The Future of Forest Wildfires in the Czech Republic

Scientific Workshop

The Czech University of Life Sciences (CZU)
Faculty of Forestry and Wood Sciences
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This presentation is based on the following open access articles and documents. More details can be obtained from the links found below.

[1] Lohmander P. (2020a). **Dynamics and control of the CO2 level via a differential equation and alternative global emission strategies**. ***Int Rob Auto J.*** 2020;6(1):7–15. DOI: 10.15406/iratj.2020.06.0019, https://medcraveonline.com/IRATJ/IRATJ-06-00197.pdf

[2] Lohmander, P. (2020b)., **Optimization of continuous cover forestry expansion under the influence of global warming**, ***International Robotics & Automation Journal***, Volume 6, Issue 3, 2020, 127-132. https://medcraveonline.com/IRATJ/IRATJ-06-00211.pdf ,

https://medcraveonline.com/IRATJ/IRATJ-06-00211A.pdf

[3] Lohmander, P. (2020c). **Fundamental principles of optimal utilization of forests with consideration of global warming**, ***Central Asian Journal of Environmental Science and Technology Innovation***, Volume 1, Issue 3, May and June 2020, 134-142. doi: 10.22034/CAJESTI.2020.03.02 http://www.cas-press.com/article_111213.html http://www.cas-press.com/article_111213_5ab21574a30f6f2c7bdc0a0733234181.pdf

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[7] Lohmander P. (2021b). **Optimization of distance between fire stations: effects of fire ignition probabilities, fire engine speed and road limitations, property values and weather conditions**. *Int Rob Auto J.* 2021;7(4):112–120. DOI: 10.15406/iratj.2021.07.0023 , https://medcraveonline.com/IRATJ/IRATJ-07-00235.pdf

[8] Lohmander, P. (2021c). **Global Stability via the Forced Global Warming Equation, Fire Control with Joint Fire Fighting Resources, and Optimal Forestry**, ***KEYNOTE at ICASE 2021***: International Conference on Applied Science & Engineering, March 31, 2021., http://www.Lohmander.com/PL_ICASE_2021_Abstract.pdf, http://www.Lohmander.com/PL_ICASE_2021_KEYNOTE.pdf

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[10] Lohmander, P., **Statistics and Mathematics of General Control Function Optimization for Continuous Cover Forestry, with a Swedish Case Study based on Picea abies**,
including Nils Fagerberg, Forest data and functions,
Statistics for Twenty-first Century 2021, (ICSTC 2021), 16-19 December 2021,
Organized by Department of Statistics, University of Kerala, Trivandrum, India,
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[12] Fagerberg, N., Olsson, J-O., Lohmander, P., Andersson, M., Bergh, J.,
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[14] Lohmander, P., (2022). **Rational Control of Global Warming Dynamics via Emission Reductions and Forestry Expansion** (*International Journal of Earth and Environmental Sciences*, Accepted for publication).

[15] Lohmander, P., Fagerberg, N. (2022). **Statistics and Mathematics of General Control Function Optimization for Continuous Cover Forestry, with a Swedish Case Study based on Picea abies** (AJSS, *Asean Journal of Statistical Sciences*, In print).

[16] Lohmander, P., **Optimal Dynamic Forest Fire Management Adapted to Stochastic Weather** and Mohammadi, Z., **Empirical data and support**, Sixth International Webinar on *RECENT TRENDS IN STATISTICAL THEORY AND APPLICATIONS-2022* (WSTA-2022), June 29 to July 02, 2022, Organized by: Department of Statistics, School of Physical and Mathematical Sciences, University of Kerala, Trivandrum in association with Indian Society for Probability and Statistics (ISPS) and Kerala Statistical Association (KSA). The first link is a presentation movie. The second and third links give the pdf and the pptx versions.
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The Future of Forest Wildfires in the Czech Republic:

This future is a function of several things.

#1. Some of these can not be very much affected by humans.

#2. Some of these can be affected by humans in all countries, in the long run.

#3. Some of these can be affected by humans, in the country, in the long run.

#4. Some of these can be affected by humans, in the country, in the short run.

Asteroids can hit the Earth. A volcanic winter can occur if volcanoes explode and increase the reflection of solar radiation. The orbit of the Earth changes, periodically and aperiodically. The planetary system contains deterministic chaos and cannot be perfectly predicted and/or proved to be stable in the long run. Temperature changes follow.

SUB-TOPIC 1: CO_2 Dynamics and Emission Control

The emissions can be changed.
Even with strong emission reductions, however, the CO_2 level in the atmosphere changes very slowly.

Global long term cooperation is necessary.

SUB-TOPIC 1: CO2 Dynamics and Emission Control
Recent results from the author:

[1] Lohmander P. (2020a). **Dynamics and control of the CO2 level via a differential equation and alternative global emission strategies**. ***Int Rob Auto J.*** 2020;6(1):7–15. DOI: 10.15406/iratj.2020.06.0019, https://medcraveonline.com/IRATJ/IRATJ-06-00197.pdf

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SUB-TOPIC 2: Environmentally acceptable forestry expansion and CO_2 effects

Environmentally acceptable expansion of optimal and active forestry, adapted to climate change and fire risk, railways and green energy, can reduce CO_2 in the atmosphere.

SUB-TOPIC 2: Environmentally acceptable forestry expansion and CO2 effects
Recent results from the author:

[2] Lohmander, P. (2020b)., **Optimization of continuous cover forestry expansion under the influence of global warming**, ***International Robotics & Automation Journal***, Volume 6, Issue 3, 2020, 127-132. https://medcraveonline.com/IRATJ/IRATJ-06-00211.pdf ,

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SUB-TOPIC 2: Environmentally acceptable forestry expansion and CO2 effects

Recent results from the author and some colleagues (continued from the earlier page):

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SUB-TOPIC 3: Fire fighting resources, infrastructure investments and cooperation

Investments in firefighting capacity and infrastructure take considerable time.
International fire cooperation plans and agreements are important.

These things are necessary to make it possible to rapidly fight fires.

SUB-TOPIC 3: Fire fighting resources, infrastructure investments and cooperation
Recent results concerning these topics from the author:

[4] Lohmander P. (2020d). **Adaptive mobile firefighting resources: stochastic dynamic optimization of international cooperation**. *Int Rob Auto J.* 2020;6(4):150–155. DOI: 10.15406/iratj.2020.06.00213, https://medcraveonline.com/IRATJ/IRATJ-06-00213.pdf

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SUB-TOPIC 4: Forest fire dynamics as a function of weather and attack time



The size of a forest fire increases rapidly if the air is hot and dry, and the wind speed is high.
Fire size functions have been estimated for 29 countries.
Detailed functions have been estimated in Czech Republic.
In CR, the fire size has also been studied as a function of the time it takes to reach the fire.

SUB-TOPIC 4: Forest fire dynamics as a function of weather and attack time

Recent results from the author and some colleagues:

[5] Lohmander, P. (2020e). Forest fire expansion under global warming conditions: -Multivariate estimation, function properties and predictions for 29 countries. *Cent. Asian J. Environ. Sci. Technol. Innov.*, 5, 262-276, https://www.cas-press.com/article_122566_c3544cd0c21d5c077f72e985a77d30e9.pdf

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A fire can start in almost any position. Before it has started, the position can be considered as stochastic.

In some cases, we may estimate probability density functions.

SUB-TOPIC 5: Optimal adaptive fire fighting decisions conditional on the latest weather information and predictions

We should dynamically adapt the positioning and readiness of fire fighting units to the latest weather information and weather forecasts.

If the temperature will be high, the air will be dry and the wind will be strong:

Make sure that more resources are ready to act, and that they are placed in the optimal initial positions!

SUB-TOPIC 5: Optimal adaptive fire fighting decisions conditional on the latest weather information and predictions

Recent results from the author and a colleague:

[7] Lohmander P. (2021b). **Optimization of distance between fire stations: effects of fire ignition probabilities, fire engine speed and road limitations, property values and weather conditions**. *Int Rob Auto J.* 2021;7(4):112–120. DOI: 10.15406/iratj.2021.07.0023 , https://medcraveonline.com/IRATJ/IRATJ-07-00235.pdf

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A general fire fighting decision optimization problem under present investigation:

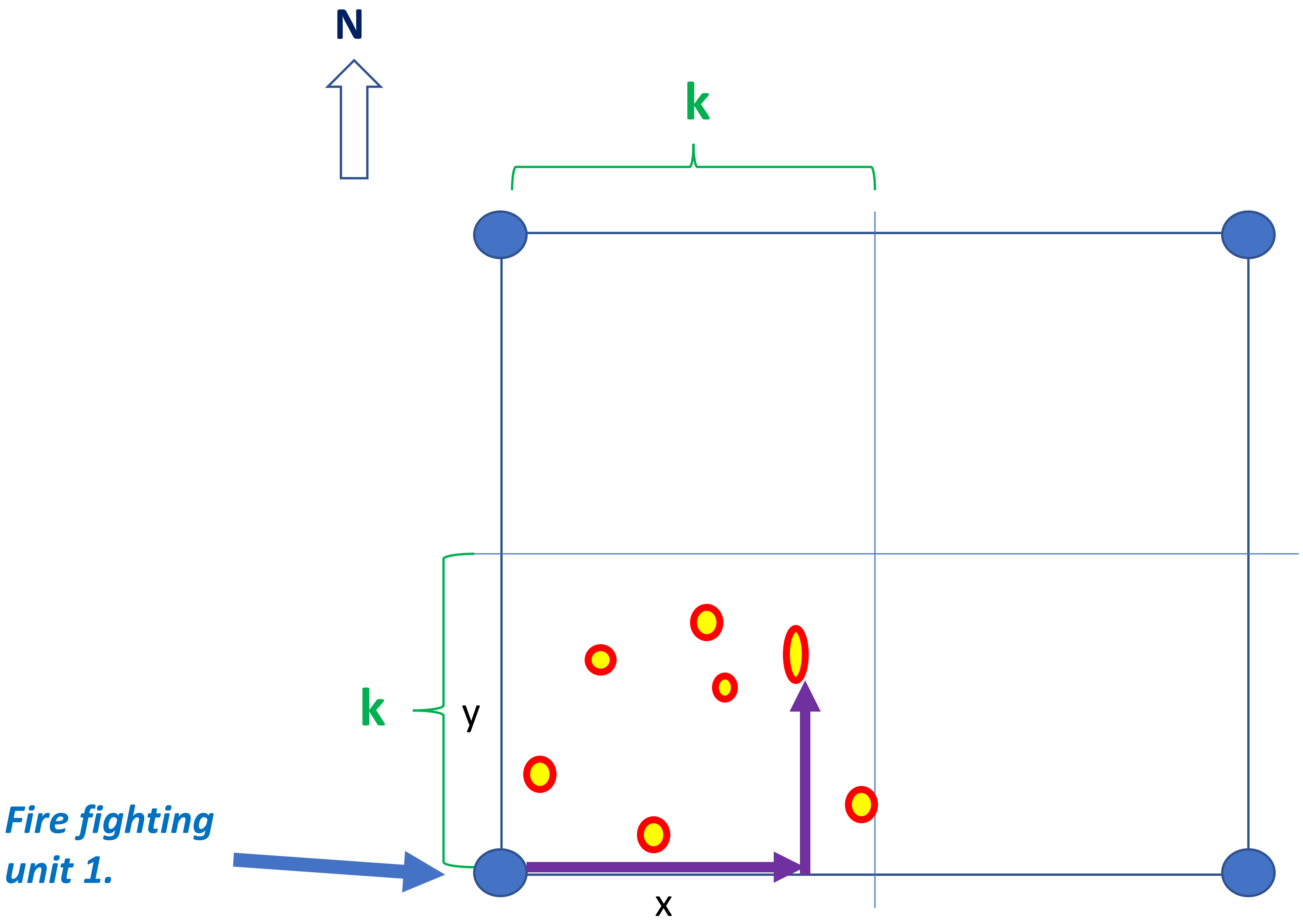
"Optimal Dynamic Forest Fire Management Adapted to Stochastic Weather ***(marginally updated)"***

Thank you Dr Zohreh Mohammadi
for very valuable data and information!

