Problem definition:

$$\pi\left(V_1,t;.\right)=\pi_A\left(V_1,t;.\right)+\pi_B\left(V_1,t;.\right) \quad , \quad V_1>0,\ t>\varepsilon>0,\varepsilon\approx 0$$

$$V(s)=V_0 \quad 0\le s<\varepsilon$$

$$V(\varepsilon)=V_0-h_0$$

$$\frac{dV(s)}{ds}>0 \quad \varepsilon\le s<t$$

$$h(0)=V_0-V_1=h_0$$

$$V(\varepsilon)=V_0-h(0)=V_1$$

$$V_2=V_2(V_1,t)=V(t)$$

OBS: Maybe a continuous time definition of h is not necessary.

$$h(kt)=V_2(V_1,t)-V_1=h_1 \quad \forall k\in\{1,2,...,n\},n\to\infty$$

$$h(s)=0 \quad \forall s\ne kt,k\in\{0,1,2,...,n\},n\to\infty$$

$$V(kt+s)=V(s) \quad \forall\varepsilon\le s\le t,k\in\{0,1,2,...,n\},n\to\infty$$

$$V(kt+\varepsilon)=V(t+\varepsilon)=V_1 \quad \forall k\in\{0,2,...,n\},n\to\infty$$

$$\pi_A=P_{A_0}\left(V_0-V_1\right)-c+\frac{P_{A_1}\left(V_2(V_1,t)-V_1\right)-c}{e^{rt}-1}$$

$$F(0)=F_0=0$$

Particular values:

$$F(\varepsilon)=F_0+h(0)=V_0-V_1=h_0$$

$$F(t+\varepsilon)-F(t)=h(t)=h_1$$

$$F(kt+\varepsilon)-F(kt)=h(t)=h_1 \quad \forall k\in\{1,2,...,n\},n\to\infty$$

Generalized results:

$$F(s)=F(\varepsilon)=h_0 \quad \varepsilon \le s < t+\varepsilon$$

$$F(kt+s)=h_0+kh_1 \quad \forall \varepsilon \le s < t+\varepsilon, k\in\{1,2,.,n\}, n\to\infty$$

$$Y(s)=V(s)+F(s)$$

$$\pi_G=\int_0^\infty e^{\phi s}e^{-rs}\left(P_{B_1}Y(s)+P_{B_2}\frac{dY(s)}{ds}\right)ds$$

$$Y(s)=V(s)+F(s)\approx V_0+\frac{h(t)}{t}s=V_0+\frac{\left(V_2(V_1,t)-V_1\right)}{t}s$$

$$\pi_B=\int_0^\infty e^{-(r-\phi)s}\left(P_{B_1}\left(V_0+\frac{\left(V_2(V_1,t)-V_1\right)}{t}s\right)+P_{B_2}\frac{\left(V_2(V_1,t)-V_1\right)}{t}\right)ds$$

$$\pi_B=\int_0^\infty e^{-r_2 s}\left(P_{B_1}V_0+\left(P_{B_1}s+P_{B_2}\right)\frac{\left(V_2(V_1,t)-V_1\right)}{t}\right)ds \quad , \quad r_2=r-\phi$$

$$\pi_B=\int_0^\infty e^{-r_2 s}\left(P_{B_1}V_0+\left(P_{B_1}s+P_{B_2}\right)\frac{\left(V_2(V_1,t)-V_1\right)}{t}\right)ds$$

$$\pi_B=\int_0^\infty e^{-r_2 s}P_{B_1}V_0 ds+\int_0^\infty e^{-r_2 s}\left(P_{B_1}s+P_{B_2}\right)\frac{\left(V_2(V_1,t)-V_1\right)}{t}ds$$

$$\pi_B=P_{B_1}V_0\int_0^\infty e^{-r_2 s}ds+\frac{\left(V_2(V_1,t)-V_1\right)}{t}\int_0^\infty e^{-r_2 s}\left(P_{B_1}s+P_{B_2}\right)ds$$

$$\pi_B\approx\pi_G$$

<u>Maximization problem:</u>

$$\max_{V_1,t}\pi\left(V_1,t;.\right)=P_{A_0}\left(V_0-V_1\right)-c+\frac{P_{A_1}\left(V_2(V_1,t)-V_1\right)-c}{e^{rt}-1}+P_{B_1}V_0\int_0^\infty e^{-r_2 s}ds+\frac{\left(V_2(V_1,t)-V_1\right)}{t}\int_0^\infty e^{-r_2 s}\left(P_{B_1}s+P_{B_2}\right)ds$$

First order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \frac{P_{A_1}}{e^{rt}-1}\left(\frac{dV_2}{dV_1}-1\right) + \frac{\left(\frac{dV_2}{dV_1}-1\right)}{t}\int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds\left(\frac{\frac{dV_2}{dt}t - \left(V_2(V_1,t)-V_1\right)}{t^2}\right) = 0$$

