

## OPTIMAL PRESENT RESOURCE EXTRACTION UNDER THE INFLUENCE OF FUTURE RISK

Peter Lohmander\*

Faculty of Forest Sciences, Swedish University of Agricultural Sciences, Umea, Sweden.

\*Professor Dr. (Speaker).  
Peter@Lohmander.com

### ABSTRACT

The analysis concerns determination of the optimal present extraction of a natural resource and how this is affected by different kinds of risk in the future. The most general definition of increasing risk, according to Rothschild and Stiglitz, is used. It can be applied to all types of statistical distributions. The approach is much more general than, for instance, increasing variance. The analysis is performed via general function multi dimensional analytical optimization and comparative dynamics analysis in discrete time. It is found that most of the analytical results can be derived via comparative dynamics in a system with three equations in combination with supporting general function analysis. The general analytical results are illustrated via computer solutions to numerically specified special cases.

**Keywords:** Optimal stochastic control; Risk; Natural resource management; Forestry; Third order derivatives.

### 1. INTRODUCTION

In real production processes, continuous adjustments of all activities are mostly not technically possible and would almost never be economically rational. The optimal extraction problems have often been studied with optimal stochastic control theory in continuous time. Then, however, the continuous time assumption and the common Wiener process assumption usually imply that derivatives of order three and higher are not needed in the derivations. The present analysis proves that, in discrete time, derivations of optimal decisions under risk have to take the third order derivatives into account. The most general definitions of increasing risk introduced and analyzed by Rothschild and Stiglitz [4] and [5] are used. Third order derivatives different from zero are often present in several parts of natural resource management problems in cost functions, growth functions and demand functions. Furthermore, third order derivatives different from zero can be very useful in approximation of capacity constraints and penalty functions. Earlier results of a similar nature have been derived via comparative dynamic analysis within stochastic dynamic programming problems. Such derivations are found in [1] and [2]. In those studies, you also find detailed analyses of the effects of stationarity in the stochastic processes. The effects of increasing future risk on the optimal present extraction level are usually also dependent on process stationarity. If the processes are stationary, the effects of increasing risk on optimal present extraction may be quite different from the case when the processes are nonstationary. These effects are also described in the present analysis.

### 2. MAIN RESULTS

In the first section, the price and/or cost risk in the next period increases. The direction of optimal adjustment of the present extraction level is then found to be a function of the third order derivatives of the profit functions in later time periods with respect to the extraction levels. If the signs of these derivatives are known and constant over time, it is possible to determine the sign of the optimal adjustment of the present extraction level. In the second section, the resource is growing. The optimal present extraction level is then studied under the influence of increasing risk in the growth process. The direction of optimal adjustment of the present

extraction is found to be a function of the third order derivatives of the profit functions in later time periods with respect to the extraction levels. In some cases, it is possible to determine the sign of the optimal adjustment of the present extraction level. In the third section, the resource contains different species, growing together. Furthermore, the total harvest in each period is constrained. The question is how the optimal present harvest of these species is affected by increasing price risk in one of the species. Again, it turns out that the direction of adjustment of the present extraction is a function of the third order derivatives. In some cases, it is possible to determine the signs of the optimal adjustments of the present extraction levels in the different species. An alternative way to optimize similar but not identical stochastic dynamic multi species management management problems is reported in [3].

### 3. REFERENCES

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