Contents:

- The firefighting capacity optimization problem
- General solution to the firefighting capacity optimization problem
- Comparative statics analysis of the optimal solution: How is the optimal solution affected by the parameters?
- Fire growth as a function of weather conditions
- Background to empirical weather and fire data
- Dynamic and stochastic properties of air temperature, relative humidity and wind speed
- Conclusions

The firefighting capacity optimization problem



Forest fires have a spatially uniform probability density function.

Fire fighting units (blue balls) have initial locations in a regular ”infinite” network with roads in directions N-S and E-W.

The distances between roads is small and fire fighting units can always use roads in the two directions to reach the different fires. (See the purple arrows.)

The decision problem is to determine the optimal value of k, where the distance between the nearest neighbours is L = 2k.

The optimal value of k is affected by many different parameters, some of which are functions of the season, for instance air temperature, relative humidity and wind speed. These parameters affect fire growth.

$$\min_{k} C(k;.) = C_I\left(k\right) + C_F(k)$$

Fire fighting groups are
located in a regular network with
roads in directions South-North
and East- West. The distance
in one of these directions, between
two neighbour groups, is L = 2k.

Expected total
costs as a
function of k.

Expected cost of
fire fighting capacity
”investment”
and use of these
resources

Expected cost of destroyed and damaged forests
including costs of CO_2 emissions

$$C_F(k) = c_D \times N \times B\left(A, H, W, t(k, m)\right)$$

Expected cost of destroyed and damaged forests
including costs of CO_2 emissions

Expected cost
per burned ha.

Expected number of fires
per time unit.

Expected size of burned
forest, per fire.

A = Air temperature (Stochastic)
H = Relative humidity (Stochastic)
W = Wind speed (Stochastic)
t(k,m) = expected time of fire life.
m = fire life time before the
fire fighting unit leaves the initial
location and starts to move towards
the fire.

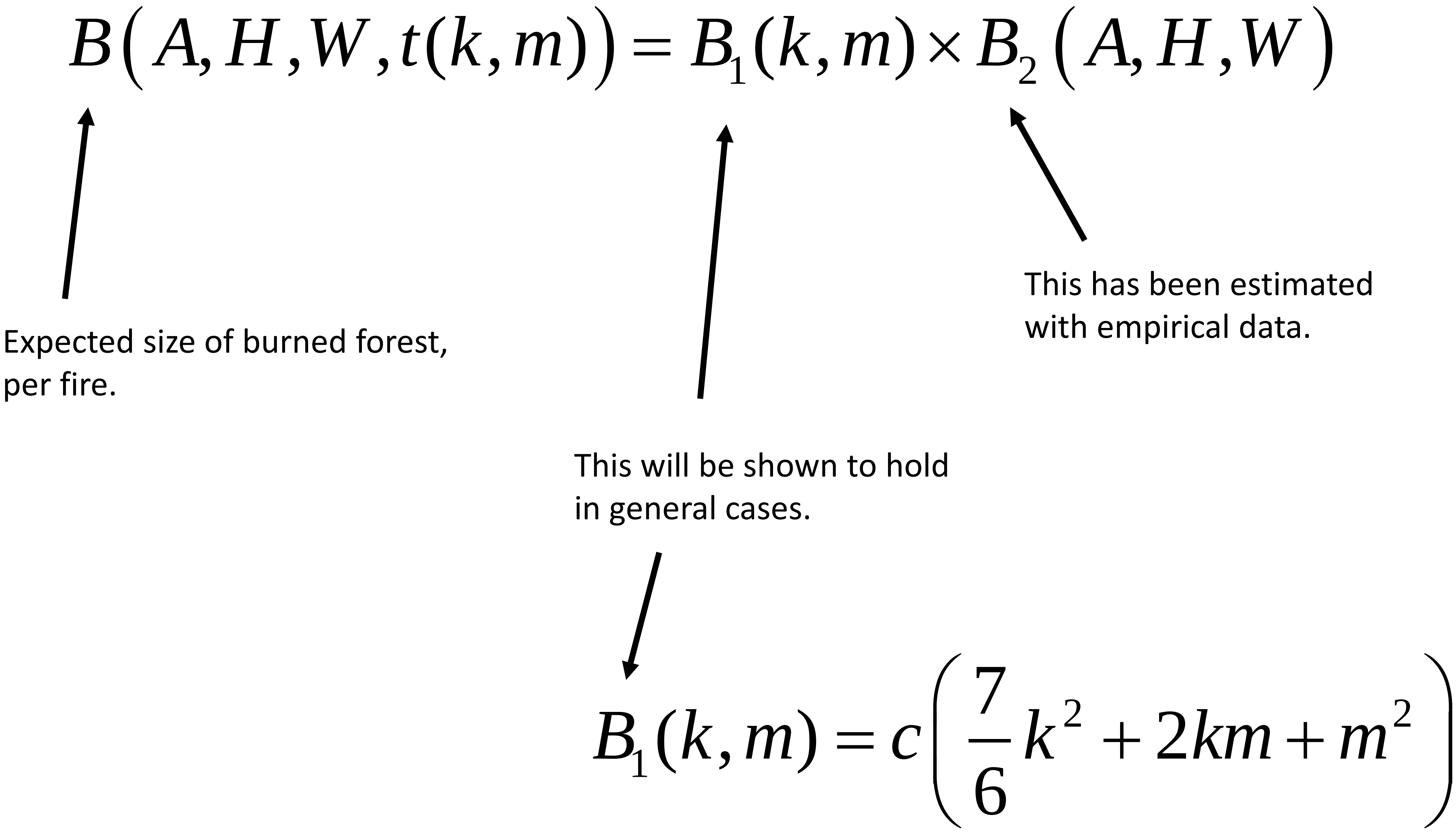
$$C_I(k) = c_u \times U$$

Number of units per
10000 square km

Expected cost of
fire fighting capacity
”investment”
and use of these
Resources.

Expected cost
per unit.

$$U = \frac{1}{L^2} = \frac{1}{(2k)^2} = \frac{1}{4}k^{-2}$$



$$B\left(A,H,W,t(k,m)\right) = B_1(k,m) \times B_2\left(A,H,W\right)$$

Expected size of burned forest, per fire.

This will be shown to hold in general cases.

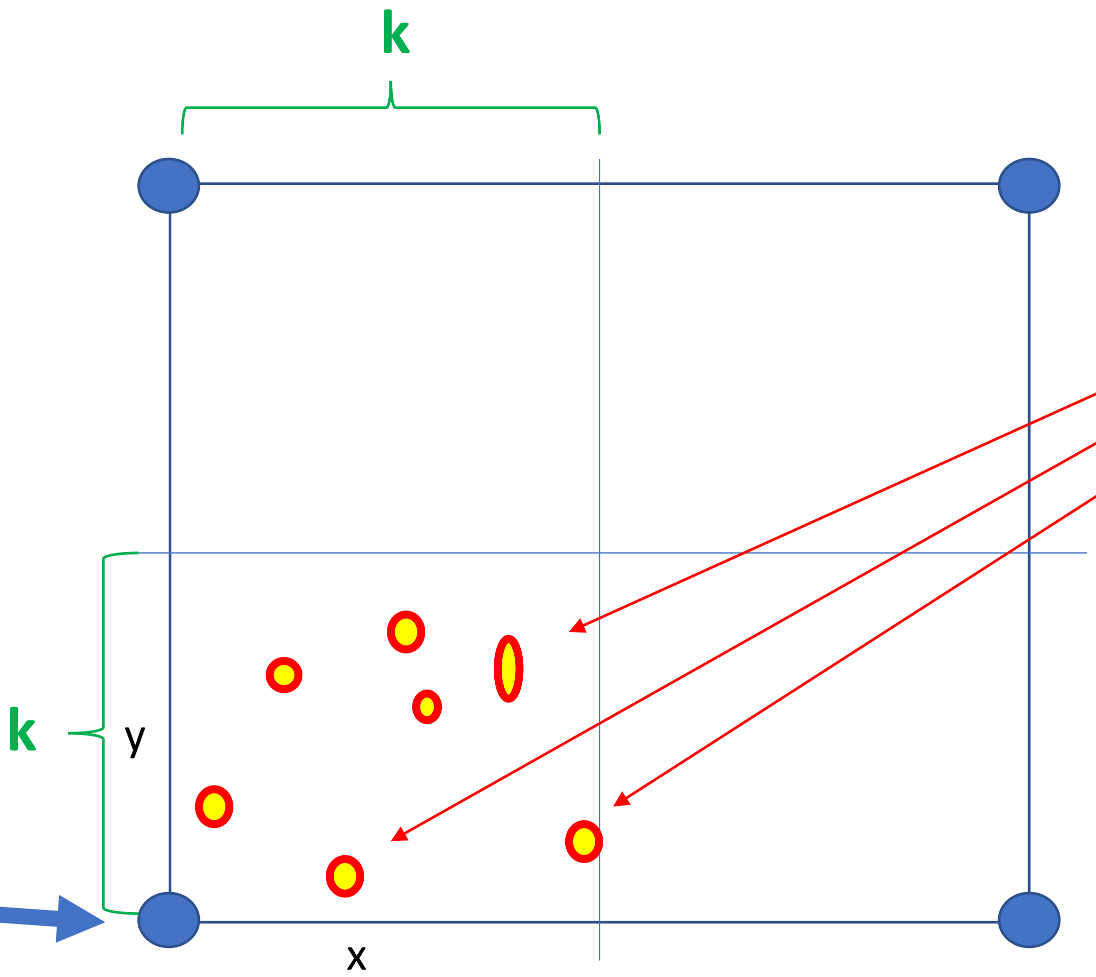
This has been estimated with empirical data.

$$B_1(k,m) = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$$

<u>Obs:</u> In the analysis of B1(k,m), k and m are both expressed in the unit ”time”. We may consider the distance between fire stations as the time it takes to go from one station to the other. Of course, this can also be considered as a function of the road quality and the capacity of the fire engines.

Why is $B_1(k,m) = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$ **?**

$$f_2(y) = \begin{cases} 0 & \text{for} \quad y < 0 \\ k^{-1} & \text{for} \quad 0 \le y \le k \\ 0 & \text{for} \quad k < y \end{cases}$$



Random fires in the square that is closest to fire fighting unit 1.