Reformulations of the first order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \frac{P_{A_1}}{e^{rt}-1}\left(\frac{dV_2}{dV_1}-1\right) + \frac{\int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dV_1}-1\right) = 0$$

$$\frac{d\pi}{dV_1} = -P_{A_0} + \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\right)\left(\frac{dV_2}{dV_1}-1\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds\left(\frac{\frac{dV_2}{dt}t - \left(V_2(V_1,t)-V_1\right)}{t^2}\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \frac{\int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2(V_1,t)-V_1\right)}{t}\right) = 0$$

Final version of the first order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t} \right)\left(\frac{dV_2}{dV_1} - 1\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dV_1}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right) = 0$$

<u>OBS:</u> Here, it is possible to introduce ”MC = MR interpretations” and ”Faustmann modification interpretations”.

Observations concerning some second order derivatives and second order maximum conditions:

$$\frac{d^2\pi}{dV_1^2} = \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t} \right)\frac{d^2V_2}{dV_1^2} < 0 \quad , \quad r>0, t>0, P_{A_1}>0, P_{B_1}\geq 0, P_{B_2}\geq 0, \frac{d^2V_2}{dV_1^2}<0$$

The sign of $\frac{d^2\pi}{dt^2}$ is more complicated to determine. However, a unique maximum will be assumed.

$$\left|\frac{d^2\pi}{dt^2}\right| < 0, \quad \begin{vmatrix} \frac{d^2\pi}{dV_1^2} & \frac{d^2\pi}{dV_1dt} \\ \frac{d^2\pi}{dtdV_1} & \frac{d^2\pi}{dt^2} \end{vmatrix} > 0$$

Comparative statics analysis in the time interval dimension:

$$\frac{d^2\pi}{dt^2}dt^* + \frac{d^2\pi}{dtdP_B}dP_B = 0$$

$$\frac{d^2\pi}{dt^2}dt^* = -\frac{d^2\pi}{dtdP_B}dP_B$$

$$\frac{dt^*}{dP_B} = \frac{-\dfrac{d^2\pi}{dtdP_B}}{\dfrac{d^2\pi}{dt^2}}$$

$$\frac{d^2\pi}{dtdP_{B_1}} = \frac{\int_0^\infty e^{-r_2 s} s\, ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$$

$$\frac{d^2\pi}{dtdP_{B_2}} = \frac{\int_0^\infty e^{-r_2 s}\, ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$$

Observation: Let $t > 0 \wedge t \to 0.$ $\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$ is strictly positive for low values of V , zero for the MSY maximizing value of V , V_{MSY} , and strictly negative for higher values of V .

$$\frac{d^2\pi}{dtdP_{B_1}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

$$\frac{d^2\pi}{dtdP_{B_2}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

$$\frac{dt^*}{dP_{B_1}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

$$\frac{dt^*}{dP_{B_2}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

Hence, if P_B increases, ceteres paribus, the optimal harvest interval increases if $V_2 < V_{MSY}$.

The optimal harvest interval is not changed if $V_2 = V_{MSY}$ and the optimal harvest decreases if $V_2 > V_{MSY}$.

It is NOT optimal to increase the stock level above V_{MSY} .

Comparative statics analysis in the volume dimension:

$$\frac{d^2\pi}{dV_1^2}dV_1^* + \frac{d^2\pi}{dV_1 dP_{B_1}}dP_{B_1} = 0$$

$$\frac{d^2\pi}{dV_1^2}dV_1^* = -\frac{d^2\pi}{dV_1 dP_{B_1}}dP_{B_1}$$

$$\frac{dV_1^*}{dP_{B_1}} = \frac{-\dfrac{d^2\pi}{dV_1 dP_{B_1}}}{\dfrac{d^2\pi}{dV_1^2}}$$

$$\frac{d^2\pi}{dV_1^2}dV_1^* + \frac{d^2\pi}{dV_1 dP_{B_2}}dP_{B_2} = 0$$

$$\frac{d^2\pi}{dV_1^2}dV_1^* = -\frac{d^2\pi}{dV_1 dP_{B_2}}dP_{B_2}$$

$$\frac{dV_1^*}{dP_{B_2}} = \frac{-\dfrac{d^2\pi}{dV_1 dP_{B_2}}}{\dfrac{d^2\pi}{dV_1^2}}$$

