Forest management optimization

- *considering biodiversity, global warming and economics goals*

Workshop at: Gorgan University of Agricultural Sciences and Natural Resources (GUASNR), November 2017

Part 3: Forest management optimization when production economics, global warming and biodiversity are considered *Version 17-11-06*

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Preparations before the workshop starts:

Lecture room preparations:

It is important that the lecture room has PC projector and necessary cables, screen etc.

It is also important that the lecture room has WiFi connection to the internet.

It is also important that the lecture room has a large whiteboard (at least 3 meters wide and one meter high) and pens with different colors. A large ruler (one meter length) makes the graphs and drawings better.

Preparations before the workshop starts:

Individual preparations:

Preparations to be made before the excercises:

During the excercices, we will use QB64.
It is important that the participants have access to laptops where QB64 has already been installed.
This software can be downloaded for free from this link: http://www.qb64.net/
It is also good if the participants have already installed Lingo.
Here is the link: http://www.lindo.com/index.php/products/lingo-and-optimization-modeling .
During the excecises, it is sufficient to have a simple version, which is free, of Lingo installed. Of course, for more advanced problems, a more advanced version is better. Advanced versions of Lingo can however be very expensive.

In the end of this document, you find the ”Workshop references”.
These references contain central theories and methods that will be discussed and used in the sessions. In the schedule, you find the references that are connected to the different sessions. All references may be downloaded from the internet. Please download the references as soon as possible and store them in your computer since internet disturbances may occur some days.

Workshop references of particular relevance to this presentation:

7,8,13,14,15,16,19,20,23,24,26

[7] Lohmander, P., The multi species forest stand, stochastic prices and adaptive selective thinning, SYSTEMS ANALYSIS - MODELLING - SIMULATION, Vol. 9, 229-250, 1992
http://www.Lohmander.com/PL_SAMS_9_1992.pdf

[8] Lohmander, P., Economic two stage multi period species management in a stochastic environment: The value of selective thinning options and stochastic growth parameters, SYSTEMS ANALYSIS - MODELLING -SIMULATION, Vol. 11, 287-302, 1993
http://www.Lohmander.com/PL_SAMS_11_1993.pdf

[13] Lohmander, P., Guidelines for Economically Rational and Coordinated Dynamic Development of the Forest and Bio Energy Sectors with CO2 constraints, Proceedings from the 16th European Biomass Conference and Exhibition, Valencia, Spain, 02-06 June, 2008 (In the version in the link, below, an earlier misprint has been corrected.)
http://www.Lohmander.com/Valencia2008.pdf

[14] Lohmander, P., Software for illustration of the CO2 and forest management issue in combination with CCS technology, 2008,
http://www.lohmander.com/co2ill2/co2ill2.htm

[15] Lu, F., Lohmander, P., Optimal Decisions for Mixed Forests under Risk, Scientia Silvae Sinicae, Vol. 45, No. 11, Nov. 2009
http://www.Lohmander.com/Lu_Lohmander_2009.pdf

[16] Lohmander, P., Zazykina, L., Rational and sustainable utilization of forest resources with consideration of recreation and tourism, bioenergy, the global warming problem, paper pulp and timber production: A mathematical approach, Proceedings of the II international workshop on Ecological tourism, Trends and perspectives on development in the global world, Saint Petersburg Forest Technical Academy, April 15-16, 2010
http://www.Lohmander.com/SPb201004/Lohmander_Zazykina_SPbFTA_2010.pdf
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[19] Lohmander, P., **With expanded bioenergy based on forest resources, we may simultaneously and sustainably reduce global warming, improve economic results, international relations and environmental conditions**, BIT'S 4th Annual World Congress of Bioenergy-2014, Qingdao International Convention Center, China
http://www.Lohmander.com/PLWCBE2014A.pptx
http://www.Lohmander.com/PLWCBE2014A.pdf
http://www.Lohmander.com/WCBE2014_Expansion.pdf
http://www.bitcongress.com/wcbe2014/