Fire fighting unit 1.

$$f_1(x) = \begin{cases} 0 & \text{for} \quad x < 0 \\ k^{-1} & \text{for} \quad 0 \le x \le k \\ 0 & \text{for} \quad k < x \end{cases}$$

Convolution

$$f_1(x) = \begin{cases} 0 & for \quad x < 0 \\ k^{-1} & for \quad 0 \le x \le k \\ 0 & for \quad k < x \end{cases}$$

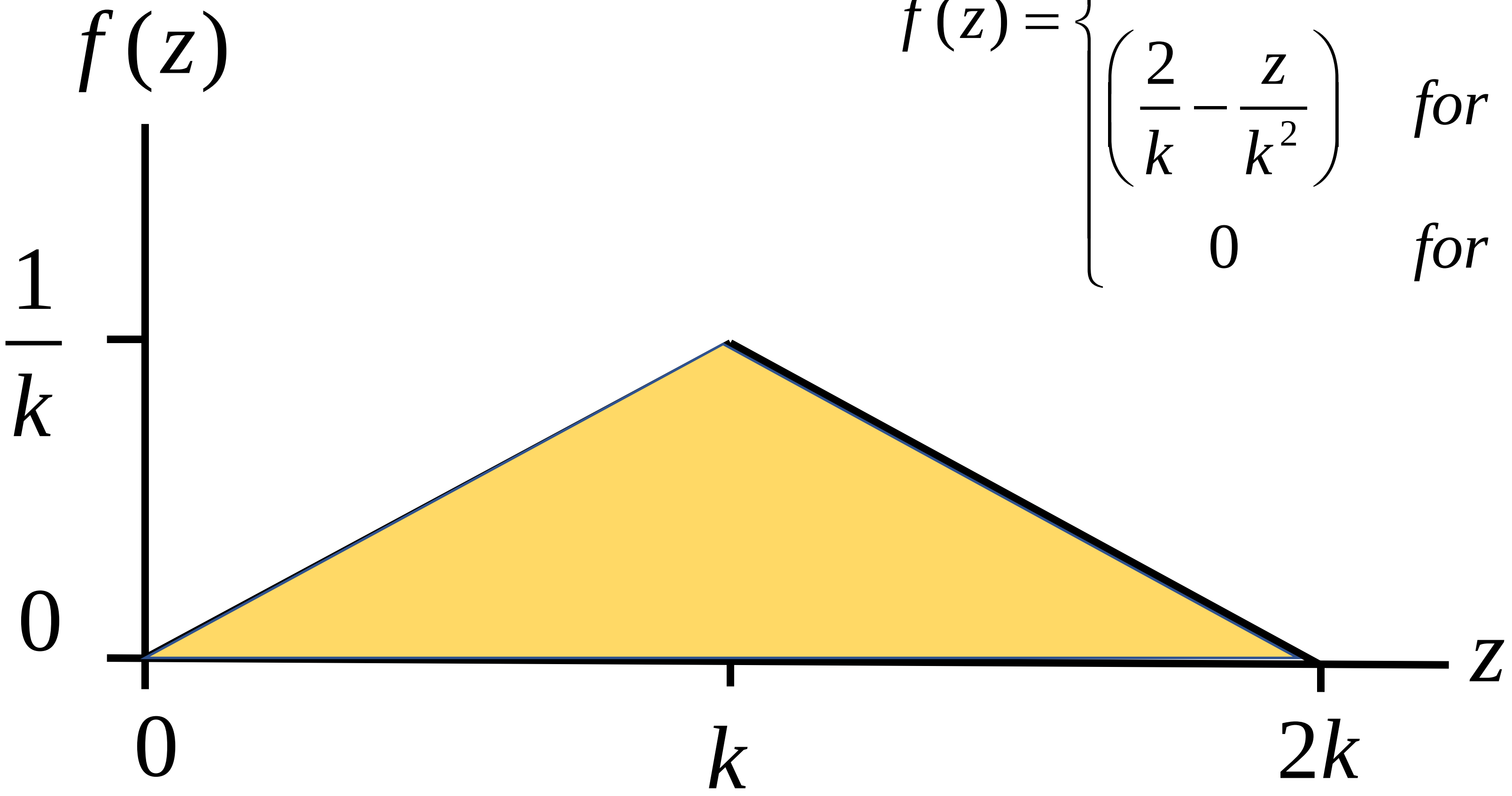
$$f_2(y) = \begin{cases} 0 & for \quad y < 0 \\ k^{-1} & for \quad 0 \le y \le k \\ 0 & for \quad k < y \end{cases}$$

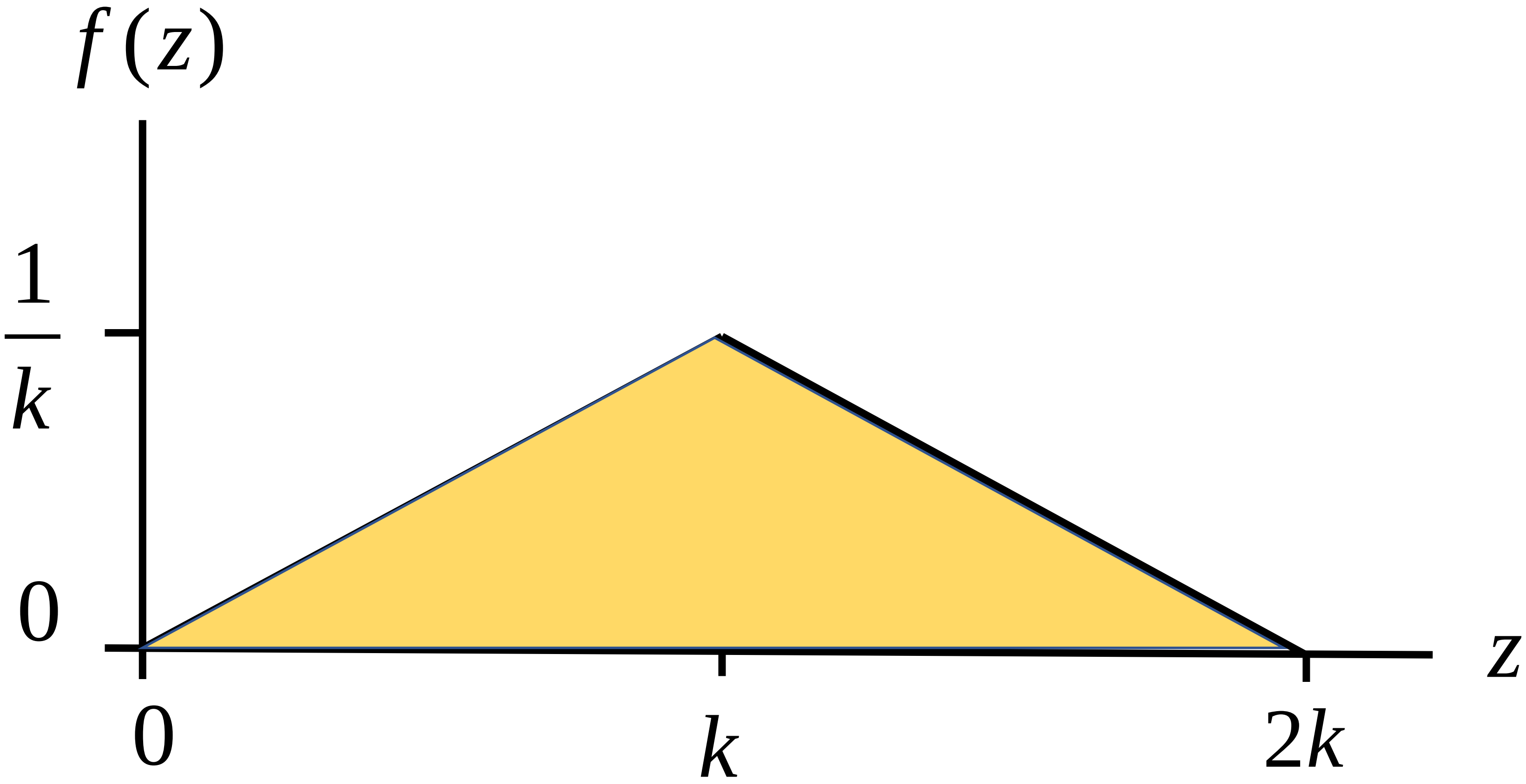
$$z = x + y$$

$$f(z) = \int_0^{2k} f_1(x)\, f_2(z - x)\, dx$$

$$f(z) = \begin{cases} 0 & for \quad z < 0 \\ \dfrac{z}{k^2} & for \quad 0 \le z < k \\ \left(\dfrac{2}{k} - \dfrac{z}{k^2}\right) & for \quad k \le z \le 2k \\ 0 & for \quad 2k < z \end{cases}$$

$$f(z) = \begin{cases} 0 & for \quad z < 0 \\ \dfrac{z}{k^2} & for \quad 0 \le z < k \\ \left(\dfrac{2}{k} - \dfrac{z}{k^2}\right) & for \quad k \le z \le 2k \\ 0 & for \quad 2k < z \end{cases}$$





$$B_1 = \int_0^k c\left(z+m\right)^2 \frac{z}{k^2}\,dz + \int_k^{2k} c\left(z+m\right)^2 \left(\frac{2}{k} - \frac{z}{k^2}\right)dz$$

$$B_1 = \int_0^k c\left(z+m\right)^2 \frac{z}{k^2}\,dz + \int_k^{2k} c\left(z+m\right)^2 \left(\frac{2}{k} - \frac{z}{k^2}\right)dz$$

$$G = \frac{B_1}{c} = k^{-2}\int_0^k \left(z^2 + 2zm + m^2\right) z\,dz$$

$$+\,2k^{-1}\int_k^{2k} \left(z^2 + 2zm + m^2\right) dz$$

$$-\,k^{-2}\int_k^{2k} \left(z^2 + 2zm + m^2\right) z\,dz$$

$$G = \frac{B_1}{c} = k^{-2}\int_0^k \left(z^2 + 2zm + m^2\right) z\,dz + 2k^{-1}\int_k^{2k} \left(z^2 + 2zm + m^2\right) dz - k^{-2}\int_k^{2k} \left(z^2 + 2zm + m^2\right) z\,dz$$

$$G = k^{-2}\int_0^k z^3\,dz + 2mk^{-2}\int_0^k z^2\,dz + m^2k^{-2}\int_0^k z\,dz$$

$$+ 2k^{-1}\int_k^{2k} z^2\,dz + 4mk^{-1}\int_k^{2k} z\,dz + 2m^2k^{-1}\int_k^{2k} 1\,dz$$

$$- k^{-2}\int_k^{2k} z^3\,dz - 2mk^{-2}\int_k^{2k} z^2\,dz - m^2k^{-2}\int_k^{2k} z\,dz$$

$$G = k^{-2}\int_0^k z^3\,dz + 2mk^{-2}\int_0^k z^2\,dz + m^2k^{-2}\int_0^k z\,dz + 2k^{-1}\int_k^{2k} z^2\,dz + 4mk^{-1}\int_k^{2k} z\,dz + 2m^2k^{-1}\int_k^{2k} 1dz - k^{-2}\int_k^{2k} z^3\,dz - 2mk^{-2}\int_k^{2k} z^2\,dz - m^2k^{-2}\int_k^{2k} z\,dz$$

$$G = k^{-2}\left(\left.\frac{z^4}{4}\right]_0^k\right) + 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_0^k\right) + m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_0^k\right)$$

$$+\,2k^{-1}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) + 4mk^{-1}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right) + 2m^2k^{-1}\left(\left.z\right]_k^{2k}\right)$$

$$-\,k^{-2}\left(\left.\frac{z^4}{4}\right]_k^{2k}\right) - 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) - m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right)$$

$$G = k^{-2}\left(\left.\frac{z^4}{4}\right]_0^k\right) + 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_0^k\right) + m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_0^k\right) + 2k^{-1}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) + 4mk^{-1}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right) + 2m^2k^{-1}\left(\left.z\right]_k^{2k}\right) - k^{-2}\left(\left.\frac{z^4}{4}\right]_k^{2k}\right) - 2mk^{-2}\left(\left.\frac{z^3}{3}\right]_k^{2k}\right) - m^2k^{-2}\left(\left.\frac{z^2}{2}\right]_k^{2k}\right)$$

$$G = k^{-2}\left(\frac{k^4}{4} - 0\right) + 2mk^{-2}\left(\frac{k^3}{3} - 0\right) + m^2k^{-2}\left(\frac{k^2}{2} - 0\right)$$

