamic programming problems together with the large, sometimes infinite, number of sequential and conditional probability distribution calculations, make it very difficult to derive qualitative and/or quantitative results analytically. Published results of this sort are mostly restricted to very special classes of problems.

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This paper starts with an analytical treatment of the two stage industrial capacity investment and production problem. Several new explicit results are derived.

When we want to treat the more general adaptive decision problems that occur in many applications, there are several ways open. Discrete time and discrete state stochastic dynamic programming can be used in finite and infinite horizon problems, infinite horizon through the use of linear programming in Markov chains. These methods however suffer from the fact that they rely on discrete state spaces and discrete control decisions. Methods have been proposed to overcome the discretization problems where the objective function recursively is approximated by smooth continuous functions. All stochastic dynamic programming algorithms have the property that they finally reach the dimensionality limit. A large table of expected objective function values at all possible multidimensional state combinations must be stored in the computer memory. Furthermore, the objective function must be additive over time, possibly multiplicative (= additive in the logarithms). When there are several stochastic processes and adaptive control options in the problem, continuous state and decision variables, and detailed answers are wanted, not many methods can be used. The adaptive control rule parameters can however be optimized with an iterative stochastic quasi-gradient method in which the objective function sequentially is approximated by large numbers of stochastic full system simulations.

This paper contains a discussion of this approach and a detailed forest industry enterprise example.

– Is there a reliable and practical method that can be used to optimize the more realistic problems of some complexity in real world enterprises?

This question will be discussed in this paper.

2. ON THE OPTIMAL CAPACITY INVESTMENT LEVEL UNDER PRICE RISK

One of the key problems in the presence of stochastic prices is to determine the optimal industrial capacity level. When this is determined, it is necessary to take the fact that future production will be adaptively optimized into consideration. The future production level will be optimized conditional on the revealed net price and the capacity level. In the following section, we will investigate a simplified problem of this kind.

McDonald and Siegel (1985) observed that the profit is a kinked convex function of price when there is an option to shut down production during low price periods. Via the Jensen inequality, it can be shown that a mean preserving increase in price risk (nonstrictly) increases the expected profit. Lohmander (1989) claimed that future price risk may be valuable to the adaptively active firms in natural resource sectors since: a. The production level can be adjusted to any level between zero and maximum capacity depending on the revealed price. b. The natural resource may be distributed between different processing plants in the most profitable way, depending on the levels of the different product prices. It is also important to note that if we stop production during a low price period in a natural resource enterprise, we save the resource for future and hopefully more profitable periods. Hence, we should not assume that the McDonald and Siegel approach to the investment problem reveals everything of interest, with respect to capacity investment, to the firm active in the natural resource sectors.

Pindyck (1988) used a continuous time model with irreversible investments. Pindyck made the assumption that production capacity can be added continuously and incrementally. Hence, he found that there is little reason to invest before the high prices have been observed. The conclusion drawn is that firms should hold less capacity when future prices can not be predicted than if they could be predicted.

In this paper, we will in the first part not only calculate the expected profit as a function of the future price risk but also optimize the level of the capacity investment. The effects of parameter changes will be explicitly derived. Of special interest is that increasing future price risk implies that the optimal capacity investment level increases. Here, it is not possible to wait for the good prices. It takes time to build the plant. We have to invest before we observe the price development. (Compare the completely different result derived by Pindyck (1988).)

In the later part of the paper, a more detailed stochastic multi decision multi period capacity investment and production problem will be discussed and optimized via a numerical approach based on a gradient method. It may be described as traditional "hill-climbing" where the best local moves are calculated via approximations of the gradient. These are estimated via large numbers of stochastic full system simulations.

3. THE CASE OF UNIFORMLY DISTRIBUTED NET PRICES

Let us start by an explicit analysis of a concrete problem. The net price (= product price − variable production costs), p, has a uniform distribution. The expected value of p is y, $y>0$. The probability density of p is $1/(2m)$ in the interval $(y-m)<p<(y+m)$ and zero elsewhere. As a reasonable assumption we introduce: $m>y$. This means that $(y-m)<0$ and that negative values of p have a strictly positive probability. The (possible) net revenue takes place in the future, at time t, since there is a delay in the investment process. The real rate of interest is r. The cost of the capacity investment is a quadratic function of the capacity level, k. The cost parameters are h, i and j. We will assume strictly positive investments costs for all strictly positive investment levels and assume increasing marginal investment costs. These assumptions can always be discussed. In any case, we assume that h, i, and j are all strictly positive. The first stage decision, which is here optimized, is the level of the capacity, k. The second stage decision is the future production level. That decision follows the following rule: If net price, p, turns out to be positive, then we produce as much as possible, which is equal to the capacity level, k. If, on the other hand the net price happens to be negative, then we do not produce anything. The expected present value of the total profit at the time of the capacity decision is the objective function. This is denoted Π.

$$\Pi = -(h + ik + jk^2) + e^{-rt} \int_0^{y+m} \frac{kp}{2m} \, dp$$

We may solve the integral and write a more concenient form of the objective function:

$$\Pi = -h - k(i + jk) + \frac{k(y+m)^2 e^{-rt}}{4m}$$

Now, we want to find the optimal level of the capacity investment k. The first order optimum condition will be derived and used:

$$\frac{\delta \Pi}{\delta k} = -i - 2jk + \frac{(y+m)^2 e^{-rt}}{4m} = 0$$

It turns out that the second order maximum condition is satisfied. We will find a local maximum which is also a unique and global maximum.

$$\frac{\delta^2 \Pi}{\delta k^2} = -2j < 0$$

We may use the first order optimum condition to calculate the optimal value of the capacity investment, k (* always indicates optimal value). We assume that the parameter values are such that the following equations always lead to strictly positive capacity levels. The fact that we could find an explicit function of the optimal investment level is very practical and will be intensively used in the following calculations.

$$k^* = -\frac{i}{2j} + \frac{(y^2 + 2my + m^2)e^{-rt}}{8jm}$$

The derivatives of the optimal investment level with respect to the parameters y, m, r, t, i and j will be given below. Thanks to the initial assumptions, most of the signs of these derivatives are known.

If the expected value of the net price increases, the optimal investment level increases.

$$\frac{\delta k^*}{\delta y} = \frac{(y+m)e^{-rt}}{4jm} > 0$$

If the risk in the net price distribution increases (m increases) and the expected price remains constant, then we should increase the investment level. Sandmo (1971) used a different but related method of uniform stretching of a distribution to describe increasing risk.

$$\frac{\delta k^*}{\delta m} = -\frac{(y+m)(y-m)e^{-rt}}{8jm^2} > 0$$

The optimal investment is a decreasing function of the rate of interest and of the length of the time period until the use of the new capacity in production.

$$\frac{\delta k^*}{\delta r} = -\frac{t(y^2 + 2my + m^2)e^{-rt}}{8jm} < 0$$

$$\frac{\delta k^*}{\delta t} = -\frac{r(y^2 + 2my + m^2)e^{-rt}}{8jm} < 0$$

If we increase the marginal investment cost with a constant amount for all capacity levels, then the optimal investment level decreases.

$$\frac{\delta k^*}{\delta i} = -\frac{1}{2j} < 0$$

It is not possible to say in what direction a positive change of the quadratic investment cost parameter will influence the optimal investment level in the general case. Of course, if we have numerically specified parameters, this is no problem.

$$\frac{\delta k^*}{\delta j} = \frac{i}{2j^2} - \frac{(y^2 + 2my + m^2)e^{-rt}}{8j^2 m} \gtreqless 0$$

Now, it is also interesting to investigate how the objective function, the expected optimal present value, is affected by parameter changes. In particular, it is important to investigate in what direction changes in the future net price risk will influence the expected profitability.

The partial derivative of Π with respect to the expected net price is easily shown to be strictly positive.

$$\frac{\delta \Pi^*}{\delta y} = \frac{k^*(y+m)e^{-rt}}{2m} > 0$$

However, the more important question is how Π is affected by changes in the parameters if optimal changes in the investment level decision k take place. Thus, we have to derive a number of total derivatives. The greek letter zeta will be used to indicate total derivatives.

$$\frac{\zeta \Pi^*}{\zeta y} = \frac{\delta \Pi^*}{\delta y} + \frac{\Pi^*}{\delta k}\frac{\delta k^*}{\delta y} > 0$$

$$>0 \qquad =0$$

We make the convenient observation that, since k is optimally chosen, the total derivative is identical to the partial derivative. Hence we know that the expected optimal present value is a strictly increasing function of the expected future price. This conclusion is reasonable.

We also find that the expected optimal present value is an increasing function of the net price risk.

$$\frac{\zeta \Pi^*}{\zeta m} = \frac{\delta \Pi^*}{\delta m} = -\frac{k^*(y-m)(y+m)e^{-rt}}{4m^2} > 0$$

As expected, the increases in r, t, i and j will decrease the value of our objective function Π. Note in particular that it was not possible to say in what direction k is affected by changes in j. Still, the qualitative effect concerning Π of changes in j could be derived.

$$\frac{\zeta\Pi^*}{\zeta r}=\frac{\delta\Pi^*}{\delta r}=-\frac{k^*t(y+m)^2e^{-rt}}{4m}<0$$

$$\frac{\zeta\Pi^*}{\zeta t}=\frac{\delta\Pi^*}{\delta t}=-\frac{k^*r(y+m)^2e^{-rt}}{4m}<0$$

$$\frac{\zeta\Pi^*}{\zeta i}=\frac{\delta\Pi^*}{\delta i}=-k^*<0$$

$$\frac{\zeta\Pi^*}{\zeta j}=\frac{\delta\Pi^*}{\delta j}=-k^{*2}<0$$

4. THE CASE OF NET PRICES WITH A TRIANGULAR DISTRIBUTION

Now, the reader may wonder if the assumption of a uniform net price distribution was critical to the strong qualitative results just obtained. One may investigate this problem in different ways: One way is to make more general probability distribution assumptions and to study the qualitative effects of some general form of increasing risk. A convenient way to handle such problems has been suggested by Rotschild and Stiglitz (1970) and (1971). Examples of how qualitative studies of increasing risk can be made this way in connection to economic multi period problems from the forest sector are found in Lohmander (1987a), (1988b) and elsewhere.

In this section, we will as an alternative, analyse the problem of increasing risk via the introduction of another kind of distribution, the triangular distribution. There are several reasons why a triangular distribution is interesting in this case:

- It may be used as an approximation to a normal distribution.
- One advantage (or disadvantage), depending on the application, compared to the normal distribution, is that the "tails" are finite.
- It is simple to integrate.
- One can obtain explicit formulae of the optimal investment level and the objective function.
- One can obtain explicit formulae of the partial derivatives of interest in this paper. It also turns out that the signs of these explicitly derived derivatives are possible to determine.

We will use the same investment cost function as in the uniform case.