Let us assume that growth, at least locally, can be approximated by the logistic equation:

$$V_2 = V_2(V_1, t)$$

The following equation is derived and explained in detail in

Lohmander, P., Zazykina, L., Methodology for optimization of continuous cover forestry with consideration of recreation and the forest and energy industries, Report and Abstract, Forests of Eurasia, Publishing House of Moscow State Forest University, September 19 - 25, 2010
http://www.lohmander.com/Moscow10/Moscow10_PL_LZ.pdf
http://www.lohmander.com/Moscow10/Moscow10_PL_LZ.doc
http://www.lohmander.com/Moscow_PL_2010.pdf
http://www.lohmander.com/Moscow_2010/Lohmander_Zazykina_Moscow_2010.ppt
http://www.lohmander.com/Moscow_2010/Programma-LE_10_01.doc

$$V_2 = V_2(V_1, t;.) = \frac{1}{\dfrac{1}{K} + \left(\dfrac{1}{V_1} - \dfrac{1}{K}\right)e^{-at}}$$

$$V_2 - V_1 = \frac{1}{\frac{1}{K} + \left(\frac{1}{V_1} - \frac{1}{K}\right)e^{-at}} - V_1$$

$$\frac{d\left(V_2 - V_1\right)}{dV_1} = \frac{dV_2}{dV_1} - 1$$

$$\frac{dV_2}{dV_1} - 1 = \frac{\left(1 - e^{at}\right)\left(V_1^2 e^{at} - \left(K - V_1\right)^2\right)}{\left(V_1 e^{at} + K - V_1\right)^2}$$

Assumptions:

$$at > 0 \Rightarrow \left(1 - e^{at}\right) < 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = \operatorname{sgn}\left(-\left(V_1^2 e^{at} - \left(K - V_1\right)^2\right)\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = \operatorname{sgn}\left(-\left(V_1^2 e^{at} - \left(K - V_1\right)^2\right)\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right) = \operatorname{sgn}\left(-V_1^2 e^{at} + \left(K - V_1\right)^2\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right) = \operatorname{sgn}\left(-V_1^2 e^{at} + K^2 - 2KV_1 + V_1^2\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right) = \operatorname{sgn}\left(V_1^2\left(1 - e^{at}\right) + K^2 - 2KV_1\right)$$

Calculations with very short time intervals:

$$t \to 0 \Rightarrow \left(1 - e^{at}\right) \to 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K^2 - 2KV_1\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K\left(K - 2V_1\right)\right)$$

Assumptions:

$$K > 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K - 2V_1\right) = \operatorname{sgn}\left(\frac{K}{2} - V_1\right)$$

Observations:

$$a.\quad V_1 < \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) > 0$$

$$b.\quad V_1 = \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = 0$$

$$c.\quad V_1 > \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) < 0$$

Calculations with longer time intervals:

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(V_1^2\left(1 - e^{at}\right) + K^2 - 2KV_1\right)$$

Hence, if t increases from zero, then $\left(1 - e^{at}\right)$ becomes more negative.

As a result, $\left(V_1^2\left(1 - e^{at}\right) + K^2 - 2KV_1\right)$ becomes more negative.

$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right)$ and $\frac{dV_2}{dV_1} - 1$ may become strictly negative even if $V_1 \le \frac{K}{2}$.

Conclusions in the volume dimension:

If the time interval is very short (approximately zero):

$\frac{dV_1^*}{dP_{B_1}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_1}}}{\frac{d^2\pi}{dV_1^2}}$ and $\frac{dV_1^*}{dP_{B_2}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_2}}}{\frac{d^2\pi}{dV_1^2}}$ are strictly positive for low values of V , zero for V that maximizes MSY and strictly negative for larger values of V .

Hence, if P_{B_1} and/or P_{B_2} increase(s), ceteres paribus, the optimal stock level converges to $V \approx V_{MSY}$.

It is NOT optimal to increase the stock level above V_{MSY} .

If the time interval is not very short:

$\frac{dV_1^*}{dP_{B_1}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_1}}}{\frac{d^2\pi}{dV_1^2}}$ and $\frac{dV_1^*}{dP_{B_2}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_2}}}{\frac{d^2\pi}{dV_1^2}}$ are strictly positive for low values of V (if the time interval is sufficiently short), equal to zero for a particular value of $V, V_1 < V_{MSY}$, and strictly negative for larger values of V.

Hence, if P_{B_1} and/or P_{B_2} increase(s), ceteres paribus, the optimal stock level converges to a value below V_{MSY}.

It is NOT optimal to increase the stock level above V_{MSY}.