[20] Mohammadi Limaei, S., Lohmander, P., Olsson, L., **Sub models for optimal continuous cover multi species forestry in Iran**,
The 8th International Conference of Iranian Operations Research Society,
Department of Mathematics, Ferdowsi University of Mashhad, Mashhad, Iran, 21-22 May 2015

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http://www.Lohmander.com/SML_PL_LO_IRAN_ABS_2015.pdf
http://www.Lohmander.com/PL_IRAN_B_2015.pdf
http://www.Lohmander.com/PL_IRAN_B_2015.pptx
http://www.Lohmander.com/PL_OR8_Abs_15.pdf
http://www.or8.um.ac.ir

[23] Lohmander, P., **A general dynamic function for the basal area of individual trees derived from a production theoretically motivated autonomous differential equation**, National Conference on the Caspian Forests of Iran, "Past, Current, Future", University of Guilan, Rasht, Iran, April 26-27, 2017
http://www.Lohmander.com/PPT_GUILAN_PL_DIFF_2017.pptx
http://www.Lohmander.com/PPT_GUILAN_PL_DIFF_2017.pdf
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http://conf.isc.gov.ir/forestnorth

[24] Hatami, N., Lohmander, P., Moayeri, M.H., Mohammadi Limaei, S., **A basal area increment model for individual trees in mixed species continuous cover stands in Iranian Caspian forests**, National Conference on the Caspian Forests of Iran, "Past, Current, Future", University of Guilan, Rasht, Iran, April 26-27, 2017
http://www.Lohmander.com/PPT_GUILAN_JOINT_2017.ppt
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Case

- The following model is used to derive general conclusions about how the optimal forest management decisions change if the importance of storing carbon increases.
- The model can also be used as a background to particular numerically specified models to be solved by Lingo.
- Later, such Lingo models will be defined and analyzed with changing parameter values.

Problem definition:

$$\pi\left(V_1,t;.\right)=\pi_A\left(V_1,t;.\right)+\pi_B\left(V_1,t;.\right) \quad , \quad V_1>0,\ t>\varepsilon>0,\varepsilon\approx 0$$

$$V(s)=V_0 \quad 0\leq s<\varepsilon$$

$$V(\varepsilon)=V_0-h_0$$

$$\frac{dV(s)}{ds} > 0 \quad \varepsilon \le s < t$$

$$h(0) = V_0 - V_1 = h_0$$

$$V(\varepsilon) = V_0 - h(0) = V_1$$

$$V_2 = V_2(V_1, t) = V(t)$$

OBS: ***Maybe a continuous time definition of h is not necessary.***

$$h(kt) = V_2(V_1, t) - V_1 = h_1 \quad \forall k \in \{1, 2, ..., n\}, n \to \infty$$

$$h(s) = 0 \quad \forall s \neq kt, k \in \{0, 1, 2, ..., n\}, n \to \infty$$

$$V(kt + s) = V(s) \quad \forall \varepsilon \leq s \leq t, k \in \{0, 1, 2, ..., n\}, n \to \infty$$

$$V(kt + \varepsilon) = V(t + \varepsilon) = V_1 \quad \forall k \in \{0, 2, ..., n\}, n \to \infty$$

$$\pi_A = P_{A_0}\left(V_0 - V_1\right) - c + \frac{P_{A_1}\left(V_2(V_1, t) - V_1\right) - c}{e^{rt} - 1}$$

Particular values:

$$F(0) = F_0 = 0$$

$$F(\varepsilon) = F_0 + h(0) = V_0 - V_1 = h_0$$

$$F(t+\varepsilon) - F(t) = h(t) = h_1$$

$$F(kt+\varepsilon) - F(kt) = h(t) = h_1 \quad \forall k \in \{1, 2, \ldots, n\}, n \to \infty$$