The triangular net price probability density function is determined by the following assumptions:

The probability density has the maximum value, $1/m$, at the expected price, y. The probability density is strictly positive in the net price interval $(y-m)<p<(y+m)$. The points $(y-m)$ and $(y+m)$ mark the intersections of the "probability density triangle" with the "base line", the net price line. As earlier, we assume that $y>0$ and that $(y-m)<0$. In the calculations, we will have to use two different integrals.

$$\Pi = -(h + ik + jk^2) + \mathrm{e}^{-rt}\left(\int_0^y kp\left(\frac{m-y}{m^2} + \frac{p}{m^2}\right)\mathrm{d}p + \int_y^{y+m} kp\left(\frac{1}{m} - \frac{1}{m^2}(p-y)\right)\mathrm{d}p\right)$$

If we solve the integrals, we obtain the following expression of the objective function:

$$\Pi = -h - k(i + jk) - \frac{k(y^3 - 3my^2 - 3m^2y - m^3)\mathrm{e}^{-rt}}{6m^2}$$

In order to make the coming results easier to interpret, we replace m by $(y + x)$. According to our earlier assumptions, since $(y - m) < 0$, we known that $x > 0$.

$$\Pi = -h - k(i + jk) - \frac{k(y^3 - 3(y+x)y^2 - 3(y+x)^2y - (y+x)^3)\mathrm{e}^{-rt}}{6(y+x)^2}$$

Let us simplify the objective function as much as possible:

$$\Pi = -h - k(i + jk) + \frac{k(x^3 + 6x^2y + 12xy^2 + 6y^3)\mathrm{e}^{-rt}}{6(x+y)^2}$$

The first order optimum condition is:

$$\frac{\delta\Pi}{\delta k} = -i - 2jk + \frac{(x^3 + 6x^2y + 12xy^2 + 6y^3)\mathrm{e}^{-rt}}{6(x+y)^2} = 0$$

From this, we may determine the optimal capacity investment level explicitly:

$$k^* = -\frac{\mathrm{e}^{-rt}(6i(x+y)^2\mathrm{e}^{rt} - x^3 - 6y(x^2 + 2xy + y^2))}{12j(x+y)^2}$$

It turns out that the second order maximum condition is satisfied. The optimum is a global and unique maximum!

$$\frac{\delta^2\Pi}{\delta k^2} = -2j < 0$$

The qualitative comparative statics results (the signs of the derivatives) are the same in the uniform and in the triangular cases. This is shown by the following six equations (and inequalities). Positive changes in the expected value of the net price or the net price risk, will increase the optimal investment level. Positive changes in r, t or i, will reduce the optimal investment level.

$$\frac{\delta k^*}{\delta y} = \frac{(2x^3 + 9x^2y + 9xy^2 + 3y^3)\mathrm{e}^{-rt}}{6j(x+y)^3} > 0$$

$$\frac{\delta k^*}{\delta x} = \frac{x^2(x + 3y)\mathrm{e}^{-rt}}{12j(x+y)^3} > 0$$

$$\frac{\delta k^*}{\delta r} = -\frac{t(x^3 + 6x^2y + 12xy^2 + 6y^3)\mathrm{e}^{-rt}}{12j(x+y)^2} < 0$$

$$\frac{\delta k^*}{\delta t} = -\frac{r(x^3 + 6x^2y + 12xy^2 + 6y^3)\mathrm{e}^{-rt}}{12j(x+y)^2} < 0$$

$$\frac{\delta k^*}{\delta i} = -\frac{1}{2j} < 0$$

$$\frac{\delta k^*}{\delta j} = \frac{i}{2j^2} - \frac{(x^3 + 6x^2y + 12xy^2 + 6y^3)e^{-rt}}{12j^2(x+y)^2} \gtreqless 0$$

The optimal objective function value is also affected in the same directions by changing parameter values as in the uniform case: Positive changes in the expected net price or the net price risk, will increase the optimal objective function value. Positive changes in r, t, i or j, will reduce the optimal objective function value.

$$\frac{\zeta\Pi^*}{\zeta y} = \frac{\delta\Pi^*}{\delta y} = \frac{k^*(2x^3 + 9x^2y + 9xy^2 + 3y^3)e^{-rt}}{3(x+y)^3} > 0$$

$$\frac{\zeta\Pi^*}{\zeta x} = \frac{\delta\Pi^*}{\delta x} = \frac{k^*x^2(x+3y)e^{-rt}}{6(x+y)^3} > 0$$

$$\frac{\zeta\Pi^*}{\zeta r} = \frac{\delta\Pi^*}{\delta r} = -\frac{k^*t(x^3 + 6x^2y + 12xy^2 + 6y^3)e^{-rt}}{6(x+y)^2} < 0$$

$$\frac{\zeta\Pi^*}{\zeta t} = \frac{\delta\Pi^*}{\delta t} = -\frac{k^*r(x^3 + 6x^2y + 12xy^2 + 6y^3)e^{-rt}}{6(x+y)^2} < 0$$

$$\frac{\zeta\Pi^*}{\zeta i} = \frac{\delta\Pi^*}{\delta i} = -k^* < 0$$

$$\frac{\zeta\Pi^*}{\zeta j} = \frac{\delta\Pi^*}{\delta j} = -k^{*2} < 0$$

5. CAPACITY INVESTMENT AND PRODUCTION IN THE FOREST INDUSTRY ENTERPRISE

Of course, it is nice to be able to handle all of the capacity investment questions analytically. Most of the questions could be answered with explicit formulae and signs. Unfortunately, many real world capacity investment problems are more complicated. Such a situation will be discussed below:

In Figure 1 we find a forest industry enterprise. It is active in the following way: The wood used as raw material comes from the own forest stock (H = harvest) and from the roundwood market (B = buy). The raw material is stored at the mill in the roundwood stock. The raw material is processed (M = manufacturing) and enters the product stock. Finally, the product (pulp, sawn wood etc.) is sold (SA = sales).

The enterprise which is discussed here and the precise economic assumptions are found in the optimization program subroutine in the numerical appendix. Since the details of the model do not help the reader to understand the fundamental concepts under discussion in this section and since the details cannot be expected to be relevant in other enterprises, they are not presented in the main text.

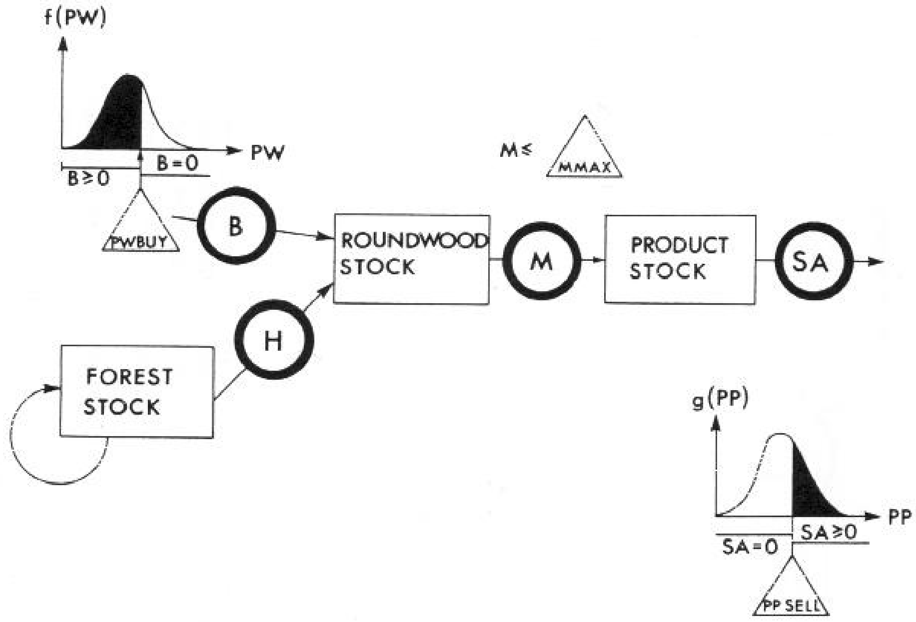

Figure 1 A forest industry enterprise. The definitions are found in the main text.

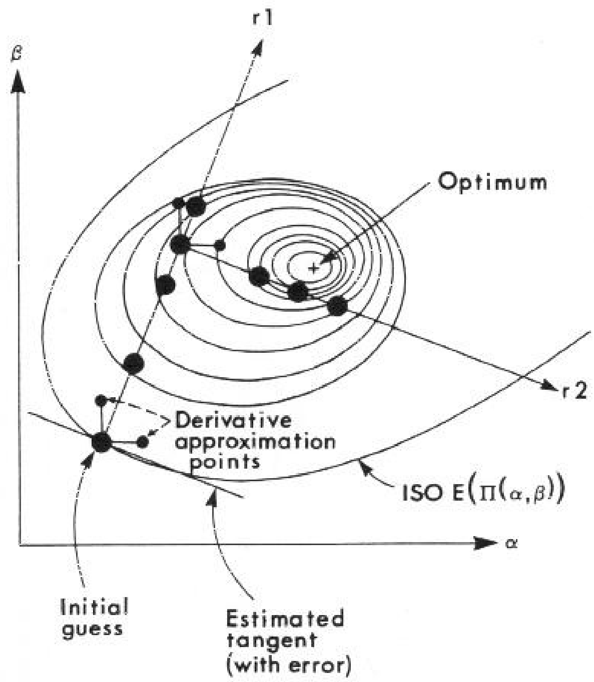

Figure 2 The hill climbing method used in the numerical optimization of the adaptive control parameters. α and β denote two such parameters (in the problem, there are three parameters, namely PPSELL, PWBUY and MMAX. The black points are positions in the control function parameter space where the objective function, $E(\Pi)$, the expected optimal present value, is estimated via large numbers of stochastic full system simulations. You start by estimating the tangent of the iso objective function curve. Then, you move "up the hill" using the bisection method discussed in Figure 3. When you can not improve the solution more in that direction, you estimate the tangent again and move further in the new direction. Finally, you approach the top of the mountain.

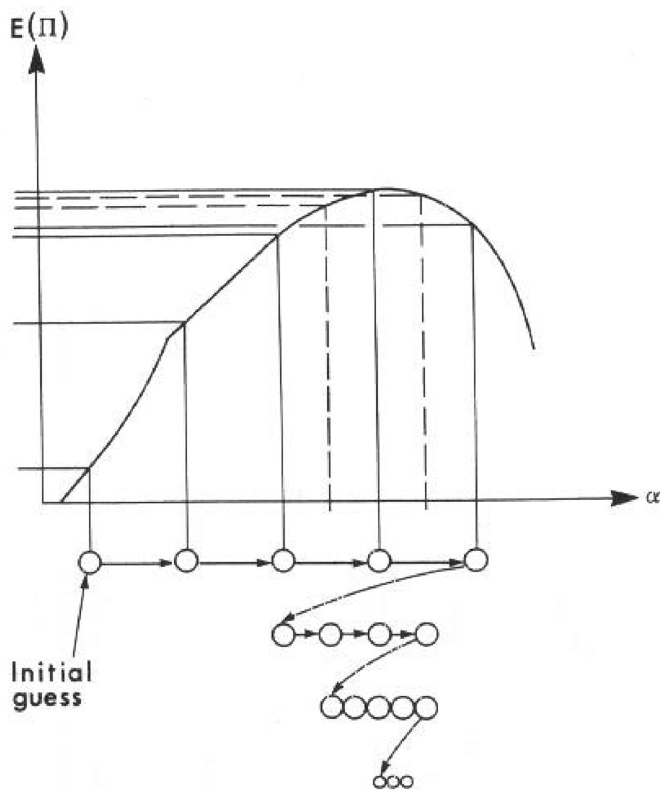

Figure 3 The bisection method illustrated in one parameter direction, α. You continue in the suggested direction as long as the latest function value is better than the second latest function value. If this is not the case, then you take two steps backward and reduce the step length by 50%. Then you continue in the first direction. Note that the method converges to the global maximum if the objective function is strictly quasi concave also if it contains convex segments. The Newton–Raphson method and/or other methods based on second order derivatives often fail in such cases.