$$+ 2k^{-1}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) + 4mk^{-1}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right) + 2m^2k^{-1}\left(2k - k\right)$$

$$- k^{-2}\left(\frac{16k^4}{4} - \frac{k^4}{4}\right) - 2mk^{-2}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) - m^2k^{-2}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right)$$

$$G = k^{-2}\left(\frac{k^4}{4} - 0\right) + 2mk^{-2}\left(\frac{k^3}{3} - 0\right) + m^2k^{-2}\left(\frac{k^2}{2} - 0\right) + 2k^{-1}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) + 4mk^{-1}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right) + 2m^2k^{-1}\left(2k - k\right) - k^{-2}\left(\frac{16k^4}{4} - \frac{k^4}{4}\right) - 2mk^{-2}\left(\frac{8k^3}{3} - \frac{k^3}{3}\right) - m^2k^{-2}\left(\frac{4k^2}{2} - \frac{k^2}{2}\right)$$

$$G = \frac{1}{4}k^2 + \frac{2}{3}mk + \frac{1}{2}m^2$$

$$+\frac{14}{3}k^2 + 6mk + 2m^2$$

$$-\frac{15}{4}k^2 - \frac{14}{3}mk - \frac{3}{2}m^2$$

$$G=\frac{1}{4}k^2+\frac{2}{3}mk+\frac{1}{2}m^2+\frac{14}{3}k^2+6mk+2m^2-\frac{15}{4}k^2-\frac{14}{3}mk-\frac{3}{2}m^2$$

$$G=\left(\frac{1}{4}+\frac{14}{3}-\frac{15}{4}\right)k^2+\left(\frac{2}{3}+6-\frac{14}{3}\right)km+\left(\frac{1}{2}+2-\frac{3}{2}\right)m^2$$

$$G=\left(\frac{3}{12}+\frac{56}{12}-\frac{45}{12}\right)k^2+\left(\frac{2}{3}+\frac{18}{3}-\frac{14}{3}\right)km+\left(\frac{1}{2}+\frac{4}{2}-\frac{3}{2}\right)m^2$$

$$G=\frac{14}{12}k^2+\frac{6}{3}km+\frac{2}{2}m^2$$

$$G=\frac{7}{6}k^2+2km+m^2$$

Hence, $$B_1 = c\left(\frac{7}{6}k^2 + 2km + m^2\right)$$

Important properties:

$$\frac{dB_1}{dk} = c\left(\frac{7}{3}k + 2m\right) > 0$$

$$\frac{d^2B_1}{dk^2} = \frac{7}{3}c > 0$$

$$\frac{d^2B_1}{dmdk} = 2c > 0$$

$$\frac{dB_1}{dm} = c(2k + 2m) > 0$$

$$\frac{d^2B_1}{dm^2} = 2c > 0$$

General solution to the firefighting capacity optimization problem

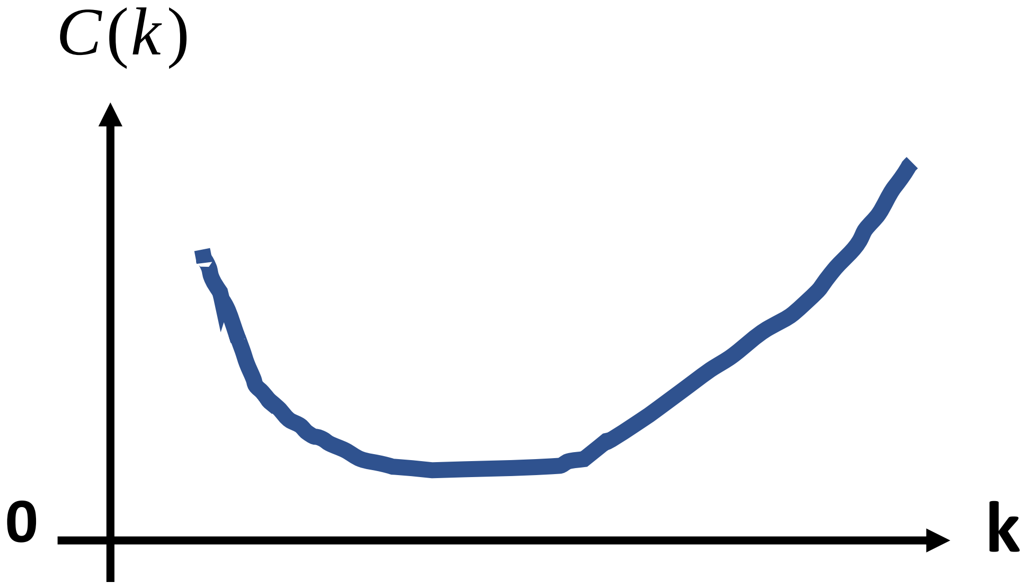
$$\min_{k} C(k;.) = C_I\left(k\right) + C_F(k)$$

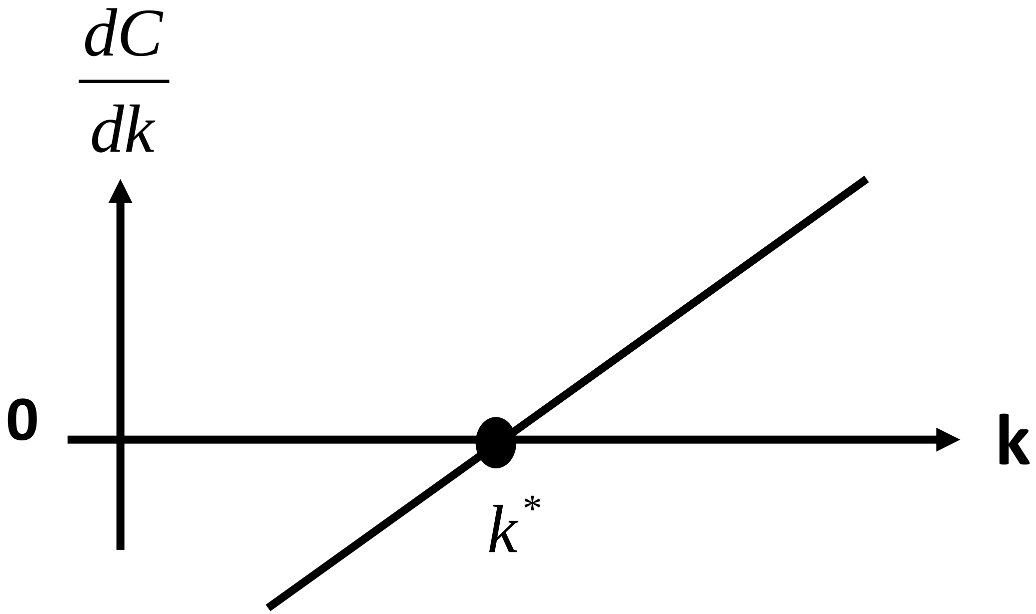
$$\min_{k} C(k;.) = \frac{c_u}{4} k^{-2} + c_D N c B_1(k,m) B_2\left(A, H, W\right)$$

$$\min_k C(k;.) = \frac{c_u}{4} k^{-2} + c_D N c B_1(k,m) B_2\left(A, H, W\right)$$

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(.\right) = 0$$

$$\frac{d^2 C}{dk^2} = \frac{3c_u}{2} k^{-4} + c_D N c \frac{d^2 B_1}{dk^2} B_2\left(.\right) > 0$$


$C(k)$
0
k


$\frac{dC}{dk}$
0
k
k^*

Comparative statics analysis of the optimal solution: How is the optimal solution affected by the parameters?

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$d\left(\frac{dC}{dk}\right) = \frac{d^2C}{dk^2}dk^* + \frac{d^2C}{dkdc_u}dc_u = 0$$

$$\left(d\left(\frac{dC}{dk}\right) = 0\right) \Rightarrow \left(\frac{d^2C}{dk^2}dk^* + \frac{d^2C}{dkdc_u}dc_u = 0\right)$$

$$\left(d\left(\frac{dC}{dk}\right) = 0\right) \Rightarrow \left(\frac{dk^*}{dc_u} = \frac{-\left(\frac{d^2C}{dkdc_u}\right)}{\left(\frac{d^2C}{dk^2}\right)}\right)$$

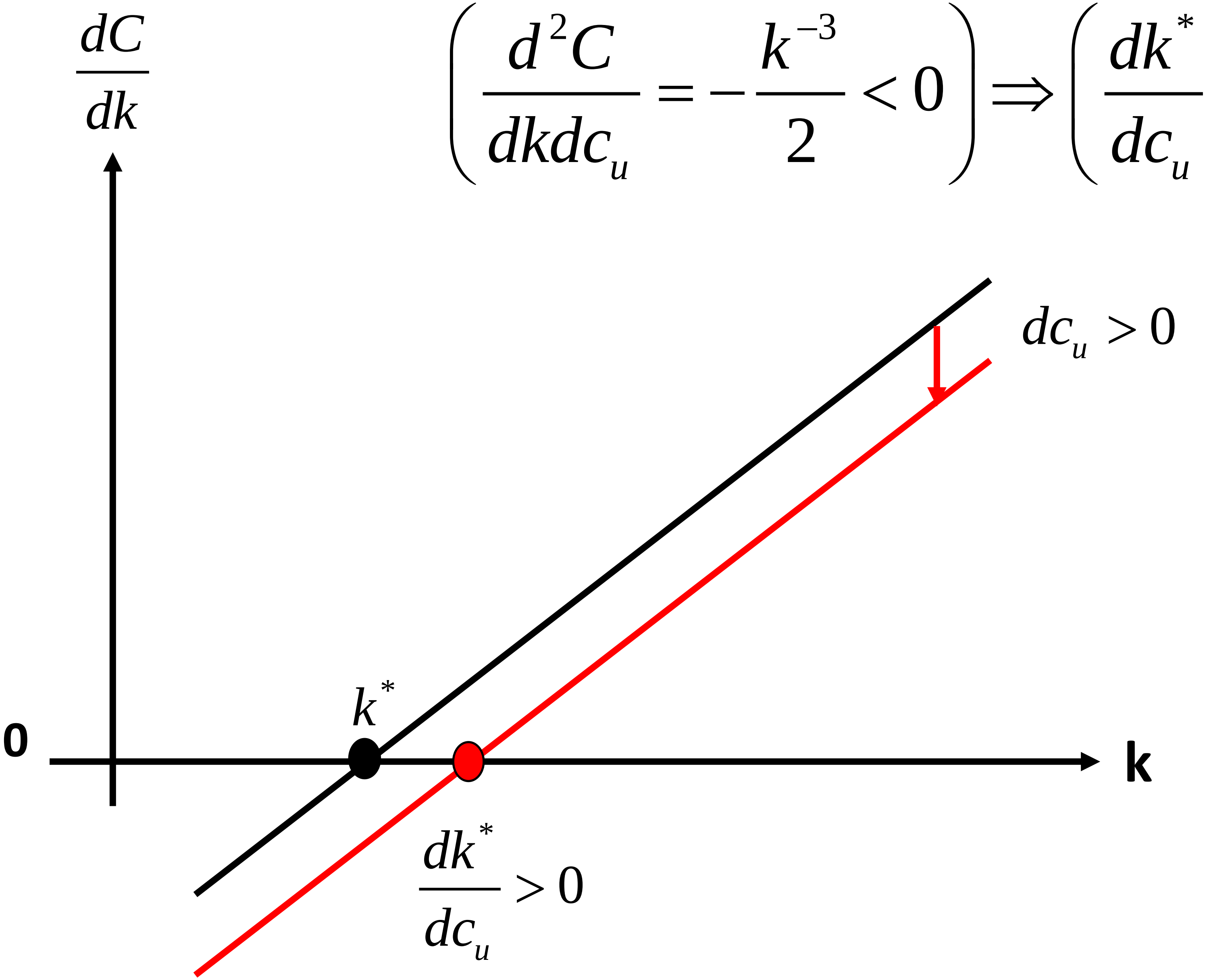
$$\left(d\left(\frac{dC}{dk} \right) = 0 \right) \Rightarrow \left(\frac{dk^*}{dc_u} = \frac{-\left(\frac{d^2C}{dkdc_u} \right)}{\left(\frac{d^2C}{dk^2} \right)} \right) \quad , \quad \frac{d^2C}{dk^2} > 0$$

$$\left(d\left(\frac{dC}{dk} \right) = 0 \right) \Rightarrow \left(\operatorname{sgn}\left(\frac{dk^*}{dc_u} \right) = -\operatorname{sgn}\left(\frac{d^2C}{dkdc_u} \right) \right)$$

$$\left(\frac{d^2C}{dkdc_u} = -\frac{k^{-3}}{2} < 0 \right) \Rightarrow \left(\frac{dk^*}{dc_u} > 0 \right)$$