Generalized expressions:

$$F(s) = F(\varepsilon) = h_0 \quad \varepsilon \le s < t + \varepsilon$$

$$F(kt + s) = h_0 + kh_1 \quad \forall \varepsilon \le s < t + \varepsilon, k \in \{1, 2, .., n\}, n \to \infty$$

$$Y(s) = V(s) + F(s)$$

$$\pi_G = \int_0^\infty e^{\phi s} e^{-rs} \left(P_{B_1} Y(s) + P_{B_2} \frac{dY(s)}{ds} \right) ds$$

$$Y(s) = V(s) + F(s) \approx V_0 + \frac{h(t)}{t} s = V_0 + \frac{\left(V_2(V_1, t) - V_1\right)}{t} s$$

$$\pi_G = \int_0^\infty e^{\phi s} e^{-rs} \left(P_{B_1} Y(s) + P_{B_2} \frac{dY(s)}{ds} \right) ds$$

$$\pi_B = \int_0^\infty e^{-(r-\phi)s} \left(P_{B_1} \left(V_0 + \frac{\left(V_2(V_1,t) - V_1\right)}{t} s \right) + P_{B_2} \frac{\left(V_2(V_1,t) - V_1\right)}{t} \right) ds$$

$$\pi_B = \int_0^\infty e^{-r_2 s} \left(P_{B_1} V_0 + \left(P_{B_1} s + P_{B_2} \right) \frac{\left(V_2(V_1,t) - V_1\right)}{t} \right) ds \quad , \quad r_2 = r - \phi$$

$$\pi_B = \int_0^\infty e^{-r_2 s}\left(P_{B_1}V_0 + \left(P_{B_1}s + P_{B_2}\right)\frac{\left(V_2(V_1,t) - V_1\right)}{t}\right)ds$$

$$\pi_B = \int_0^\infty e^{-r_2 s} P_{B_1}V_0 ds + \int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)\frac{\left(V_2(V_1,t) - V_1\right)}{t}ds$$

$$\pi_B = P_{B_1}V_0\int_0^\infty e^{-r_2 s} ds + \frac{\left(V_2(V_1,t) - V_1\right)}{t}\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds$$

$$\pi_B \approx \pi_G$$

$$\pi_G = \int_0^\infty e^{\phi s} e^{-rs} \left(P_{B_1} Y(s) + P_{B_2} \frac{dY(s)}{ds} \right) ds$$

$$\pi_B = P_{B_1} V_0 \int_0^\infty e^{-r_2 s} ds + \frac{\left(V_2(V_1, t) - V_1\right)}{t} \int_0^\infty e^{-r_2 s} \left(P_{B_1} s + P_{B_2}\right) ds$$

$$\pi(V_1, t;.) = \pi_A(V_1, t;.) + \pi_B(V_1, t;.) \quad , \quad V_1 > 0,\ t > \varepsilon > 0, \varepsilon \approx 0$$

Maximization problem:

$$\max_{V_1,t} \pi(V_1, t;.) = P_{A_0}(V_0 - V_1) - c + \frac{P_{A_1}\left(V_2(V_1,t) - V_1\right) - c}{e^{rt} - 1} + P_{B_1} V_0 \int_0^\infty e^{-r_2 s} ds + \frac{\left(V_2(V_1,t) - V_1\right)}{t} \int_0^\infty e^{-r_2 s}\left(P_{B_1} s + P_{B_2}\right) ds$$

First order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \frac{P_{A_1}}{e^{rt}-1}\left(\frac{dV_2}{dV_1}-1\right) + \frac{\left(\frac{dV_2}{dV_1}-1\right)}{t}\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds\left(\frac{\frac{dV_2}{dt}t - \left(V_2\left(V_1,t\right)-V_1\right)}{t^2}\right) = 0$$