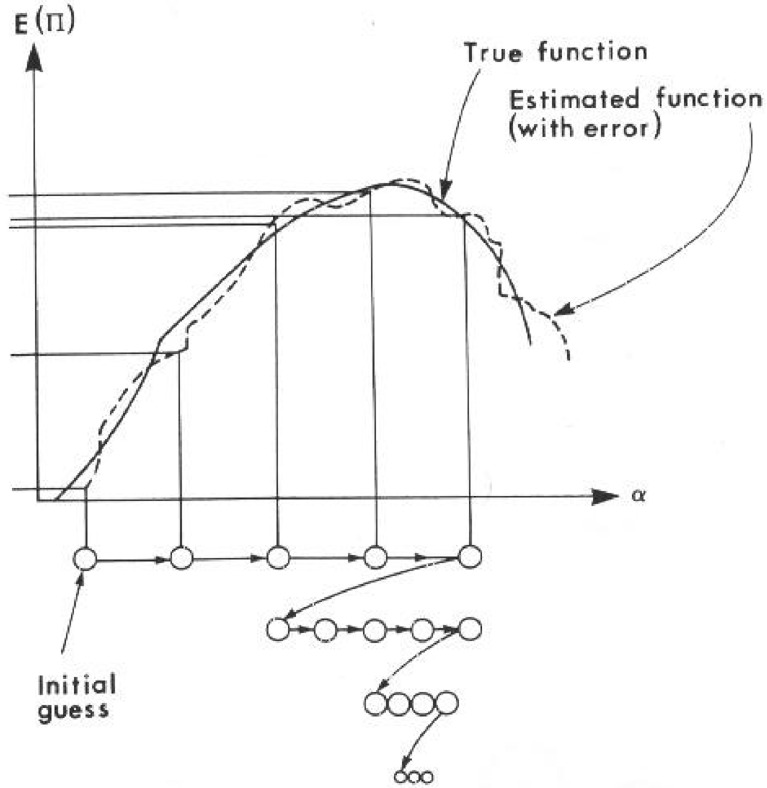

Figure 4 We should be aware that Figure 3 shows an ideal case. Since we are dealing with stochastic systems, we can not expect to get perfect estimates of the objective function value. Hence, even if the true objective function is smooth and strictly quasi concave, we will observe something else. Note that the bisection method works well also in many such cases. If the initial step length is large, the probability is low that we are trapped in local optima (that are only artificial optima caused by estimation errors) far away from the true optimum.

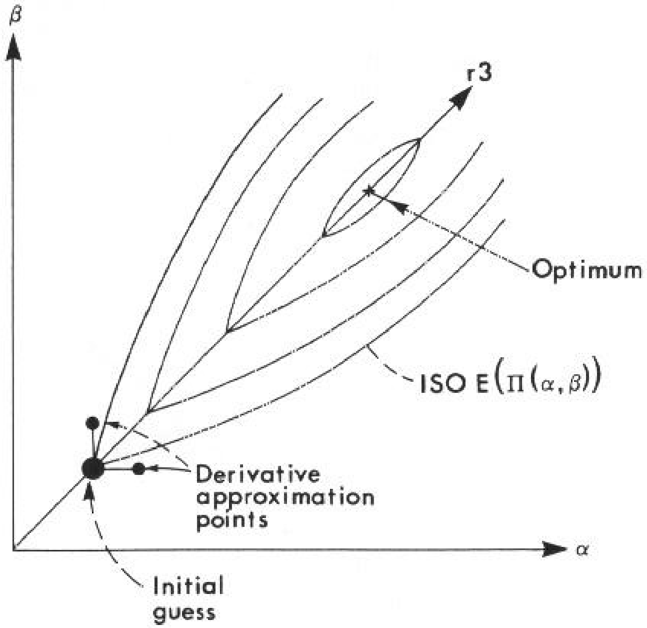

Figure 5 In some problems, not in the particular problem under study here, the objective function may have the shape shown in this graph. One such case is when α and β denote capacity levels of different machines that are arranged in a series. (A product must most of the time first be processed in one machine and then in the other.) Then, if capacity expansion should be profitable, both capacity levels must be increased simultaneously. One way to improve the speed of convergence in such cases is to introduce a new variable, Γ. Then, we optimize the variables α and Γ. $\beta = \alpha * \Gamma$.

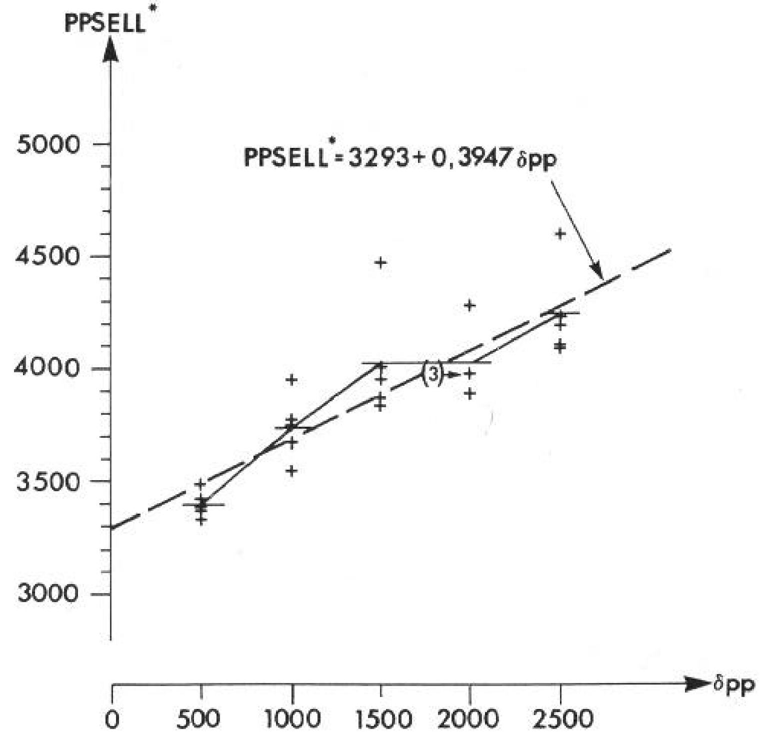

Figure 6 The sales reservation price PPSELL as a function of the standard deviation of the product price, PP. The function is estimated from 25 different optimizations of the parameter PPSELL. The individual optimization results are shown in the graph. Obviously, it is optimal to be more product price selective and to wait for higher prices, if the price variation increases. We may call this "product price speculation".

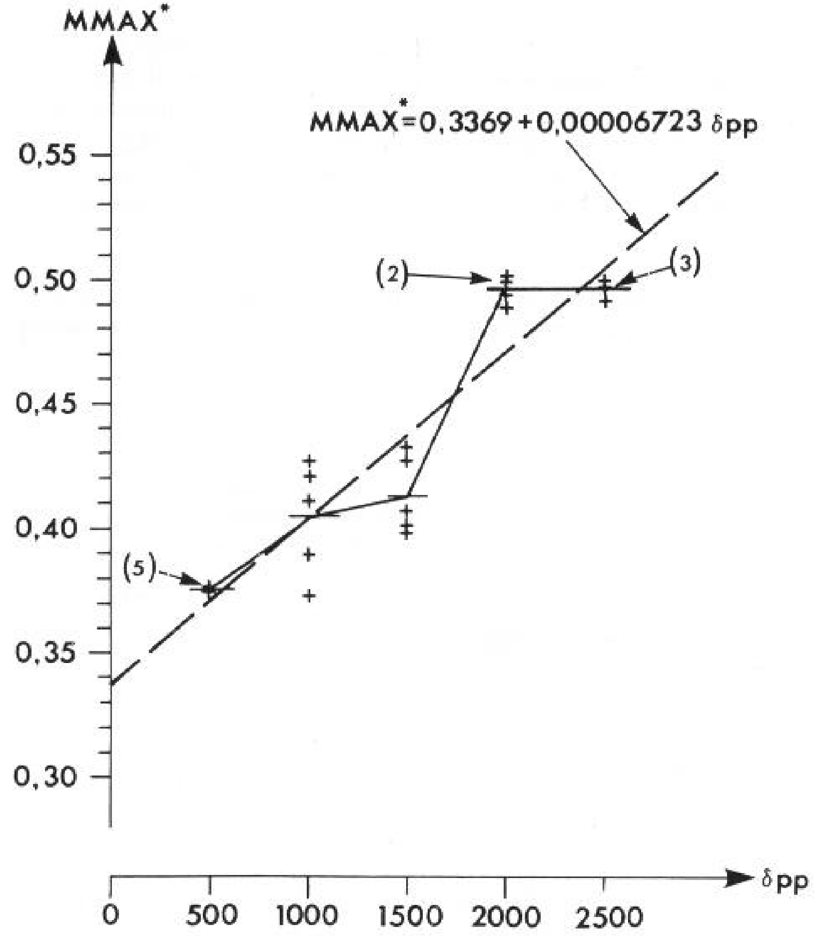

Figure 7 The optimal production capacity MMAX as a function of the standard deviation of the product price, PP. Compare Figure 6. If the product price variation increases, we should increase the production capacity. Clearly, it is more important than in a low risk case to be able to respond rapidly to good price options. Compare the similar analytical results reported in this paper.

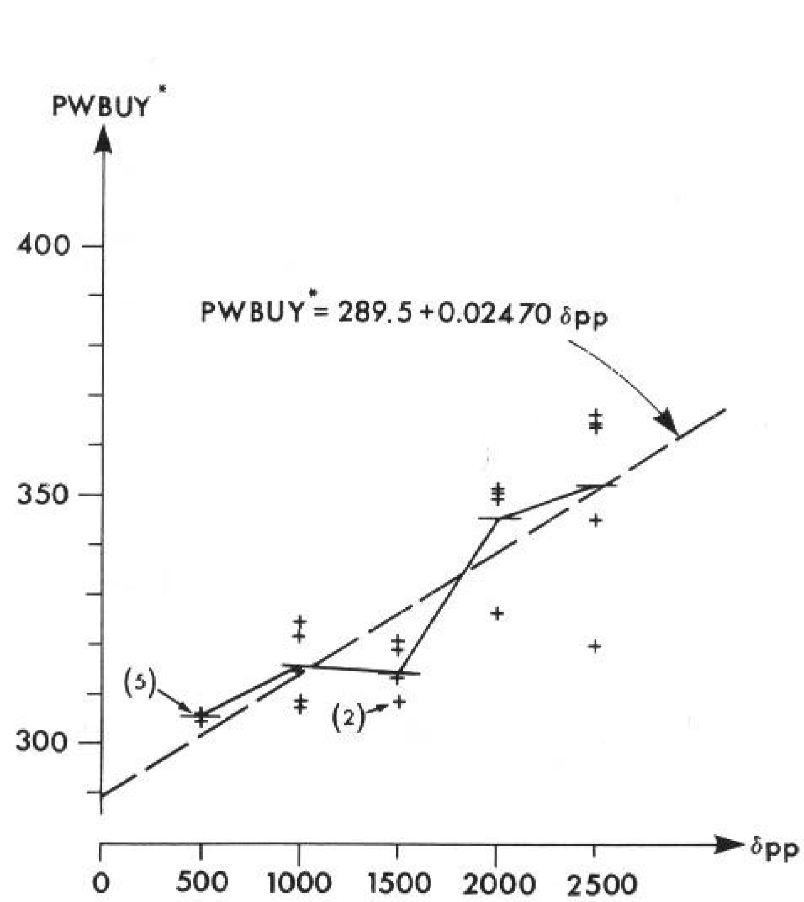

Figure 8 The optimal roundwood reservation price PWBUY as a function of the standard deviation of the product price, PP. Compare Figure 6. We should accept higher roundwood prices if the product price risk increases. It is more important to be able to deliver products if there are sometimes very good product prices than otherwise.