If the cost per fire fighting unit increases,
the optimal distance between fire fighting units increases.

$$\left(\frac{d^2C}{dk\,dc_u} = -\frac{k^{-3}}{2} < 0\right) \Rightarrow \left(\frac{dk^*}{dc_u} > 0\right)$$

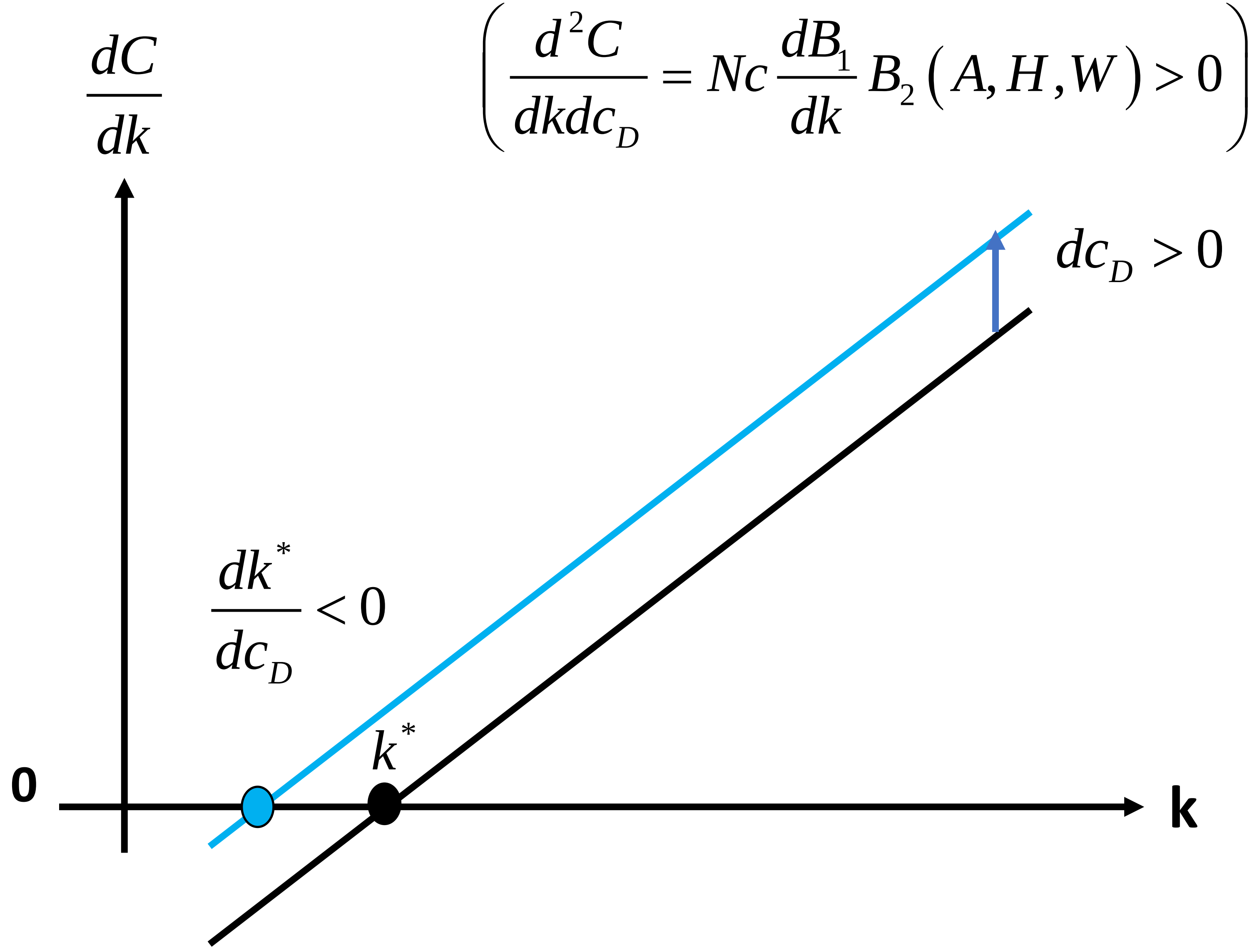


$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2C}{dkdc_D} = Nc\frac{dB_1}{dk} B_2\left(A, H, W\right) > 0\right) \Rightarrow \left(\frac{dk^*}{dc_D} < 0\right)$$

If the cost per fire damaged hectare of forest increases, the optimal distance between fire fighting units decreases. Note that, if the cost of CO_2 emissions is considered, the optimal value of k is lower than if we do not care about CO_2 emissions.

$$\left(\frac{d^2C}{dk\,dc_D} = Nc\frac{dB_1}{dk}B_2\left(A,H,W\right) > 0\right) \Rightarrow \left(\frac{dk^*}{dc_D} < 0\right)$$



$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2C}{dkdN} = c_D c \frac{dB_1}{dk} B_2\left(A, H, W\right) > 0\right) \Rightarrow \left(\frac{dk^*}{dN} < 0\right)$$

If the expected number of fires per area unit increases,
the optimal distance between fire fighting units decreases.

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2 C}{dk dc} = c_D N \frac{dB_1}{dk} B_2\left(A, H, W\right) > 0 \right) \Rightarrow \left(\frac{dk^*}{dc} < 0 \right)$$

If the general fire speed parameter c increases,
the optimal distance between fire fighting units decreases.
Possible reasons: More open terrain and/or
larger amounts of dry grass and other fuels.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1(m,k)}{dk} B_2(A,H,W) = 0$$

$$\left(\frac{d^2C}{dkdm} = c_D N c \frac{d^2 B_1}{dkdm} B_2(A,H,W) > 0 \right) \Rightarrow \left(\frac{dk^*}{dm} < 0 \right)$$

If m increases, the fires get more time to grow before fire supression starts, and the optimal distance between fire fighting units decreases.
Possible reasons: Less efficient and/or intensive fire surveillance, fog, smoke or worse road conditions, forcing the fire fighting units to decrease the travel speed.

Fire growth as a function of weather conditions

The comparative statics analysis now needs some empirical information.

$$B = 5.40687 \cdot H_R{}^{-4.40357} e^{0.027112T} e^{1.88944 \cdot 10^{-6} W^3}$$

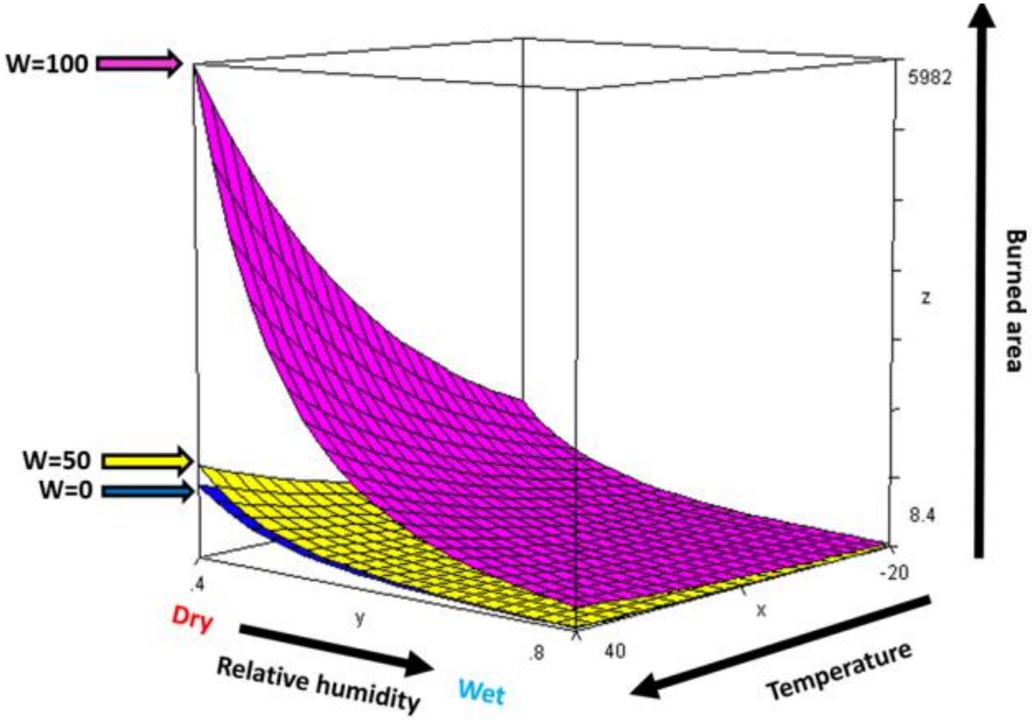


Fig. 7 Individual burned area (m^2) as a function of relative humidity (a value between 0 and 1), air temperature (°C) and wind speed (km h^{-1}) based on Eq. 19. The figure shows the simultaneous effects of relative humidity, air temperature and wind speed on forest wildfire size in the Prague-East District of the Czech Republic. Increases in air temperature and wind speed and a reduction in humidity increase the size of the wildfire

Mohammadi, Z., Lohmander, P., Kašpar, J. *et al.* **The effect of climate factors on the size of forest wildfires (case study: Prague-East district, Czech Republic)**. *J. For. Res.* (2021). https://doi.org/10.1007/s11676-021-01413-w

The empirical estimations gave these results:

$$B_2\left(A, H, W\right)$$

$$\frac{dB_2}{dA} > 0$$

$$\frac{dB_2}{dH} < 0$$

$$\frac{dB_2}{dW} > 0$$

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2 C}{dk dA} = c_D N c \frac{dB_1}{dk} \frac{dB_2\left(A, H, W\right)}{dA} > 0 \right) \Rightarrow \left(\frac{dk^*}{dA} < 0 \right)$$

If the air temperature increases, the speed of fire growth increases, and the optimal distance between fire fighting units decreases.

$$\frac{dC}{dk} = -\frac{c_u}{2} k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2 C}{dk dH} = c_D N c \frac{dB_1}{dk} \frac{dB_2\left(A, H, W\right)}{dH} < 0 \right) \Rightarrow \left(\frac{dk^*}{dH} > 0 \right)$$

If the relative humidity increases, the speed of fire growth decreases, and the optimal distance between fire fighting units increases.

$$\frac{dC}{dk} = -\frac{c_u}{2}k^{-3} + c_D N c \frac{dB_1}{dk} B_2\left(A, H, W\right) = 0$$

$$\left(\frac{d^2C}{dkdW} = c_D N c \frac{dB_1}{dk} \frac{dB_2\left(A, H, W\right)}{dW} > 0 \right) \Rightarrow \left(\frac{dk^*}{dW} < 0 \right)$$

If the wind speed increases, the speed of fire growth increases, and the optimal distance between fire fighting units decreases.