Reformulations of the first order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \frac{P_{A_1}}{e^{rt}-1}\left(\frac{dV_2}{dV_1}-1\right) + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dV_1}-1\right) = 0$$

$$\frac{d\pi}{dV_1} = -P_{A_0} + \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\right)\left(\frac{dV_2}{dV_1}-1\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right)-\left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \int_0^\infty e^{-r_2 s}\left(P_{B_1}s+P_{B_2}\right)ds\left(\frac{\frac{dV_2}{dt}t-\left(V_2\left(V_1,t\right)-V_1\right)}{t^2}\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dt}\left(e^{rt}-1\right)-\left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s+P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dt}-\frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right) = 0$$

Final version of the first order optimum conditions:

$$\frac{d\pi}{dV_1} = -P_{A_0} + \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t} \right)\left(\frac{dV_2}{dV_1} - 1\right) = 0$$

$$\frac{d\pi}{dt} = \frac{P_{A_1}\frac{dV_2}{dV_1}\left(e^{rt}-1\right) - \left(P_{A_1}\left(V_2(V_1,t)-V_1\right)-c\right)re^{rt}}{\left(e^{rt}-1\right)^2} + \frac{\int_0^\infty e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right) = 0$$

OBS: *Here, it is possible to introduce ”MC = MR interpretations” and modified ”Faustmann type interpretations”.*

Observations concerning some second order derivatives and second order maximum conditions:

$$\frac{d^2\pi}{dV_1^{\,2}} = \left(\frac{P_{A_1}}{e^{rt}-1} + \frac{\int_0^{\infty} e^{-r_2 s}\left(P_{B_1}s + P_{B_2}\right)ds}{t} \right) \frac{d^2V_2}{dV_1^{\,2}} < 0 \quad , \quad r>0, t>0, P_{A_1}>0, P_{B_1}\geq 0, P_{B_2}\geq 0, \frac{d^2V_2}{dV_1^{\,2}}<0$$

The sign of $\frac{d^2\pi}{dt^2}$ **is more complicated to determine.**

However, a unique maximum will be assumed.

$$\left|\frac{d^2\pi}{dt^2}\right| < 0 \qquad \begin{vmatrix} \frac{d^2\pi}{dV_1^2} & \frac{d^2\pi}{dV_1 dt} \\ \frac{d^2\pi}{dt dV_1} & \frac{d^2\pi}{dt^2} \end{vmatrix} > 0$$

Comparative statics analysis in the time interval dimension:

$$\frac{d^2\pi}{dt^2}dt^* + \frac{d^2\pi}{dtdP_B}dP_B = 0$$

$$\frac{d^2\pi}{dt^2}dt^* = -\frac{d^2\pi}{dtdP_B}dP_B$$

$$\frac{dt^*}{dP_B} = \frac{-\frac{d^2\pi}{dtdP_B}}{\frac{d^2\pi}{dt^2}}$$

$$\frac{d^2\pi}{dtdP_{B_1}} = \frac{\int_0^\infty e^{-r_2 s} s\, ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$$

$$\frac{d^2\pi}{dtdP_{B_2}} = \frac{\int_0^\infty e^{-r_2 s}\, ds}{t}\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$$

Observation: Let $t > 0 \wedge t \to 0.$ $\left(\frac{dV_2}{dt} - \frac{\left(V_2\left(V_1,t\right)-V_1\right)}{t}\right)$ is strictly positive for low values of V , zero for the MSY maximizing value of V , V_{MSY} , and strictly negative for higher values of V .

$$\frac{d^2\pi}{dtdP_{B_1}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases} \qquad \frac{dt^*}{dP_{B_1}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

$$\frac{d^2\pi}{dtdP_{B_2}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases} \qquad \frac{dt^*}{dP_{B_2}} = \begin{cases} >0 & if \quad V_2 < V_{MSY} \\ =0 & if \quad V_2 = V_{MSY} \\ <0 & if \quad V_2 > V_{MSY} \end{cases}$$

Hence, if P_B increases, ceteres paribus, the optimal harvest interval increases if $V_2 < V_{MSY}$.