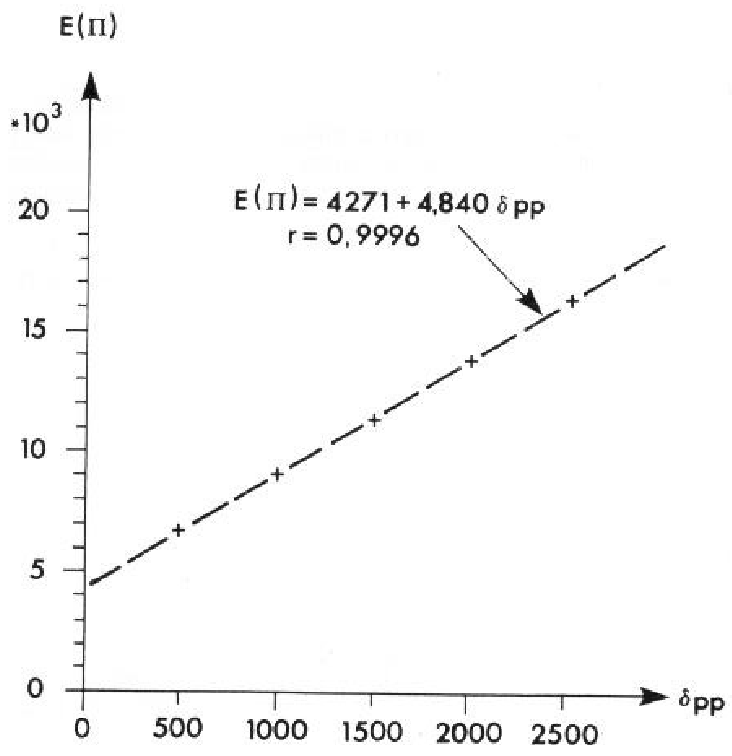

Figure 9 The expected optimal present value as a function of the standard deviation of the product price, PP. The higher the product price risk, the higher is the expected present value. Similar analytical results have been derived earlier in this paper. The values of PPSELL, MMAX and PWBUY have been determined from the equations found in Figures 6, 7 and 8. Note that we benefit from the good price periods and do not loose very much during the low price periods (since we do not sell then).

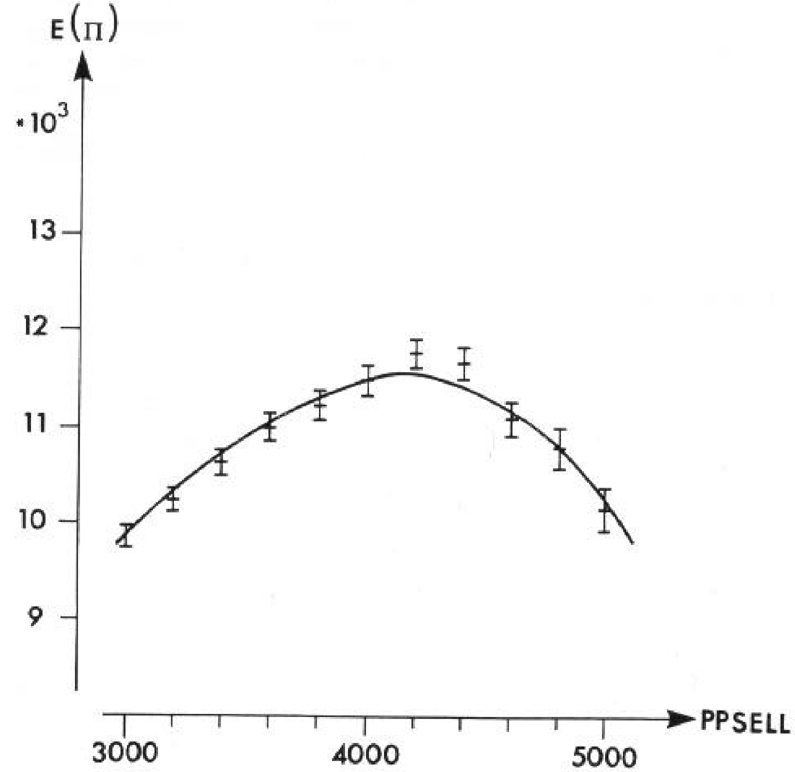

Figure 10 A one dimensional investigation of the objective function with a large number of stochastic simulations with new pseudo random numbers. The estimation errors (one standard deviation) are marked in the graph. There seems to be a unique maximum in this parameter direction. This optimum is close to the optimum found via the optimized equation shown in Figure 6. A complete numerical three dimensional parameter investigation is too complicated in this problem. An analytical test of the objective function concavity is practically impossible. The standard deviation of PP is 1500. MMAX and PWBUY are determined from the optimized equations.

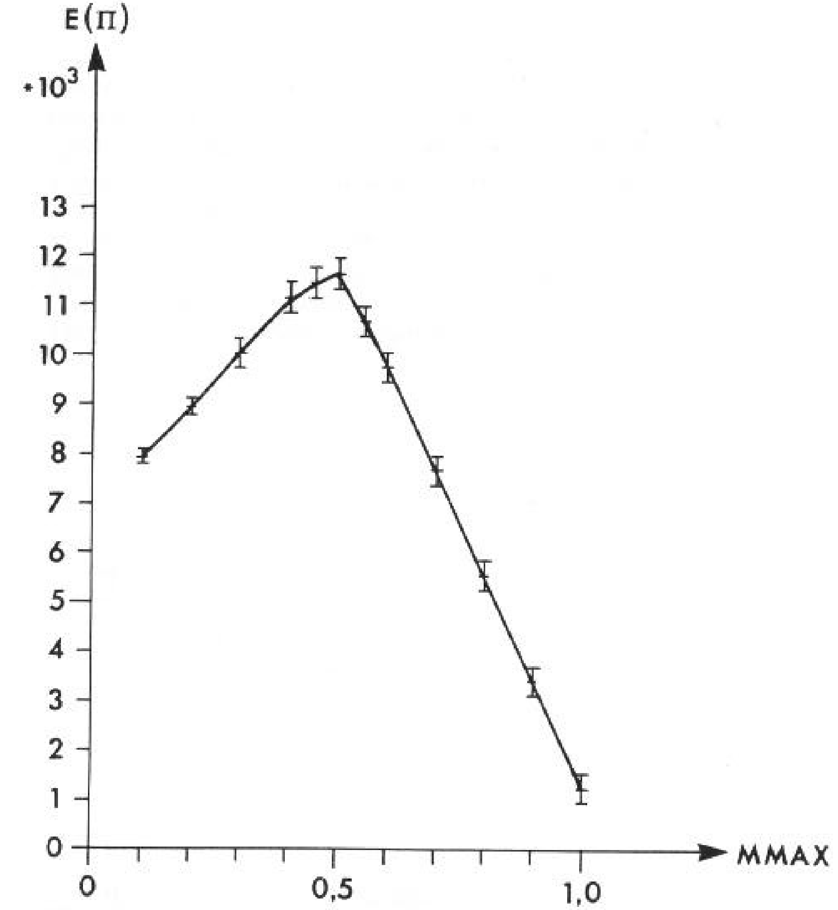

Figure 11 A one dimensional investigation in the capacity dimension. The test is made in the same way as in Figure 10. There seems to be a unique maximum in the production capacity dimension.

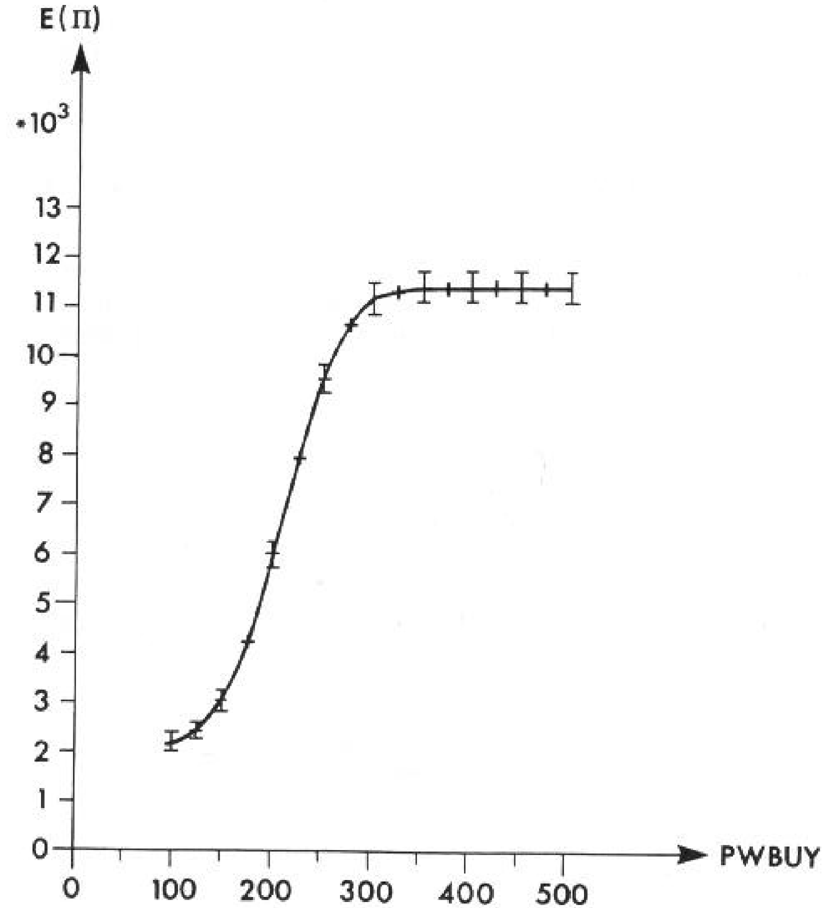

Figure 12 A one dimensional investigation in the roundwood reservation price dimension. The test is made in the same way as in Figure 10. There seems to be no unique optimum in this graph. This is most likely an effect of the fact that very high roundwood prices do not occur in the stochastic simulations because of the random number parameter selection. In any case, the value of PWBUY suggested in Figure 8 as a result of the optimization, is optimal (but not uniquely optimal) also according to this graph.

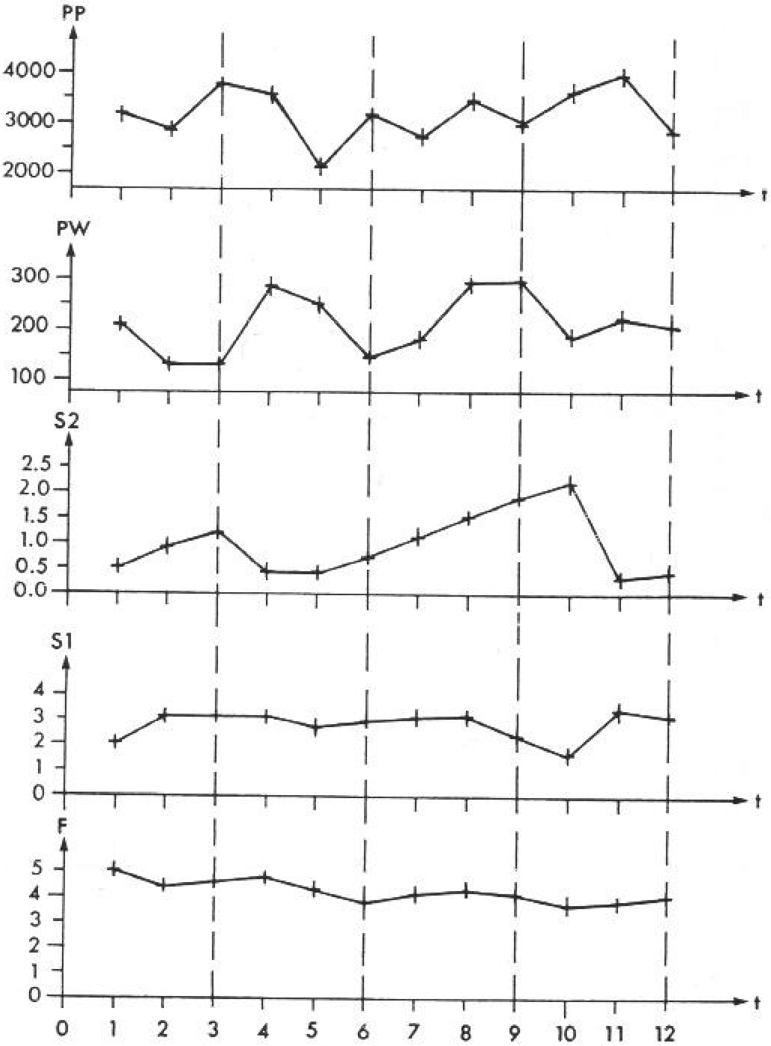

Figure 13 Five time series of state variables. The time unit is one month. One year is shown in the graphs. PP = product price, PW = roundwood price, S2 = product stock, S1 = roundwood stock, F = forest stock. PP and PW are exogenous stochastic processes. S2, S1 and F are adaptively controlled. Note that the product stock decreases only when the product price is very high. Hence, the product stock varies considerably over the year, with a stochastic "saw-tooth pattern".