Comments on the optimimization problem

Above, we have assumed that labor employment constraints are applied, that require that the numbers of fire fighting units at different points in time are determined before the true weather component residuals are known.

Optimality conditions of the dynamically changing firefighting capacity levels have been analytically determined.

The solutions have been found to be unique minima.

Comparative statics analysis has been used to determine the directions of change of the optimal capacity levels under the influence of alternative parameter changes.

The expected fire sizes can also be numerically approximated via random numbers with relevant correlations, based on Cholesky factorization.

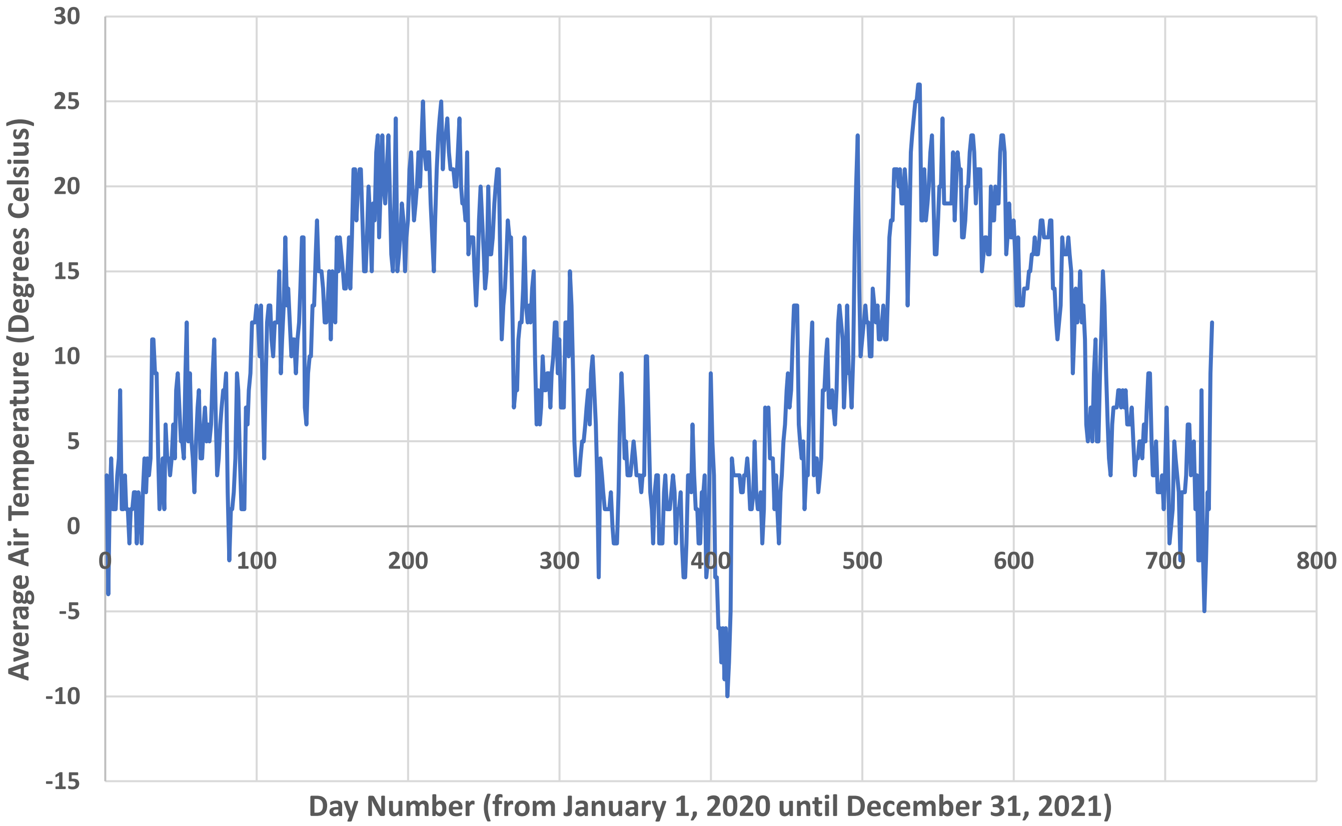
A stochastic dynamic programming version of the dynamic investment decision problem is also possible to present, based on a very flexible labor market, where the number of fire fighting units rapidly can be adapted to the sequentially revealed weather situation. However, since time is limited and since such labour conditions are not relevant in many countries, this will not be presented here.

Background to empirical weather and fire data

Thank you Dr Zohreh Mohammadi
for very valuable data and information!

Dynamic and stochastic properties of air temperature, relative humidity and wind speed

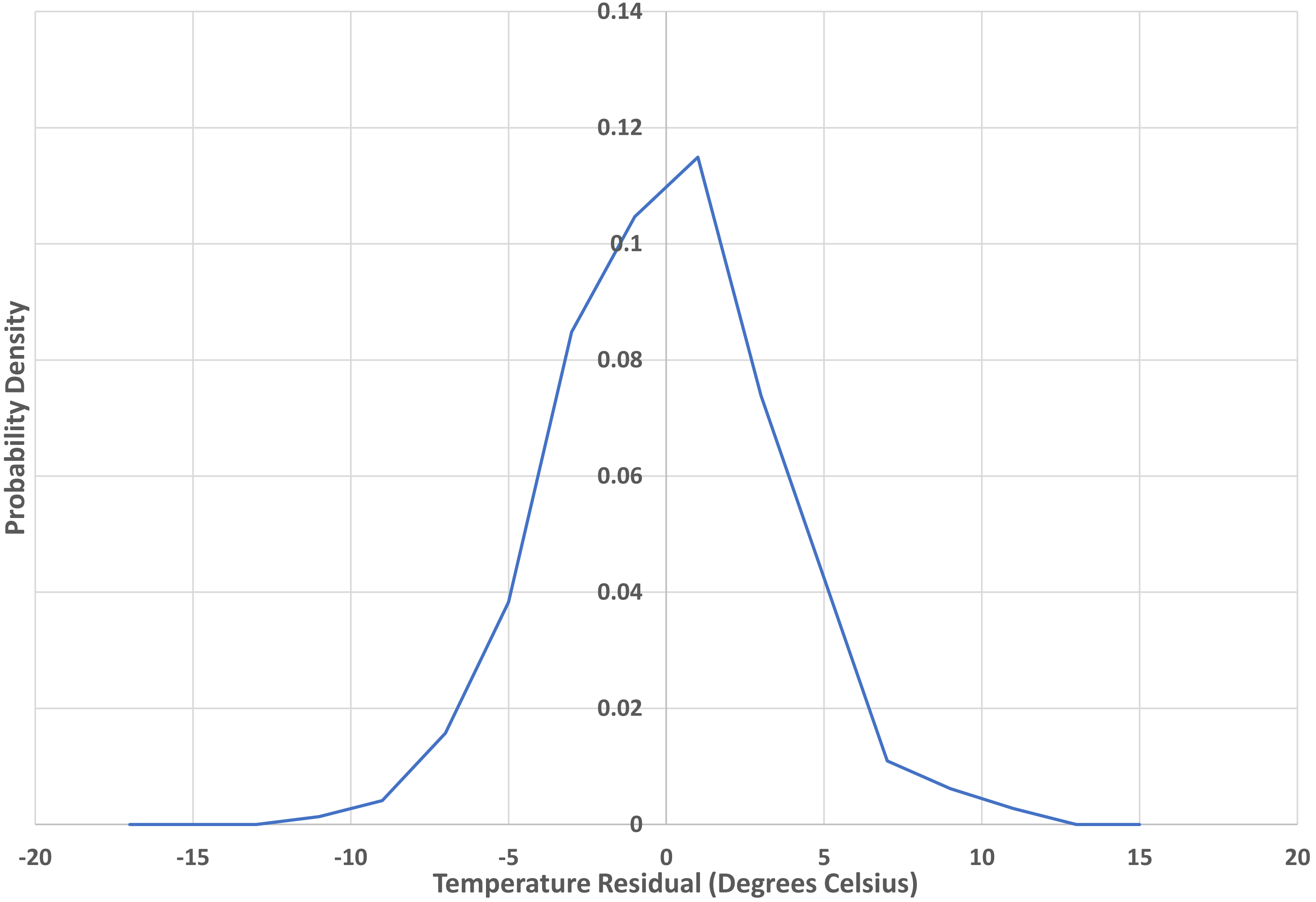
The following parameter estimations related to dynamic and stochastic properties of air temperature, relative humidity and wind speed, can be used in the fire fighting capacity optimization problem.

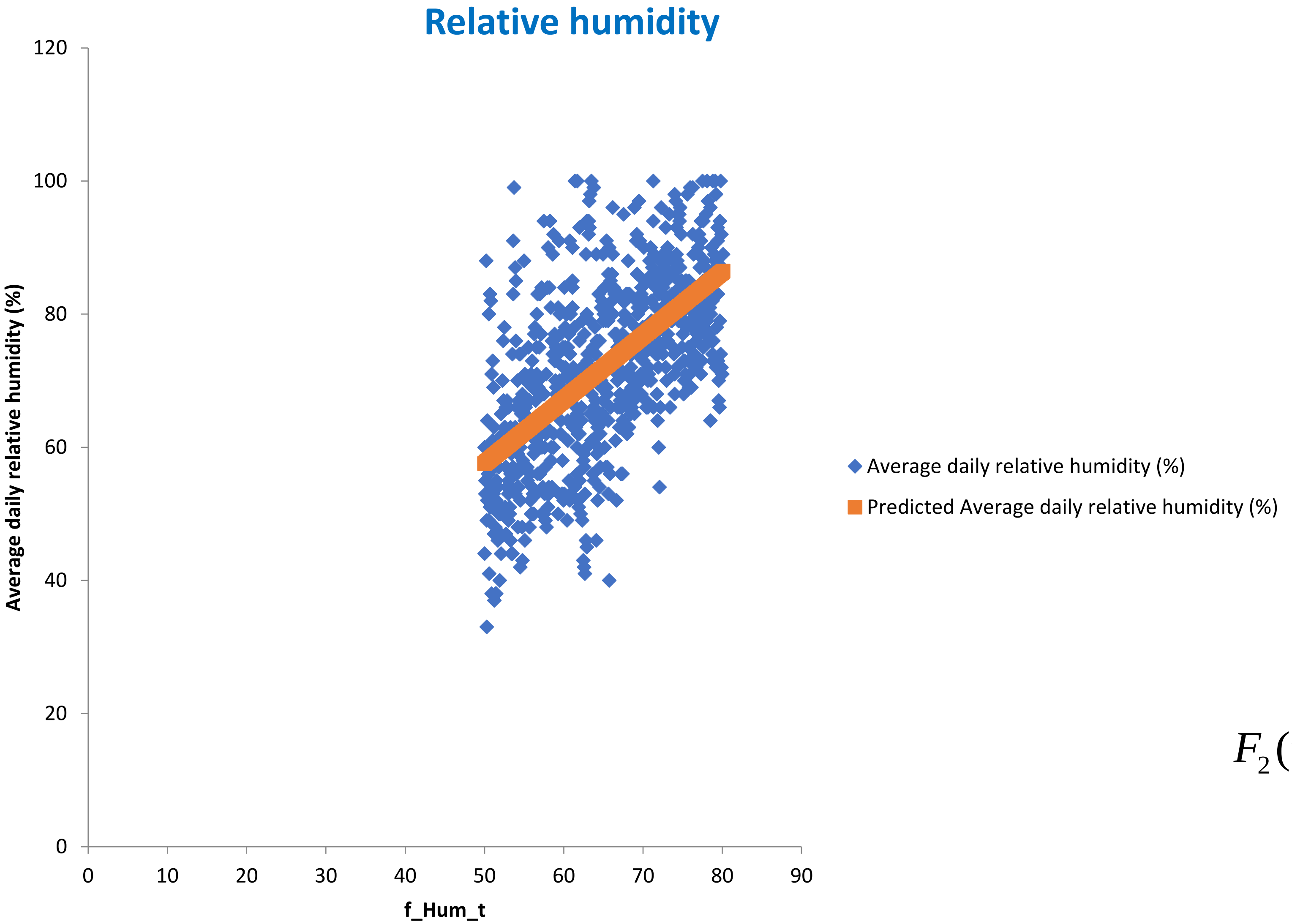


$$A(t) = a_0 + a_1 F_1(t) + \varepsilon_A(t)$$

$$s.t. \begin{cases} E(a_0) \approx 10.068 & \sigma_{a_0} \approx 0.131 \\ E(a_1) \approx \ \ 9.153 & \sigma_{a_1} \approx 0.185 \\ E(\varepsilon_A(t)) = 0 & \sigma_A \approx 3.531 \end{cases}$$