The optimal harvest interval is not changed if $V_2 = V_{MSY}$ and the optimal harvest decreases if $V_2 > V_{MSY}$.

It is NOT optimal to increase the stock level above V_{MSY} .

Comparative statics analysis in the volume dimension:

$$\frac{d^2\pi}{dV_1^2}dV_1^* + \frac{d^2\pi}{dV_1 dP_{B_1}}dP_{B_1} = 0$$

$$\frac{d^2\pi}{dV_1^2}dV_1^* = -\frac{d^2\pi}{dV_1 dP_{B_1}}dP_{B_1}$$

$$\frac{dV_1^*}{dP_{B_1}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_1}}}{\frac{d^2\pi}{dV_1^2}}$$

$$\frac{d^2\pi}{dV_1^2} dV_1^* + \frac{d^2\pi}{dV_1 dP_{B_2}} dP_{B_2} = 0$$

$$\frac{d^2\pi}{dV_1^2} dV_1^* = -\frac{d^2\pi}{dV_1 dP_{B_2}} dP_{B_2}$$

$$\frac{dV_1^*}{dP_{B_2}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_2}}}{\frac{d^2\pi}{dV_1^2}}$$

Let us assume that growth, at least locally, can be approximated by the logistic equation:

$$V_2 = V_2(V_1, t)$$

The equation is derived and explained in detail in

Lohmander, P., Zazykina, L., Methodology for optimization of continuous cover forestry with consideration of recreation and the forest and energy industries, Report and Abstract, Forests of Eurasia, Publishing House of Moscow State Forest University, September 19 - 25, 2010
http://www.lohmander.com/Moscow10/Moscow10_PL_LZ.pdf
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http://www.lohmander.com/Moscow_2010/Programma-LE_10_01.doc

$$V_2 = V_2(V_1, t; .) = \frac{1}{\frac{1}{K} + \left(\frac{1}{V_1} - \frac{1}{K}\right) e^{-at}}$$

$$V_2 - V_1 = \frac{1}{\frac{1}{K} + \left(\frac{1}{V_1} - \frac{1}{K}\right) e^{-at}} - V_1$$

$$\frac{d\left(V_2 - V_1\right)}{dV_1} = \frac{dV_2}{dV_1} - 1$$

$$\frac{dV_2}{dV_1} - 1 = \frac{\left(1 - e^{at}\right)\left(V_1^{\,2} e^{at} - \left(K - V_1\right)^2\right)}{\left(V_1 e^{at} + K - V_1\right)^2}$$

Assumptions:

$$at > 0 \Rightarrow \left(1 - e^{at}\right) < 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = \operatorname{sgn}\left(-\left(V_1^2 e^{at} - \left(K - V_1\right)^2\right)\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = \operatorname{sgn}\left(-\left(V_1^2 e^{at} - \left(K - V_1\right)^2\right)\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right)=\operatorname{sgn}\left(-V_1^{\,2}e^{at}+\left(K-V_1\right)^2\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right)=\operatorname{sgn}\left(-V_1^{\,2}e^{at}+K^2-2KV_1+V_1^{\,2}\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right)=\operatorname{sgn}\left(V_1^{\,2}\left(1-e^{at}\right)+K^2-2KV_1\right)$$

Calculations with very short time intervals:

$$t \to 0 \Rightarrow \left(1 - e^{at}\right) \to 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K^2 - 2KV_1\right)$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K\left(K - 2V_1\right)\right)$$

Assumptions:

$$K > 0 \Rightarrow$$

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(K - 2V_1\right) = \operatorname{sgn}\left(\frac{K}{2} - V_1\right)$$

Observations:

$$a.\quad V_1 < \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) > 0$$

$$b.\quad V_1 = \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) = 0$$

$$c.\quad V_1 > \frac{K}{2} \quad \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(\frac{dV_2}{dV_1} - 1\right) < 0$$

Calculations with longer time intervals:

$$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right) = \operatorname{sgn}\left(V_1^2\left(1-e^{at}\right) + K^2 - 2KV_1\right)$$

Hence, if t increases from zero, then $\left(1-e^{at}\right)$ becomes more negative.