The economic environment of the enterprise can be described the following way: The product price (PP) is exogenous to the enterprise. It is determined by the world market and contains dramatic fluctuations. It may be described as a stochastic process. The roundwood price (PW) is determined by the roundwood market where the enterprise is only a marginal buyer. Also this price can be described as a stochastic process. PP and PW may or may not be strongly correlated. According to Lohmander (1991b) the correlation is sometimes close to zero.

The objective function is the expected present value. Let us discuss some of the important economic decisions taken by the enterprise. First of all, we should optimize the manufacturing capacity level. Note that it is not always easy to define manufacturing capacity in a practical way. A factory is often a complicated

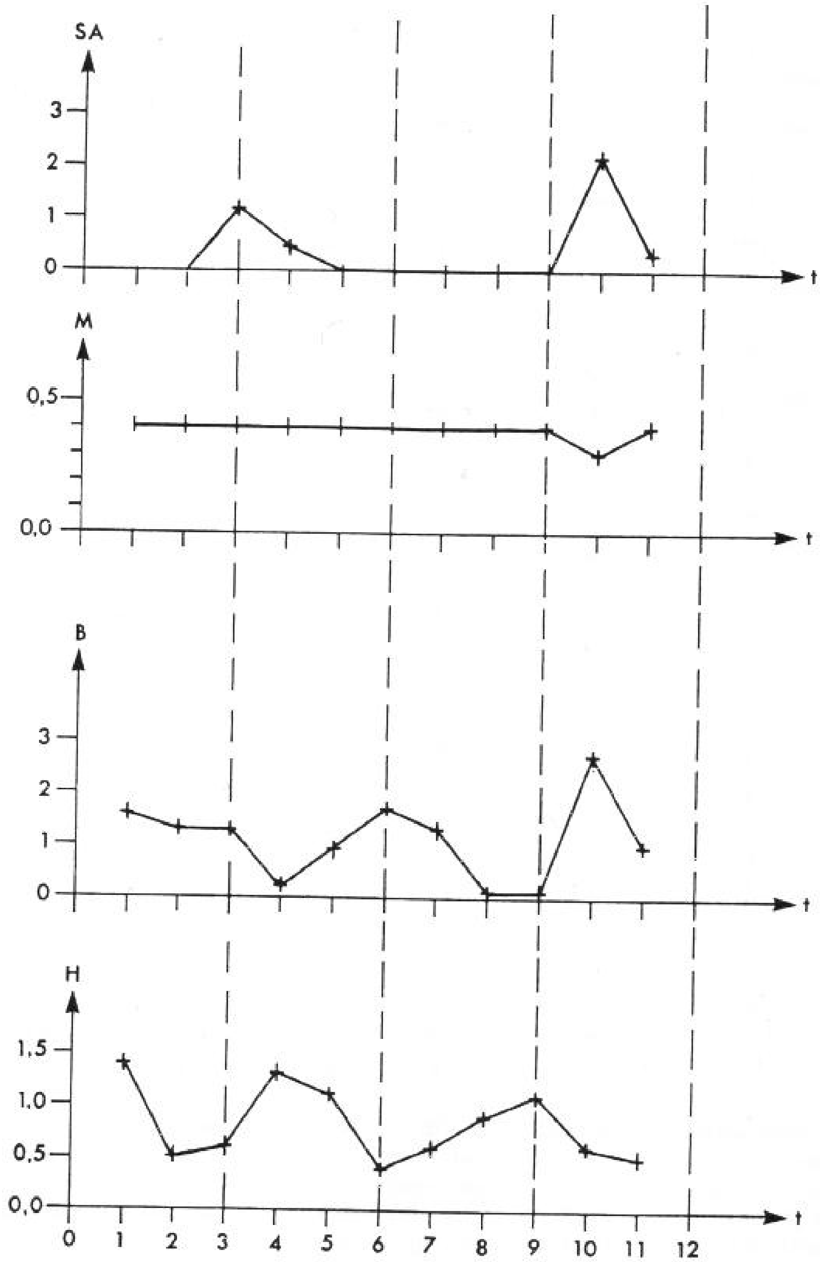

Figure 14 Four time series of adaptive control decisions. The series belong to the same example as the series shown in Figure 13. SA = product sales, M = manufacturing (production), *B* = amount of roundwood bought, H = harvest. We note that the sales is a highly discontinuous activity. Production, on the other hand, is almost constant over time. B and H take care of the roundwood consumption, which is almost constant, thanks to the stable production. Hence, B and H are negatively correlated. B is negatively correlated to the roundwood price PW. As a consequence, H is positively correlated to the roundwood price (even if no wood is sold from the enterprise)!

unit containing several different kinds of buildings, machines, labour and transportation facilities. In any case, we assume that we can compress the capacity level into one variable, MMAX.

However, MMAX cannot be optimized without explicitly taking the other decisions into consideration. We let MMAX be a variable which is held constant during a time interval, namely from the point in time when the plant starts to

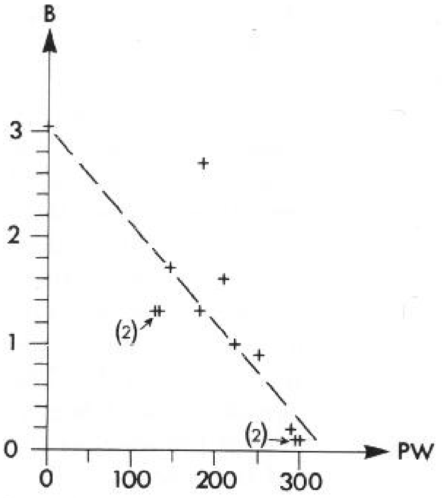

Figure 15 The roundwood demand function of the enterprise as estimated by an observer who just looks at the observations of the year shown in Figures 13 and 14. The function has the traditional slope.

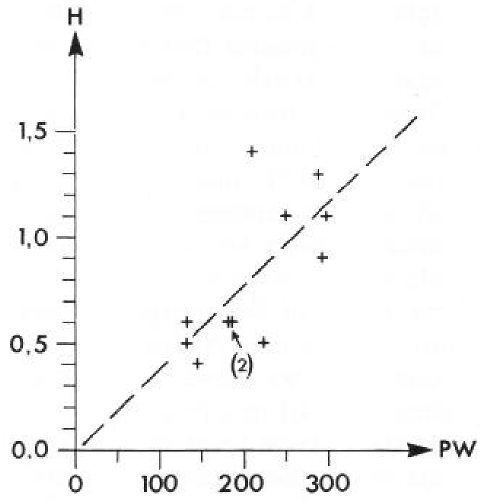

Figure 16 The harvest function of the enterprise. Compare Figure 15.

function until the horizon of our calculations. In the numerically defined model, we assume that the horizon is one year and that each period is one month.

Let us introduce a variable to optimize, PPSELL. This is the lowest product price that we accept in order to sell from the product stock to the market. If the revealed product price happens to be above PPSELL, then we sell everything that we have in the product stock. Otherwise, we sell nothing and the product stock level increases until the next month. Note that we implicitly assume that the "bang-bang" sales policy (sell everything or nothing, depending on the price) is

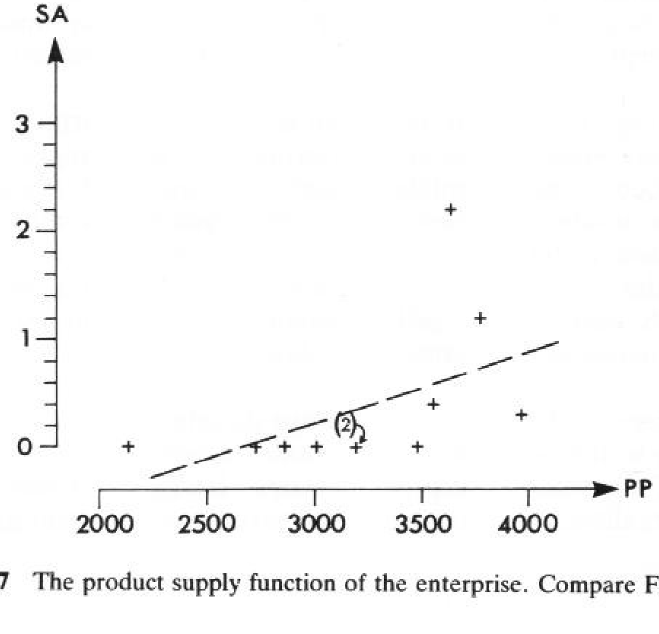

Figure 17 The product supply function of the enterprise. Compare Figure 15.

optimal. This is not obviously true. Depending on the "sales problem environment", it is possible that more "smooth" sales strategies are optimal, in particular if there are strictly convex cost functions involved.

Finally, we introduce PWBUY. PWBUY is the maximum price that we accept if we should buy roundwood from the market, a kind of reservation price. Of course, PWBUY must be optimized. We should be aware that more complicated input market strategies may turn out to be more economical than the pure reservation price strategy. Maybe we should also take the stock level of finished products into account when we decide how much wood to buy? We always have to be aware of the marginal costs and marginal revenues of more detailed and rapid information. At least, the amount that we buy should be a function of the amount that we have in the rounwood stock and the harvest level (H). The precise form of this relationship may be discussed in every specific case. Clearly, when the harvest level should be determined, we have to consider the level of the forest stock, the roundwood market price and the roundwood stock level.

A practical way to handle these questions is to introduce an internal raw material market with a roughly approximated stock dependent "shadow price function". This is done in the numerical model and the details can be found in the algorithm.

The reader should be aware that Figure 1 could be interpreted as an enterprise from some other sector of the economy, if the notation is changed accordingly. The specific notation will however be useful since it makes it easier to explicitly discuss different properties of the enterprise and the optimization problem.

6. DISCUSSION

In this paper, analytical and numerical methods to handle the typical economical optimization problems of the enterprise have been presented. It was found that analytical methods may be used in some special cases but that much more realistic and detailed enterprise problems could be handled via a stochastic quasi gradient method.

The suggested and introduced algorithm is a gradient method, "hill climbing", where the bisection method is used to determine the step length. In the objective function estimations, the complete adaptively controlled stochastic system is simulated a large number of times. Hence, the algorithm is not very fast. On the other hand, the size of the needed computer memory is not very large. A typical optimization of the enterprise example takes 30 minutes on a 386 computer.

Other methods such as stochastic dynamic programming were initially discussed. The dimensionality problem is however often strongly reducing the applicability.

The author suggests that future efforts are directed towards the application of the methodology to real world enterprise optimizations. Presently, one such study is undertaken of a sawmill enterprise by Roger Berggren. There are preliminary indications that realistic and practically interesting management guidelines will be obtained.

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NUMERICAL APPENDIX

General

This appendix contains the computer program which has been used in the optimization of the adaptive decision rules (IADAC8.BAS). The stochastic enterprise problem is defined in detail in the objective function subroutine of the optimization program. There is also a computer program which is used to simulate the adaptively controlled system (ISIM5.BAS). Both programs are written in QuickBasic, which is a modern version of standard basic. QuickBasic is very similar to Fortran. The programs may be executed on a personal computer with DOS. Examples of input and output files, screens and command files are included.