$$F_1(t) = \sin\left(\frac{4}{3} + 2\left(\frac{t}{365}\right)\pi\right)$$

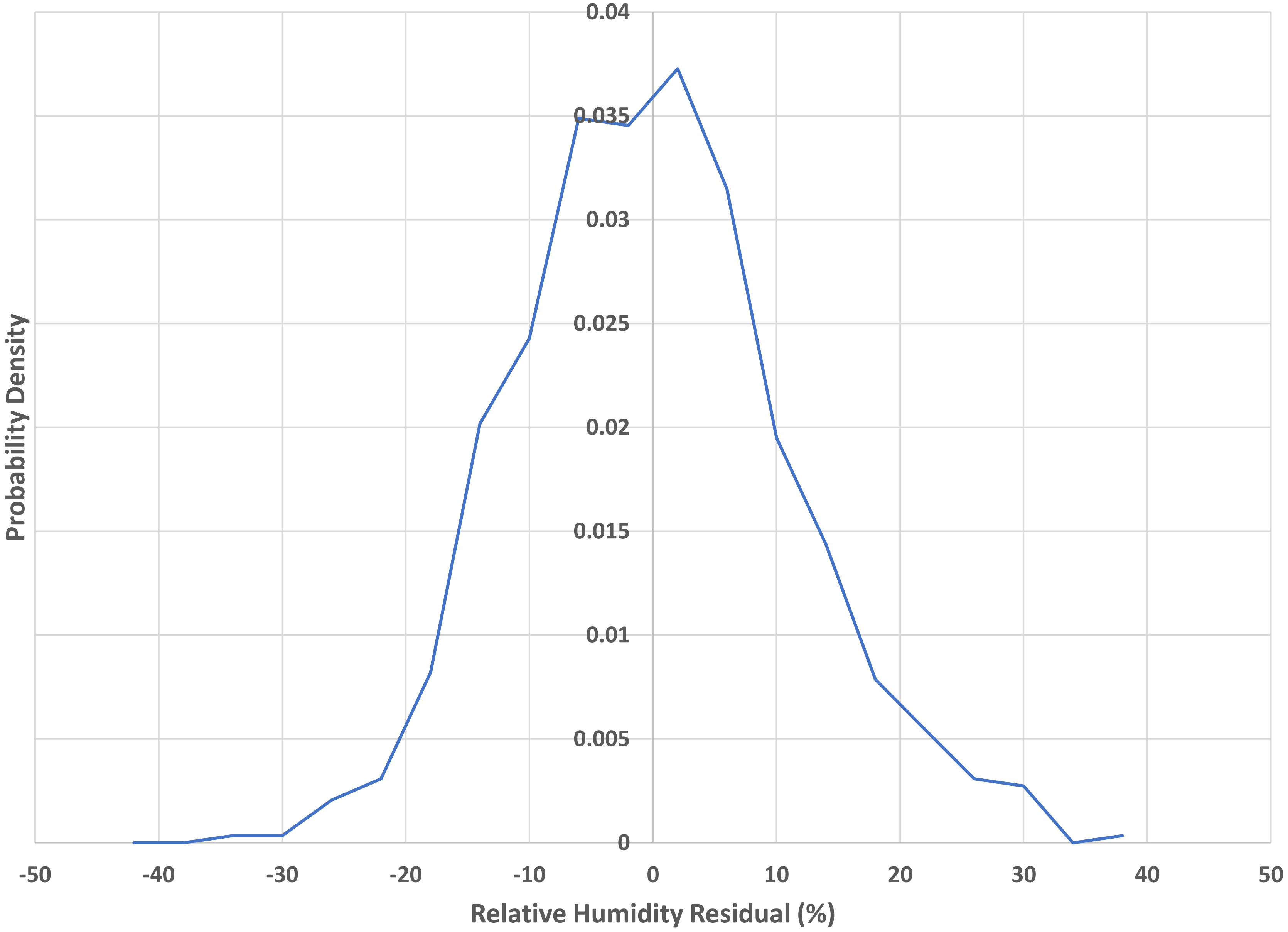

0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Probability Density
-20
-15
-10
-5
0
5
10
15
20
Temperature Residual (Degrees Celsius)



$$\bar{H}(t) = \bar{h}_0 + \bar{h}_1\, F_2(t) + \bar{\varepsilon}_H(t)$$

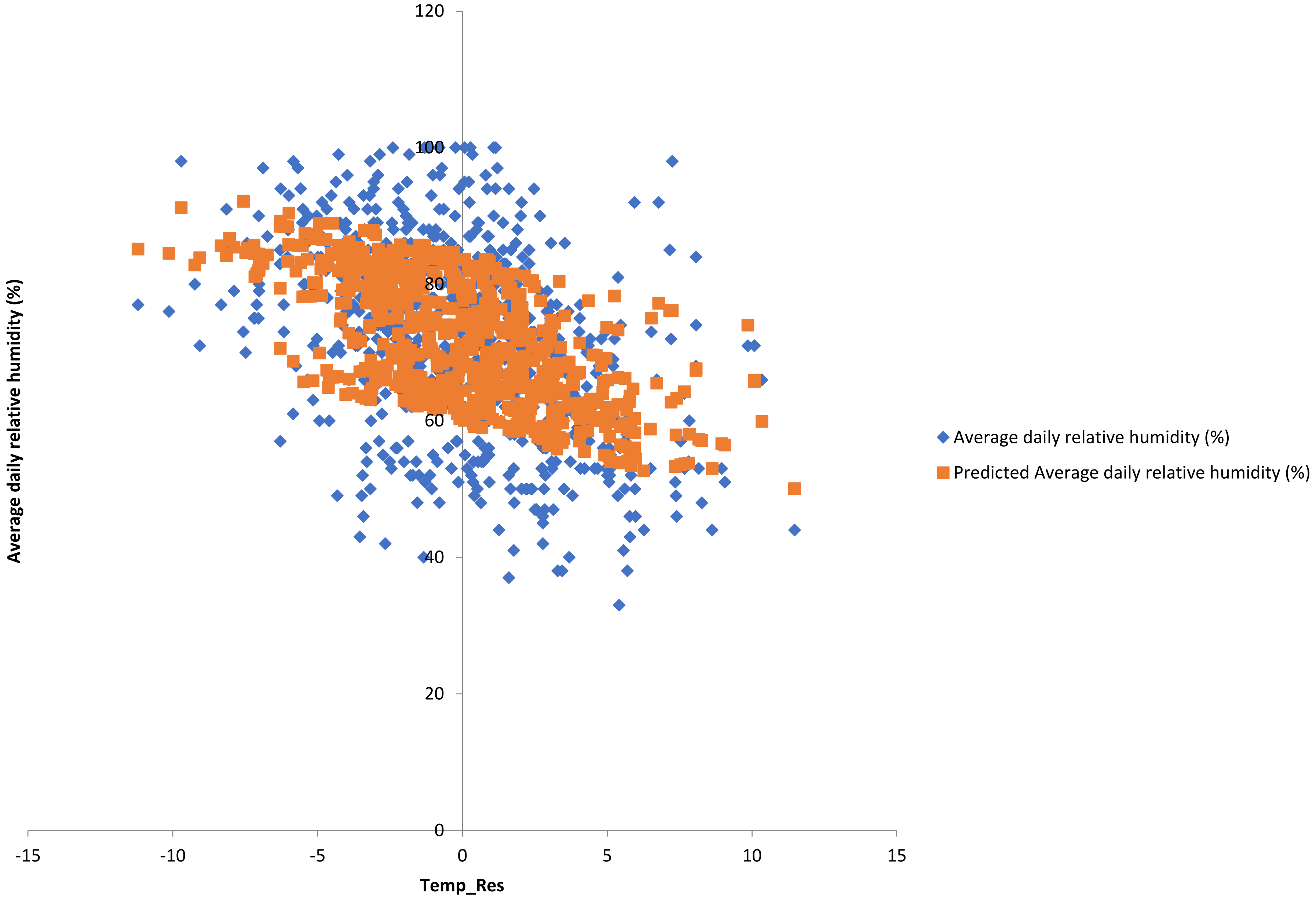
$$s.t.\begin{cases} E(\bar{h}_0) \approx 9.494 & \sigma_{\bar{h}_0} \approx 3.062 \\ E(\bar{h}_1) \approx 0.961 & \sigma_{\bar{h}_1} \approx 0.0467 \\ E(\bar{\varepsilon}_H(t)) = 0 & \sigma_{\bar{H}} \approx 10.945 \end{cases}$$