Comparative statics analysis in two dimensions:

In the derivations below, we investigate the qualitative effects (directions of changes) of the decision variables in case the imaginary parameter P_B increases. Since $\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right)$ and $\operatorname{sgn}\left(\frac{d^2\pi}{dt dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dt dP_{B_2}}\right)$, and since complementary assumptions are made (below), the results derived and reported below with respect to the imaginary parameter P_B, are relevant to both of the the real parameters P_{B_1} and P_{B_2}.

$$\begin{bmatrix} \frac{d^2\pi}{dV_1^2} & \frac{d^2\pi}{dV_1 dt} \\ \frac{d^2\pi}{dt dV_1} & \frac{d^2\pi}{dt^2} \end{bmatrix} \begin{bmatrix} dV_1^* \\ dt^* \end{bmatrix} = \begin{bmatrix} -\frac{d^2\pi}{dV_1 dP_B} dP_B \\ -\frac{d^2\pi}{dt dP_B} dP_B \end{bmatrix}$$

$$[D] = \begin{bmatrix} \frac{d^2\pi}{dV_1^2} & \frac{d^2\pi}{dV_1 dt} \\ \frac{d^2\pi}{dt dV_1} & \frac{d^2\pi}{dt^2} \end{bmatrix}$$

$$\left|D\right| = \begin{vmatrix} \frac{d^2\pi}{dV_1^2} & \frac{d^2\pi}{dV_1 dt} \\ \frac{d^2\pi}{dt dV_1} & \frac{d^2\pi}{dt^2} \end{vmatrix} > 0$$

$$\frac{dV_1^*}{dP_B} = \frac{\begin{vmatrix} -\frac{d^2\pi}{dV_1 dP_B} & \frac{d^2\pi}{dV_1 dt} \\ -\frac{d^2\pi}{dt dP_B} & \frac{d^2\pi}{dt^2} \end{vmatrix}}{\left|D\right|} = \frac{-\frac{d^2\pi}{dV_1 dP_B}\frac{d^2\pi}{dt^2} + \frac{d^2\pi}{dt dP_B}\frac{d^2\pi}{dV_1 dt}}{\left|D\right|}$$

$$\frac{dt^*}{dP_B} = \frac{\begin{vmatrix} \frac{d^2\pi}{dV_1^2} & -\frac{d^2\pi}{dV_1 dP_B} \\ \frac{d^2\pi}{dt dV_1} & -\frac{d^2\pi}{dt dP_B} \end{vmatrix}}{\left|D\right|} = \frac{-\frac{d^2\pi}{dV_1^2}\frac{d^2\pi}{dt dP_B} + \frac{d^2\pi}{dt dV_1}\frac{d^2\pi}{dV_1 dP_B}}{\left|D\right|}$$

Special assumptions of relevance to the two dimensional comparative statics analysis:

$\frac{d^2\pi}{dt dV_1} = \varepsilon < 0$. $ABS(\varepsilon)$ is very small. We have a unique maximum. $\left|D\right| > 0$.

As a consequence,

$$\frac{dV_1^*}{dP_B} = \frac{\begin{vmatrix} -\frac{d^2\pi}{dV_1 dP_B} & \frac{d^2\pi}{dV_1 dt} \\ -\frac{d^2\pi}{dt dP_B} & \frac{d^2\pi}{dt^2} \end{vmatrix}}{\left|D\right|} = \frac{-\frac{d^2\pi}{dV_1 dP_B}\frac{d^2\pi}{dt^2} + \frac{d^2\pi}{dt dP_B}\varepsilon}{\left|D\right|} \approx \frac{-\frac{d^2\pi}{dV_1 dP_B}\frac{d^2\pi}{dt^2}}{\left|D\right|}$$

$$\frac{dt^*}{dP_B} = \frac{\begin{vmatrix} \frac{d^2\pi}{dV_1^2} & -\frac{d^2\pi}{dV_1 dP_B} \\ \frac{d^2\pi}{dt dV_1} & -\frac{d^2\pi}{dt dP_B} \end{vmatrix}}{\left|D\right|} = \frac{-\frac{d^2\pi}{dV_1^2}\frac{d^2\pi}{dt dP_B} + \varepsilon\frac{d^2\pi}{dV_1 dP_B}}{\left|D\right|} \approx \frac{-\frac{d^2\pi}{dV_1^2}\frac{d^2\pi}{dt dP_B}}{\left|D\right|}$$

$$\operatorname{sgn}\left(\frac{dV_1^*}{dP_B}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right)$$

$$\operatorname{sgn}\left(\frac{dt^*}{dP_B}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dt dP_B}\right)$$

Conclusions from the one and two dimensional comparative statics analyses:

If the initial value of V_1 is lower than V_{MSY} and the time interval t is sufficiently short, then V_1 and t increase if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is lower than V_{MAX} and the time interval t is sufficiently long, then V_1 and t may be unchanged or decrease if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is equal to V_{MSY} and the time interval t is very short, then V_1 and t are not changed if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is higher than V_{MSY} and/or the time interval t is sufficiently long, then V_1 and t decrease if P_B (P_{B_1} and/or P_{B_2}) increase(s).