Contents *page*

```
REM ************************************************************
REM IADAC8.BAS
REM LOHMANDER PETER 92-03-25, 00.27
REM PROGRAM FOR STOCHASTIC QUASIGRADIENT OPTIMIZATION OF SEVERAL
REM ADAPTIVE FOREST INDUSTRY ENTERPRISE DECISIONS.
REM ************************************************************

REM -----------------------------------------------------------------
REM The objective is to find the adaptive decision rules
REM that maximize the expected present value of all activities over
REM time. A complete Markov decision treatment with linear programming
REM and/or policy iteration is not possible since the state space in
REM that case must be discrete and becomes very large and numerically
REM difficult to handle if a high state space resolution is needed.
REM The problem with the Newton Raphson method in this application
REM is that the objective function is not everywhere
REM two times continuously differentiable with respect
REM to all variables of interest.
REM -----------------------------------------------------------------

CLS
REM RANDOMIZE

REM *********************************************************
REM SECTION 1. Dimensions, openings, inputs and definitions.
REM *********************************************************

DEFDBL A-H, O-Z
DIM X(10), XBEST(10), FX(10), DX(10), XOLD(10), XCHANGE(10)
DIM Y(1, 100), U(1, 100), PRO(100), DISC(100)
DIM XTOT(10, 10), AVERS(10), AVERH(10)
DIM PP(12, 30), PW(12, 30), HC(12, 30), UTSTATE(12, 4)
DIM UTDEC(12, 4), UTPARA(12, 4)

OPEN "INIAD8.DAT" FOR INPUT AS #1
OPEN "UTIAD8.DAT" FOR OUTPUT AS #2

CLS
PRINT "YOU NOW ENTER THE ADAPTIVE FOREST INDUSTRY "
PRINT "ENTERPRISE CONTROL FUNCTION OPTIMIZER"
REM BEEP
PRINT "by Peter Lohmander 1992"
PRINT "": PRINT ""
REM INPUT "If you want detailed state and decision tables, type 1. ", UTTAB
UTTAB = 0
REM BEEP
PRINT "----------------------------------------------------"
REM INPUT "If you want detailed iteration information, type 1. ", UTITER
UTITER = 0
PRINT ""
INPUT #1, INFOTEX$
INPUT #1, AUTH$
INPUT #1, TWHEN$
PRINT DATE$, TIME$
PRINT #2, DATE$, TIME$
PRINT TWHEN$

REM -------------------------------------------------------------
REM The number of parameters to be optimized, IXMAX, is determined.
REM -------------------------------------------------------------
INPUT #1, IXMAX, T$
IMAX = IXMAX
JMAX = IMAX + 1
```

```
REM ----------------------------------------------------------------
REM The initial values of the variables to be optimized are determined.
REM ----------------------------------------------------------------
INPUT #1, X(1), T$
INPUT #1, X(2), T$
INPUT #1, X(3), T$

REM ----------------------------------
REM The rate of interest is determined.
REM ----------------------------------
INPUT #1, RATEINT, T$
R = RATEINT / 100

REM ------------------------------------------------------------
REM STA = step size when the partial derivatives are approximated.
REM ------------------------------------------------------------
INPUT #1, STA, T$

REM -------------------------------------------------------
REM DIST0 = Initial step size before reduction via bisection.
REM -------------------------------------------------------
INPUT #1, DIST0, T$

REM -----------------------------------------------
REM The lenght of each period (months) is determined.
REM -----------------------------------------------
INPUT #1, PERL, T$

REM ---------------------------------------
REM The time horizon is determined (months).
REM ---------------------------------------
INPUT #1, TMAX, T$

REM ----------------------------------------------------------------
REM The initial state is determined. F = wood resource in the forest,
REM S1 = raw materials stock and S2 = stock of products at the industry.
REM ----------------------------------------------------------------
INPUT #1, F0, T$: UTSTATE(1, 1) = F0
INPUT #1, S10, T$: UTSTATE(1, 2) = S10
INPUT #1, S20, T$: UTSTATE(1, 3) = S20

REM -------------------------------------------------------------
REM Distribution function parameters of wood price, harvest cost and
REM product price.
REM -------------------------------------------------------------
INPUT #1, PWMEAN, T$
INPUT #1, PWSTDEV, T$
INPUT #1, HCMEAN, T$
INPUT #1, HCSTDEV, T$
INPUT #1, PPMEAN, T$
INPUT #1, PPSTDEV, T$

REM -----------------------------------------------
REM Industrial stock and production capacity levels.
REM -----------------------------------------------
INPUT #1, S1MAX, T$
INPUT #1, S2MAX, T$
REM INPUT #1, MMAX, T$
REM THE PRODUCTION CAPACITY LEVEL IS NOW DETERMINED ENDOGENOUSLY.
INPUT #1, CMMAX, T$

REM -------------------------------------------------------------
REM The number of experiments and the number of price vector series
```

```
REM (complete histories) in each experiment are determined.
REM ---------------------------------------------------------------
INPUT #1, NUMEX, T$
INPUT #1, NUMPRI, T$
PRINT "NUMPRI = "; NUMPRI; T$

REM ************************************************************
REM SECTION 2. The experiment loop starts here. NUMEX experiments
REM            will take place where each experiment has its own
REM            set of NUMPRI stochastic price series. For each
REM            set of price series, the harvest function is
REM            optimized. The price series are generated from an
REM            application of the central limit theorem. They
REM            are approximately N(PAV,STDEV^2). Compare Rde and
REM            Westergren (1988) pg. 316.
REM ************************************************************

FOR EPER = 1 TO NUMEX

REM ---------------------------------------------
REM Here, the stochastic price series are calculated.
REM ---------------------------------------------
FOR T = 1 TO TMAX
FOR NPRI = 1 TO NUMPRI

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
PW(T, NPRI) = PWMEAN + (PRAN - 6) * PWSTDEV

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
HC(T, NPRI) = HCMEAN + (PRAN - 6) * HCSTDEV

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
PP(T, NPRI) = PPMEAN + (PRAN - 6) * PPSTDEV

NEXT NPRI
DISC(T) = EXP(-R * T * PERL / 12)
NEXT T

REM -------------------------------------
REM The start values are saved in XOLD(.).
REM -------------------------------------
FOR I = 1 TO IXMAX: XOLD(I) = X(I): NEXT I

REM ------------------------------------------------------------------
REM The number of step directions, N, the "best objective function
REM value", PIO, and the "latest objective function value", AOBJ, are
REM given initial values that make the iteration leave the initial guess.
REM ------------------------------------------------------------------
N = 0
PIO = 0
AOBJ = -1000000


430 REM ----------------------------------------------------------
REM The "latest objective function value", AOBJ, is calculated in the
REM subroutine where the complete dynamical system is included.
REM ----------------------------------------------------------------
GOSUB 2190

REM ----------------------------------------------------------------
REM If the latest solution is better than the old solution, then we
REM move on to 422 and test a new direction and more movements from the
REM latest solution.
REM ----------------------------------------------------------------
```

```
420 IF PIO < AOBJ THEN GOTO 422

REM -----------------------------------------------------------------
REM Now, the old solution clearly was not worse than the old solution.
REM Hence, we accept the solution and move on the the next experiment.
REM -----------------------------------------------------------------
423 NEXT EPER

REM -------------------------------------------------------------------
REM Now, all experiments have been completed. It is time to send the
REM solutions to the result file UTADG.DAT. This is done from line 1000.
REM -------------------------------------------------------------------
421 GOTO 1000

422 REM
N = N + 1
REM -----------------------------------------------------------------
REM If more than 200 directions have been tested in this experiment,
REM then the iterations are stopped and the next experiment is started.
REM -----------------------------------------------------------------
IF N > 200 THEN GOTO 423
STELOC = 0
CLS

REM ********************************************************
REM SECTION 3. The latest solution is printed on the screen.
REM ********************************************************

PRINT "***************************************************************"
PRINT "EXPER = "; EPER;
PRINT " N = "; N; " interest = "; RATEINT; " TMAX = "; TMAX; "PERL = "; PERL
PRINT "PRICE VECTOR SERIES IN EACH EXPERIMENT = "; NUMPRI
PRINT "S1MAX, S2MAX, MMAX = "; S1MAX; S2MAX; MMAX
PRINT "PARAM. (PW, HC, PP) = "; PWMEAN; PWSTDEV;
PRINT HCMEAN; HCSTDEV; PPMEAN; PPSTDEV

PRINT ""

PRINT "   VARIABLE      VALUE     CHANGE   REL. CH."

FOR IX = 1 TO IXMAX
XCHANGE(IX) = X(IX) - XOLD(IX)
RECH = 0
IF N = 1 THEN GOTO 77
RECH = XCHANGE(IX) / X(IX)
77 PRINT USING "#####.#####"; IX; X(IX); XCHANGE(IX); RECH
XTOT(EPER, IX) = X(IX)
XOLD(IX) = X(IX)
NEXT IX
GOSUB 2190
PIO = AOBJ
PROFBEST = PIO
PRINT ""
PRINT " ****** OBJECTIVE FUNCTION VALUE = "; PROFBEST; " ******"
PRINT ""

REM *****************************************************************
REM SECTION 4. THE PARTIAL DERIVATIVES OF THE OBJECTIVE FUNCTION
REM            ARE CALCULATED AND STEPS ARE TAKEN IN THE MOST
REM            "PROFITABLE" DIRECTION.
REM *****************************************************************

REM -----------------------------------------------------------------
REM The partial derivatives of the objective function are calculated.
```

```
REM ------------------------------------------------------------
IF UTITER = 1 THEN PRINT "-------NEW ITERATION DIRECTION ------------"