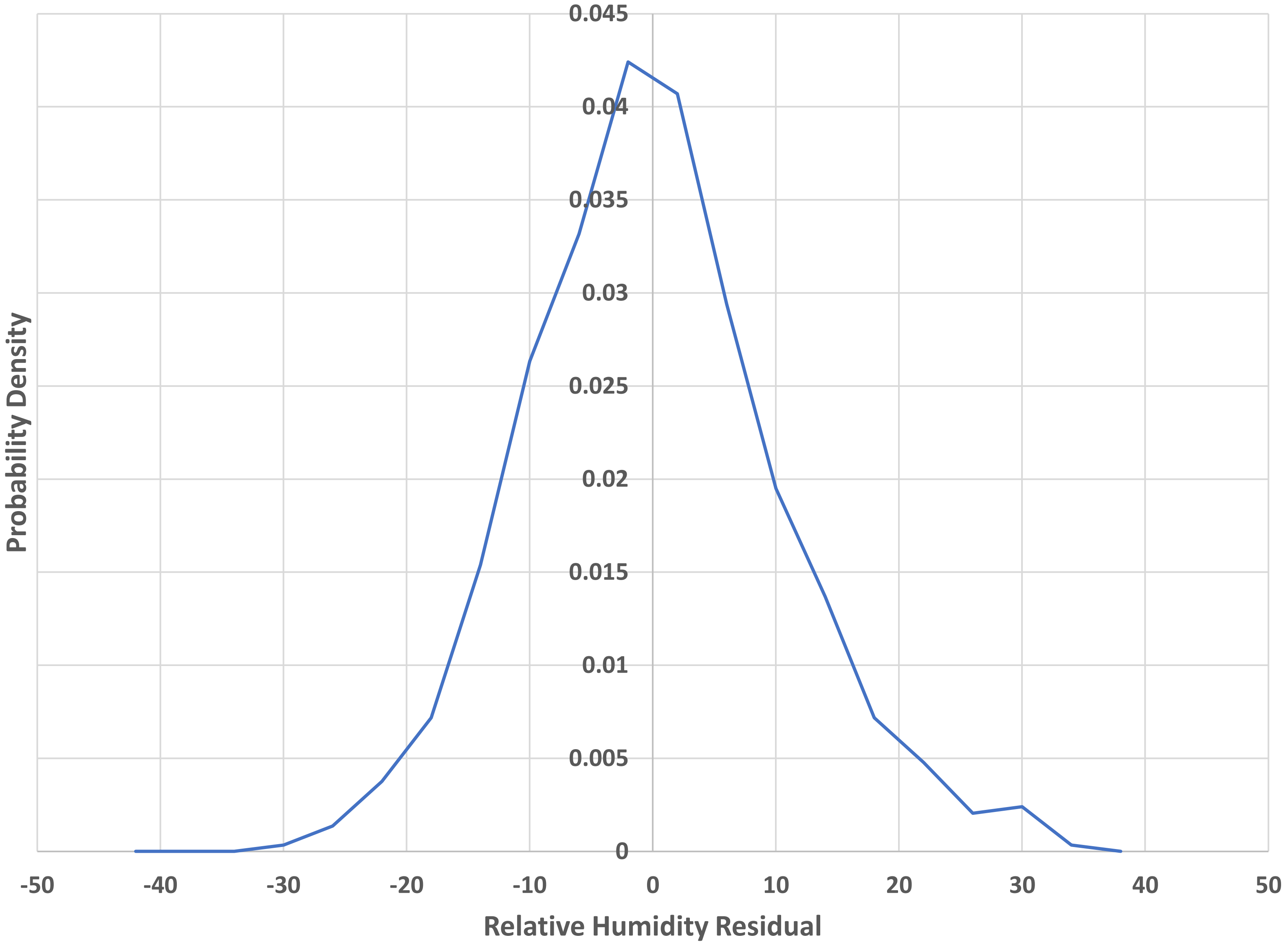
$$F_2(t) = \begin{cases} 80 - 0.3t & 0 \le t \le 100 \\ 50 + \dfrac{30}{265}\left(t - 100\right) & 100 < t \le 366 \end{cases}$$

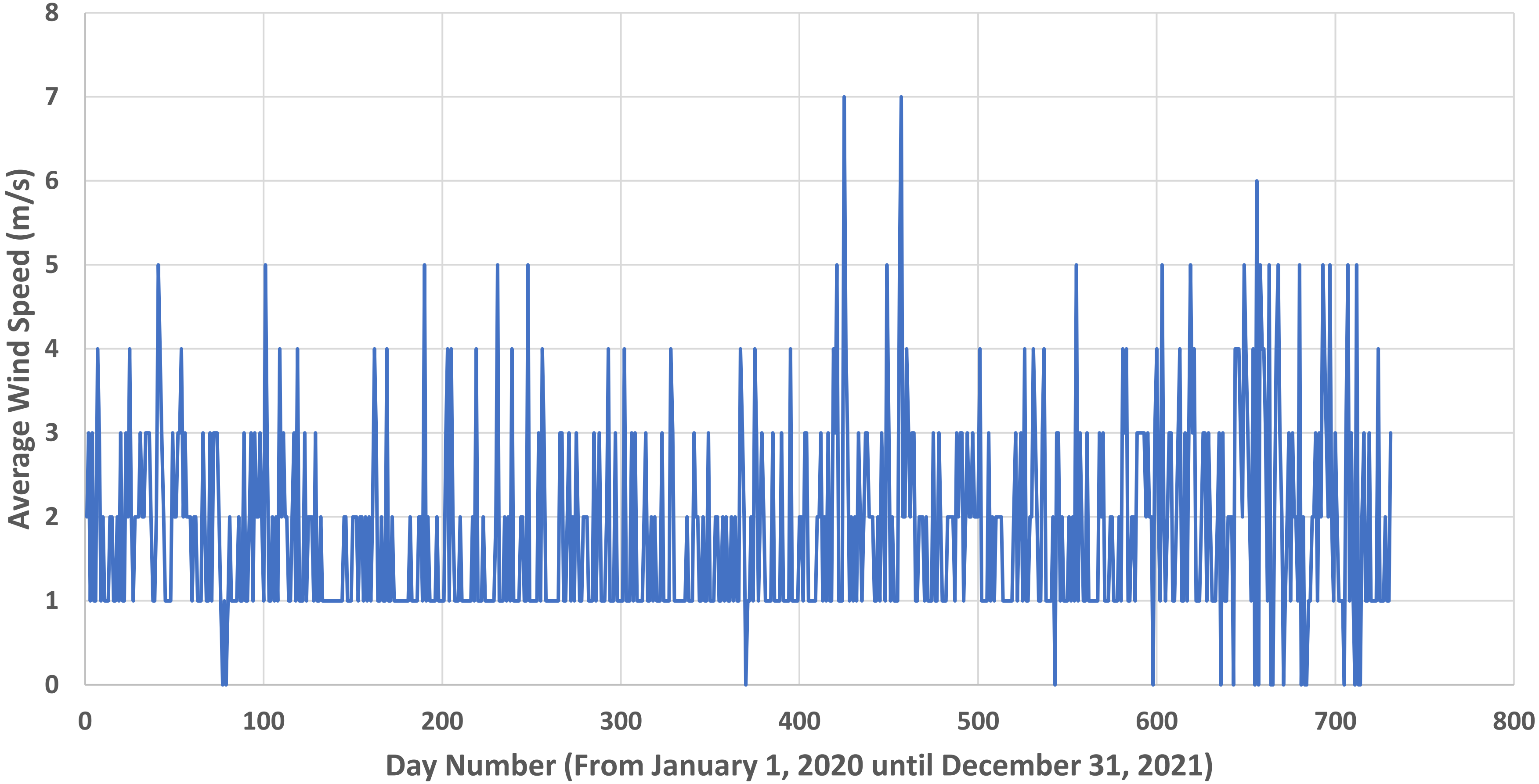

0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
-50
-40
-30
-20
-10
0
10
20
30
40
50
Probability Density
Relative Humidity Residual (%)

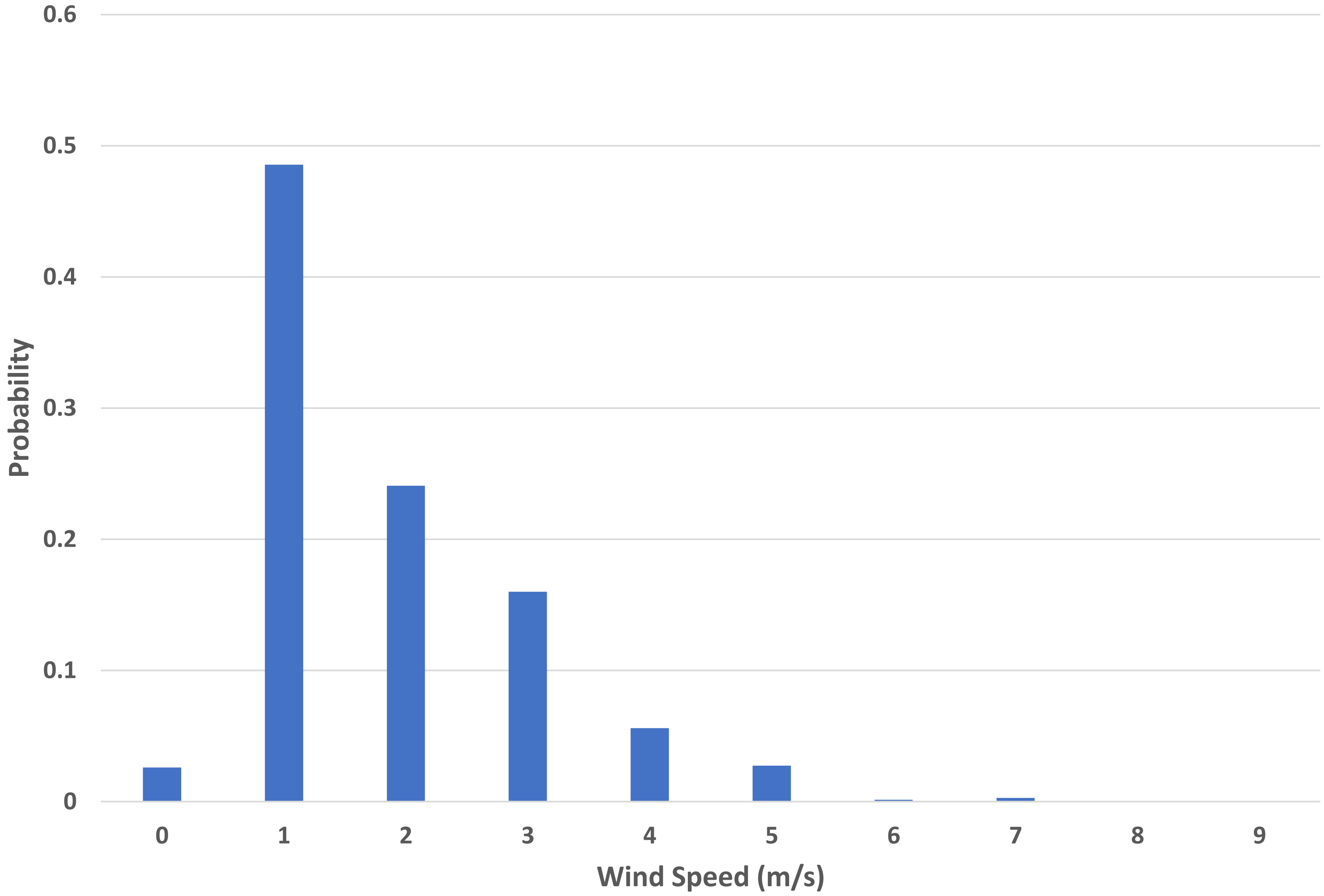
$$H(t) = h_0 + h_1 F_2(t) + h_2 \varepsilon_A(t) + \varepsilon_H(t)$$

$$s.t. \begin{cases} E(h_0) \approx 17.395 & \sigma_{h_0} \approx 3.010 \\ E(h_1) \approx 0.839 & \sigma_{h_1} \approx 0.0459 \\ E(h_2) \approx -1.067 & \sigma_{h_2} \approx 0.113 \\ E(\varepsilon_H(t)) = 0 & \sigma_H \approx 10.337 \end{cases}$$


120
100
80
60
40
20
0
-15
-10
-5
0
5
10
15
Average daily relative humidity (%)
Temp_Res
◆ Average daily relative humidity (%)
■ Predicted Average daily relative humidity (%)


0.045
0.04
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
Probability Density
-50
-40
-30
-20
-10
0
10
20
30
40
50
Relative Humidity Residual


Average Wind Speed (m/s)
0
1
2
3
4
5
6
7
8
0
100
200
300
400
500
600
700
800
Day Number (From January 1, 2020 until December 31, 2021)


0.6
0.5
0.4
0.3
0.2
0.1
0
Probability
0
1
2
3
4
5
6
7
8
9
Wind Speed (m/s)

Conclusions

The optimal fire fighting capacity is a dynamically changing function of several parameters. The optimal solution has been derived and presented in general form. Comparative statics analysis has been used to show how the optimal decisions are affected by parameter changes.

Since the optimal decisions are functions of many local cost parameters and since different regions have different and dynamically changing weather parameters, the optimal capacity solutions are not the same in different regions.

The general approach presented here should be possible to use in most countries and regions of our world.

THANK YOU VERY MUCH FOR YOUR TIME AND FOR INVITING ME TO PRAGUE!

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http://www.lohmander.com/Information/Ref.htm