FOR IX = 1 TO IXMAX
X(IX) = X(IX) + STA
GOSUB 2190
PI1 = AOBJ
X(IX) = X(IX) - STA
FX(IX) = (PI1 - PI0) / STA
IF UTITER = 1 THEN PRINT "ix, fx(ix) = "; IX; FX(IX)
NEXT IX
IF UTITER = 1 THEN PRINT ""

REM ----------------------------------------------------------
REM The distance PRELDIS is the Euclidian distance of the movement
REM if all parameters (variables) are moved the distances equal
REM to the partial derivatives of the objective function to
REM the respective parameters (variables).
REM ----------------------------------------------------------
PRELDIS = 0
FOR IX = 1 TO IXMAX
PRELDIS = PRELDIS + FX(IX) * FX(IX)
NEXT IX
PRELDIS = PRELDIS ^ .5
IF UTITER = 1 THEN PRINT "preldis = "; PRELDIS
IF UTITER = 1 THEN PRINT ""
IF PRELDIS < .0001 THEN PRINT "STEP=0"

REM ------------------------------------------------------------
REM Determination of the optimal step length in the suggested direction
REM via a modified bisection method.
REM ------------------------------------------------------------
PROMDIR = PI0
DIST = DIST0

7 REM
STELOC = STELOC + 1

REM ------------------------------------------------------------
REM If we have taken more than 100 steps in the suggested direction, some
REM numerical problem may be present or we may have an objective function
REM that is not locally concave. We keep the latest objective function
REM value and leave the local steps and iterations.
REM ------------------------------------------------------------
IF STELOC > 100 THEN GOTO 430

REM ----------------------------------------------------
REM If the steps are still sufficiently large, go to line 20.
REM ----------------------------------------------------
IF DIST > (DIST0 / 3000) THEN GOTO 20

REM ------------------------------------------------------------
REM Now, the steps are too short. The old solution is selected and we
REM finally leave the iterations.
REM ------------------------------------------------------------

FOR IX = 1 TO IXMAX
X(IX) = XBEST(IX)
NEXT IX

GOTO 430

20 REM ---------------------------------------------------------
REM The different parameters (variables), X(IX), are changed in the
REM suggested direction in a way that makes the Euclidian distance of
```

```
REM the movement equal to the step length.
REM ----------------------------------------------------------------
FOR IX = 1 TO IXMAX
DX(IX) = FX(IX) * DIST / PRELDIS
NEXT IX

FOR IX = 1 TO IXMAX
X(IX) = X(IX) + DX(IX)
IF UTITER = 1 THEN PRINT "IX = "; IX; "  X(IX) = "; X(IX)
NEXT IX

GOSUB 2190
PROMEV = AOBJ
IF UTITER = 1 THEN PRINT "OBJECTIVE FUNCTION = "; PROMEV
IF UTITER = 1 THEN PRINT ""
IF PROMEV > PROMDIR THEN GOTO 8

FOR IX = 1 TO IXMAX
X(IX) = X(IX) - 2 * DX(IX)
NEXT IX

DIST = DIST / 2
GOSUB 2190
PROMDIR = AOBJ
GOTO 7

8 PROMDIR = PROMEV
IF PROMDIR <= PROFBEST THEN GOTO 7
PROFBEST = PROMDIR
FOR IX = 1 TO IXMAX
XBEST(IX) = X(IX)
NEXT IX

GOTO 7
END

1000 REM ***********************************************************
REM      SECTION 5. RESULTS ARE SENT TO THE OUTPUT FILE UTIAD.DAT
REM ***************************************************************

PRINT #2, "FILE = UTIAD8.DAT"
PRINT #2, "RESULTS FROM IADAC8.BAS"
PRINT #2, TWHEN$
PRINT #2, "NUMBER OF EXPERIMENTS = "; NUMEX
PRINT #2, "VECTOR PRICE SERIES IN EACH EXPERIMENT = "; NUMPRI
PRINT #2, "INTEREST RATE = "; RATEINT; " NUMBER OF PERIODS = "; TMAX
PRINT #2, "LENGTH OF EACH PERIOD = "; PERL; " DIST0 = "; DIST0
PRINT #2, "INITIAL FOREST STOCK = "; F0
PRINT #2, "INITIAL RAW MATERIALS STOCK = "; S10
PRINT #2, "INITIAL PRODUCT STOCK = "; S20
PRINT #2, "MEAN WOOD PRICE = "; PWMEAN; " AND ST. DEV. = "; PWSTDEV
PRINT #2, "MEAN HARVEST COST = "; HCMEAN; " AND ST. DEV. = "; HCSTDEV
PRINT #2, "MEAN PROD. PRICE = "; PPMEAN; " AND ST. DEV. = "; PPSTDEV
PRINT #2, "MAXIMUM RAW MATERIALS STOCK CAPACITY = "; S1MAX
PRINT #2, "MAXIMUM PRODUCT STOCK CAPACITY = "; S2MAX
PRINT #2, "MARGINAL PROD. CAP. COST PER PERIOD = "; CMMAX

PRINT #2, ""
PRINT #2, "MAXIMUM OBJECTIVE FUNCTION VALUE = "; PROFBEST
PRINT #2, "THE OPTIMIZED ADAPTIVE CONTROL FUNCTION PARAMETERS ARE: "
FOR EPER = 1 TO NUMEX
PRINT #2, "EXPERIMENT = "; EPER
PRINT #2, "PARAMETER LIST = ";
PRINT #2, "( = PPSELL, PWBUY*10, OPTMMAX*10000) "
```

```
PRINT #2, ""
FOR IX = 1 TO IXMAX
PRINT #2, USING "#######.#####"; XTOT(EPER, IX);
NEXT IX
PRINT #2, ""

NEXT EPER

PRINT #2, ""
PRINT #2, "************** END OF LIST *****************"
CLOSE #2

IF SGNL = 0 THEN END
FOR I = 1 TO 2: SOUND (1000 + I * 50), 5
SOUND (3000 - I * 50), 5: NEXT I

END

2190 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM HERE, THE OBJECTIVE FUNCTION, AOBJ, IS "WRITTEN".
REM NOTE THAT THE "OBJECTIVE FUNCTION" MUST BE CALCULATED VIA THE
REM SIMULATION OF THE COMPLETE SYSTEM DURING A LONG TIME PERIOD!
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS

PROFIT = 0

REM ------------------------------------------------------------------------
REM The calculations are repeated for each price series in the experiment.
REM ------------------------------------------------------------------------
FOR NPRI = 1 TO NUMPRI

REM ------------------------------
REM The parameters are determined.
REM ------------------------------
PPSELL = X(1)
PWBUY = X(2) / 10
MMAX = X(3) / 10000

REM ----------------------------------------------
REM The path and the objective function of the
REM system under adaptive control are calculated.
REM ----------------------------------------------
FOR T = 1 TO (TMAX - 1)
T2 = T + 1

REM ---------------------------
REM Adaptive control decisions.
REM ---------------------------

F = UTSTATE(T, 1): S1 = UTSTATE(T, 2): S2 = UTSTATE(T, 3)

SA = 0
IF PP(T, NPRI) > PPSELL THEN SA = S2
M = .2 * S1
IF M > MMAX THEN M = MMAX
L = (5 - S1) / 5
H = 2 * (F - 3) * L
IF H > (F - 3) THEN H = F - 3
IF H < 0 THEN H = 0
B = .04 * (PWBUY - PW(T, NPRI)) * L
IF B < 0 THEN B = 0
Z = H + B
IF Z > (5 - S1) THEN H = H * (5 - S1) / Z
```

```
IF Z > (5 - S1) THEN B = B * (5 - S1) / Z

UTDEC(T, 1) = H: UTDEC(T, 2) = B: UTDEC(T, 3) = M: UTDEC(T, 4) = SA

REM ---------------------------------------
REM Difference equation path calculation.
REM ---------------------------------------

S2 = S2 + M - SA
S1 = S1 + B + H - 5 * M
F = F + .3 * F * (1 - F / 10) - H

UTSTATE(T2, 1) = F: UTSTATE(T2, 2) = S1: UTSTATE(T2, 3) = S2

REM -------------------------------------
REM Local (in time) profit calculation.
REM -------------------------------------
PRO(T) = PP(T, NPRI) * SA - PW(T, NPRI) * B - HC(T, NPRI) * H
PRO(T) = PRO(T) - CMMAX * MMAX

NEXT T

REM -----------------------------------------------------------------
REM Calculation of the total present value of the profits except for
REM the value of the finally available resources. The value PROFIT
REM includes the profits of all price series in the experiment.
REM The values of the finally available forest resource and stocks
REM are added to the profit function in the end.
REM -----------------------------------------------------------------
FOR T = 1 TO (TMAX - 1)
PROFIT = PROFIT + DISC(T) * PRO(T)
NEXT T

FT = UTSTATE(TMAX, 1): S1T = UTSTATE(TMAX, 2): S2T = UTSTATE(TMAX, 3)

PROFIT = PROFIT + DISC(TMAX) * (PWMEAN * FT + PPMEAN * (.2 * S1T + 1 * S2T))
IF UTTAB = 1 THEN GOSUB 4000
NEXT NPRI
AOBJ = PROFIT
RETURN


4000 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE FOR DETAILED OUTPUT.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
PRINT " T    F    S1    S2    H    B    M    SA    PW    HC    PP"
FOR T = 1 TO TMAX
PRINT USING "##"; T;
PRINT USING "###.#"; UTSTATE(T, 1); UTSTATE(T, 2); UTSTATE(T, 3);
PRINT USING "###.#"; UTDEC(T, 1); UTDEC(T, 2); UTDEC(T, 3); UTDEC(T, 4);
PRINT USING "#####."; PW(T, NPRI); HC(T, NPRI); PP(T, NPRI)
NEXT T
INPUT "NEW TABLE? THEN PRINT 1", XXXX
RETURN
```

```
"INIAD8.DAT, THE INPUT FILE OF IADAC8.BAS."
"INPUT FILE AUTHOR =  PETER LOHMANDER."
"INPUT FILE VERSION: DATE 92-03-24. TIME = 10.48"
3          "IXMAX"
3452       "PPSEL GUESS, THE SALES RESERVATION PRICE"
2180       "PWBUY*10, THE WOOD BUY RESERVATION PRICE"
4000       "MMAX*10000, THE MAXIMUM PRODUCTION CAPACITY"
5          "RATEINT"
100        "STA, derivative calculations step size"
100        "DIST0, initial step size before step reductions"
1          "PERL, MONTHS"
12         "TMAX, MONTHS"
5          "F0"
2          "S10"
.5         "S20"
200        "MEAN WOOD PRICE"
50         "STDEV WOOD PRICE"
100        "MEAN HARVEST COST"
20         "STDEV HARVEST COST"
3000       "MEAN PRODUCT PRICE"
1000       "STDEV PRODUCT PRICE"
5          "S1MAX"
3          "S2MAX"
2000       "CMMAX, THE MARGINAL PRODUCTION CAPACITY COST PER PERIOD"
5          "NUMEX = NUMBER OF EXPERIMENTS"
30         "NUMPRI = NUMBER OF COMPLETE PRICE VECTOR SERIES, HISTORIES"
```

```
rem This is a command file CIND2.bat
rem by Peter Lohmander 92-03-25.
rem It controls the adaptive forest industry
rem optimizations via the program IADAC8.EXE.
BREAK=ON
COPY IN6.DAT INIAD8.DAT
IADAC8
COPY UTIAD8.DAT UT6.DAT
COPY IN7.DAT INIAD8.DAT
IADAC8
COPY UTIAD8.DAT UT7.DAT
COPY IN8.DAT INIAD8.DAT
IADAC8
COPY UTIAD8.DAT UT8.DAT
COPY IN9.DAT INIAD8.DAT
IADAC8
COPY UTIAD8.DAT UT9.DAT
COPY IN10.DAT INIAD8.DAT
IADAC8
COPY UTIAD8.DAT UT10.DAT
```

```
03-25-1992    00:56:23
FILE = UTIAD8.DAT
RESULTS FROM IADAC8.BAS
INPUT FILE VERSION: DATE 92-03-24. TIME = 10.48
NUMBER OF EXPERIMENTS =  5
VECTOR PRICE SERIES IN EACH EXPERIMENT =  30
INTEREST RATE =  5  NUMBER OF PERIODS =  12
LENGTH OF EACH PERIOD =  1  DIST0 =  100
INITIAL FOREST STOCK =  5
INITIAL RAW MATERIALS STOCK =  2
INITIAL PRODUCT STOCK =  .5
MEAN WOOD PRICE =  200  AND ST. DEV. =  50
MEAN HARVEST COST =  100  AND ST. DEV. =  20
MEAN PROD. PRICE =  3000  AND ST. DEV. =  1000
MAXIMUM RAW MATERIALS STOCK CAPACITY =  5
MAXIMUM PRODUCT STOCK CAPACITY =  3
MARGINAL PROD. CAP. COST PER PERIOD =  2000

MAXIMUM OBJECTIVE FUNCTION VALUE =  285776.161970818
THE OPTIMIZED ADAPTIVE CONTROL FUNCTION PARAMETERS ARE:
EXPERIMENT =  1
PARAMETER LIST = ( = PPSELL, PWBUY*10, OPTMMAX*10000)

   3545.14894   3076.53789   3736.19923
EXPERIMENT =  2
PARAMETER LIST = ( = PPSELL, PWBUY*10, OPTMMAX*10000)

   3770.50435   3086.99836   3895.01180
EXPERIMENT =  3
PARAMETER LIST = ( = PPSELL, PWBUY*10, OPTMMAX*10000)

   3948.14838   3145.56166   4115.89434
EXPERIMENT =  4
PARAMETER LIST = ( = PPSELL, PWBUY*10, OPTMMAX*10000)

   3673.97044   3219.59962   4217.74301
EXPERIMENT =  5
PARAMETER LIST = ( = PPSELL, PWBUY*10, OPTMMAX*10000)

   3743.35123   3246.65698   4268.48824

************** END OF LIST *****************
```

```
REM ***********************************************************
REM ISIM5.BAS
REM LOHMANDER PETER 92-03-26, 14.42
REM PROGRAM FOR SIMULATION AND TEST OF ADAPTIVE DECISION RULES
REM IN THE FOREST INDUSTRY ENTERPRISE.
REM ***********************************************************

CLS
REM RANDOMIZE
PRINT ""
PRINT "You now enter the program ISIM"
PRINT "****************************** "
PRINT "by Peter Lohmander 92-03-26"
PRINT ""
BEEP

REM *******************************************************
REM SECTION 1. Dimensions, openings, inputs and definitions.
REM *******************************************************

DEFDBL A-H, O-Z
DIM X(10), PREVM(30), PREVMT(1000)
DIM Y(1, 100), U(1, 100), PRO(100), DISC(100)
DIM XTOT(10, 10), AVERS(10), AVERH(10)
DIM PP(12, 30), PW(12, 30), HC(12, 30), UTSTATE(12, 4)
DIM UTDEC(12, 4), UTPARA(12, 4)

REM ----------------------
REM Parameters are defined.
REM ----------------------

R = .05
PERL = 1
TMAX = 12
F0 = 5: UTSTATE(1, 1) = F0
S10 = 2: UTSTATE(1, 2) = S10
S20 = .5: UTSTATE(1, 3) = S20
PWMEAN = 200: PWSTDEV = 50
HCMEAN = 100: HCSTDEV = 20
PPMEAN = 3000
S1MAX = 5: S2MAX = 3
CMMAX = 2000
NUMEX = 5: NUMPRI = 30

INPUT "Standard deviation of product price = ", PPSTDEV

REM -------------------------------------
REM The optimal capacity decision and the
REM adaptive rules are determined
REM via the optimized equations.
REM -------------------------------------

MMAX = .3369 + 6.723 / 100000 * PPSTDEV
PPSELL = 3293 + .3947 * PPSTDEV
PWBUY = 289.5 + .0247 * PPSTDEV

REM X(1) = PPSELL: X(2) = 10 * PWBUY: X(3) = MMAX * 10000

100 PRINT ""
PRINT "The following decisions are made: "
PRINT "MMAX = ";
PRINT USING "###.###"; MMAX;
PRINT ", PPSELL = ";
```

```
PRINT USING "#####.##"; PPSELL;
PRINT ", PWBUY = ";
PRINT USING "####.##"; PWBUY
PRINT ""
INPUT "Are the decisions satisfactory? Then print 0.", SVAR
IF SVAR = 0 THEN GOTO 200
INPUT "New value of MMAX? ( 0 = no.)", MMAXEV
INPUT "New value of PPSELL? (0 = no.)", PPSELLEV
INPUT "New value of PWBUY ? (0 = no.)", PWBUYEV
IF MMAXEV > .01 THEN MMAX = MMAXEV
IF PPSELLEV > .01 THEN PPSELL = PPSELLEV
IF PWBUYEV > .01 THEN PWBUY = PWBUYEV
GOTO 100
200 REM

X(1) = PPSELL: X(2) = 10 * PWBUY: X(3) = MMAX * 10000


REM ***************************************************************
REM SECTION 2. The experiment loop starts here. NUMEX experiments
REM            will take place where each experiment has its own
REM            set of NUMPRI stochastic price series. For each
REM            set of price series, the harvest function is
REM            optimized. The price series are generated from an
REM            application of the central limit theorem. They
REM            are approximately N(PAV,STDEV^2). Compare Rde and
REM            Westergren (1988) pg. 316.
REM ***************************************************************

FOR EPER = 1 TO NUMEX
PRINT "": PRINT ""
PRINT "Experiment nr. "; EPER; " starts now."

REM ---------------------------------------------------
REM Here, the stochastic price series are calculated.
REM ---------------------------------------------------
FOR T = 1 TO TMAX
FOR NPRI = 1 TO NUMPRI

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
PW(T, NPRI) = PWMEAN + (PRAN - 6) * PWSTDEV

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
HC(T, NPRI) = HCMEAN + (PRAN - 6) * HCSTDEV

PRAN = 0: FOR I = 1 TO 12: PRAN = PRAN + RND: NEXT I
PP(T, NPRI) = PPMEAN + (PRAN - 6) * PPSTDEV

NEXT NPRI

DISC(T) = EXP(-R * T * PERL / 12)
NEXT T

REM
GOSUB 2190

FOR I = 1 TO NUMPRI
GINDEX = (EPER - 1) * NUMPRI + I
PREVMT(GINDEX) = PREVM(I)
NEXT I

EPROFIT = AOBJ / NUMPRI
```

```
VARI = 0
FOR I = 1 TO NUMPRI
VARI = VARI + (PREVM(I) - EPROFIT) ^ 2
NEXT I
STDEV = (VARI / (NUMPRI - 1)) ^ .5
STDEVM = STDEV / (NUMPRI ^ .5)
PRINT "EXPECTED PROFIT = ";
PRINT USING "######.##"; EPROFIT;
PRINT ",  ESTIMATION ERROR = ";
PRINT USING "#####.##"; STDEVM
PRINT "STANDARD DEVIATION OF DIFFERENT OUTCOMES = ";
PRINT USING "######.##"; STDEV

FOR I = 1 TO NUMPRI
PRINT USING "######.###"; PREVM(I);
NEXT I

NEXT EPER

TEPROF = 0
NTOT = NUMPRI * NUMEX
FOR I = 1 TO NTOT
TEPROF = TEPROF + PREVMT(I)
NEXT I
TEPROF = TEPROF / NTOT

TEVAR = 0
FOR I = 1 TO NTOT
TEVAR = TEVAR + (PREVMT(I) - TEPROF) ^ 2
NEXT I
TESTDEV = (TEVAR / (NTOT - 1)) ^ .5
TEPROFE = TESTDEV / NTOT ^ .5

CLS : BEEP
PRINT "****************** RESULTS FROM ISIM ***********************"
PRINT ""
PRINT "PPSTDEV = "; PPSTDEV
PRINT "       MMAX     PPSELL      PWBUY"
PRINT USING "######.###"; MMAX; PPSELL; PWBUY
PRINT ""
PRINT "Expected present value    Estimation error    Sample st.dev."
PRINT "      "
PRINT USING "###############.##"; TEPROF; TEPROFE; TESTDEV
PRINT ""
PRINT "Number of trials = "; NTOT
PRINT ""
PRINT "***************************************************************"



END

2190 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM HERE, THE OBJECTIVE FUNCTION, AOBJ, IS "WRITTEN".
REM NOTE THAT THE "OBJECTIVE FUNCTION" MUST BE CALCULATED VIA THE
REM SIMULATION OF THE COMPLETE SYSTEM DURING A LONG TIME PERIOD!
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS

PROFIT = 0
```

```
REM The calculations are repeated for each price series in the experiment.
REM -------------------------------------------------------------------------
FOR NPRI = 1 TO NUMPRI
PR = 0

REM ------------------------------
REM The parameters are determined.
REM ------------------------------
PPSELL = X(1)
PWBUY = X(2) / 10
MMAX = X(3) / 10000

REM --------------------------------------------
REM The path and the objective function of the
REM system under adaptive control are calculated.
REM --------------------------------------------
FOR T = 1 TO (TMAX - 1)
T2 = T + 1

REM --------------------------
REM Adaptive control decisions.
REM --------------------------

F = UTSTATE(T, 1): S1 = UTSTATE(T, 2): S2 = UTSTATE(T, 3)

SA = 0
IF PP(T, NPRI) > PPSELL THEN SA = S2
M = .2 * S1
IF M > MMAX THEN M = MMAX
L = (5 - S1) / 5
H = 2 * (F - 3) * L
IF H > (F - 3) THEN H = F - 3
IF H < 0 THEN H = 0
B = .04 * (PWBUY - PW(T, NPRI)) * L
IF B < 0 THEN B = 0
Z = H + B
IF Z > (5 - S1) THEN H = H * (5 - S1) / Z
IF Z > (5 - S1) THEN B = B * (5 - S1) / Z

UTDEC(T, 1) = H: UTDEC(T, 2) = B: UTDEC(T, 3) = M: UTDEC(T, 4) = SA

REM ------------------------------------
REM Difference equation path calculation.
REM ------------------------------------

S2 = S2 + M - SA
S1 = S1 + B + H - 5 * M
F = F + .3 * F * (1 - F / 10) - H

UTSTATE(T2, 1) = F: UTSTATE(T2, 2) = S1: UTSTATE(T2, 3) = S2

REM ----------------------------------
REM Local (in time) profit calculation.
REM ----------------------------------
PRO(T) = PP(T, NPRI) * SA - PW(T, NPRI) * B - HC(T, NPRI) * H
PRO(T) = PRO(T) - CMMAX * MMAX

NEXT T

REM ------------------------------------------------------------------
REM Calculation of the total present value of the profits except for
REM the value of the finally available resources. The value PROFIT
REM includes the profits of all price series in the experiment.
```

```
REM The values of the finally available forest resource and stocks
REM are added to the profit function in the end.
REM ---------------------------------------------------------------

FOR T = 1 TO (TMAX - 1)
PR = PR + DISC(T) * PRO(T)
NEXT T

FT = UTSTATE(TMAX, 1): S1T = UTSTATE(TMAX, 2): S2T = UTSTATE(TMAX, 3)

PR = PR + DISC(TMAX) * (PWMEAN * FT + PPMEAN * (.2 * S1T + 1 * S2T))
PREVM(NPRI) = PR
PROFIT = PROFIT + PREVM(NPRI)
IF UTTAB = 1 THEN GOSUB 4000
NEXT NPRI
AOBJ = PROFIT
RETURN


4000 REM
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
REM SUBROUTINE FOR DETAILED OUTPUT.
REM SSSSSSSSSSSSSSSSSSSSSSSSSSSSSSS
PRINT " T     F    S1    S2     H     B     M    SA    PW    HC    PP"
FOR T = 1 TO TMAX
PRINT USING "##"; T;
PRINT USING "###.#"; UTSTATE(T, 1); UTSTATE(T, 2); UTSTATE(T, 3);
PRINT USING "###.#"; UTDEC(T, 1); UTDEC(T, 2); UTDEC(T, 3); UTDEC(T, 4);
PRINT USING "#####."; PW(T, NPRI); HC(T, NPRI); PP(T, NPRI)
NEXT T
INPUT "NEW TABLE? THEN PRINT 1", XXXX
RETURN
```

```
You now enter the program ISIM
********************************
by Peter Lohmander 92-03-26

Standard deviation of product price = 1000

The following decisions are made:
MMAX =    0.404, PPSELL =  3687.70, PWBUY =  314.20

Are the decisions satisfactory? Then print 0.0


Experiment nr.  1  starts now.
EXPECTED PROFIT =    8843.80,  ESTIMATION ERROR =    473.83
STANDARD DEVIATION OF DIFFERENT OUTCOMES =    2595.25
 10153.736 10710.850  9424.870  7103.411  9244.336  8997.190  7551.357 10085.9
  7111.577  8614.843  6197.491  9421.084  7459.553  8791.364  7019.114 10592.3
  6013.337 11533.178  4563.287  8464.365  4690.432  7457.663 10817.733  9802.3
  9571.252 12424.298  4077.613  8467.354 16388.458 12563.518

Experiment nr.  2  starts now.
```

```
****************** RESULTS FROM ISIM ***********************

PPSTDEV =  1000
      MMAX     PPSELL      PWBUY
     0.404  3687.700   314.200

Expected present value    Estimation error    Sample st.dev.

            9090.88              184.48            2259.35

Number of trials =  150

************************************************************
```