As a result, $\left(V_1^2\left(1-e^{at}\right) + K^2 - 2KV_1\right)$ becomes more negative.

$\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right)$ and $\frac{dV_2}{dV_1} - 1$ may become strictly negative even if $V_1 \leq \frac{K}{2}$.

Conclusions in the volume dimension:

If the time interval is very short (approximately zero):

$$\frac{dV_1^*}{dP_{B_1}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_1}}}{\frac{d^2\pi}{dV_1^2}} \text{ and } \frac{dV_1^*}{dP_{B_2}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_2}}}{\frac{d^2\pi}{dV_1^2}}$$

are strictly positive for low values of V, zero for V that maximizes MSY and strictly negative for larger values of V.

Hence, if P_{B_1} and/or P_{B_2} increase(s), ceteres paribus, the optimal stock level converges to $V \approx V_{MSY}$.

It is NOT optimal to increase the stock level above V_{MSY}.

If the time interval is not very short:

$$\frac{dV_1^*}{dP_{B_1}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_1}}}{\frac{d^2\pi}{dV_1^2}} \text{ and } \frac{dV_1^*}{dP_{B_2}} = \frac{-\frac{d^2\pi}{dV_1 dP_{B_2}}}{\frac{d^2\pi}{dV_1^2}}$$

are strictly positive for low values of V (if the time interval is sufficiently short), equal to zero for a particular value of $V, V_1 < V_{MSY}$, and strictly negative for larger values of V.

Hence, if P_{B_1} and/or P_{B_2} increase(s), ceteres paribus, the optimal stock level converges to a value below V_{MSY}.

It is NOT optimal to increase the stock level above V_{MSY}.

Comparative statics analysis in two dimensions:

In the derivations below, we investigate the qualitative effects (directions of changes) of the decision variables in case the imaginary parameter P_B increases. Since $\operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dV_1 dP_{B_2}}\right)$ and $\operatorname{sgn}\left(\frac{d^2\pi}{dt dP_{B_1}}\right) = \operatorname{sgn}\left(\frac{d^2\pi}{dt dP_{B_2}}\right)$, and since complementary assumptions are made (below), the results derived and reported below with respect to the imaginary parameter P_B, are relevant to both of the the real parameters P_{B_1} and P_{B_2}.

$$\begin{bmatrix} \dfrac{d^2\pi}{dV_1^2} & \dfrac{d^2\pi}{dV_1dt} \\ \dfrac{d^2\pi}{dtdV_1} & \dfrac{d^2\pi}{dt^2} \end{bmatrix} \begin{bmatrix} dV_1^* \\ dt^* \end{bmatrix} = \begin{bmatrix} -\dfrac{d^2\pi}{dV_1dP_B}dP_B \\ -\dfrac{d^2\pi}{dtdP_B}dP_B \end{bmatrix}$$

$$\left[D\right] = \begin{bmatrix} \dfrac{d^2\pi}{dV_1^2} & \dfrac{d^2\pi}{dV_1dt} \\ \dfrac{d^2\pi}{dtdV_1} & \dfrac{d^2\pi}{dt^2} \end{bmatrix}$$

$$|D| = \begin{vmatrix} \dfrac{d^2\pi}{dV_1^2} & \dfrac{d^2\pi}{dV_1 dt} \\ \dfrac{d^2\pi}{dt dV_1} & \dfrac{d^2\pi}{dt^2} \end{vmatrix} > 0$$

$$\frac{dV_1^*}{dP_B} = \frac{\begin{vmatrix} -\dfrac{d^2\pi}{dV_1 dP_B} & \dfrac{d^2\pi}{dV_1 dt} \\ -\dfrac{d^2\pi}{dt dP_B} & \dfrac{d^2\pi}{dt^2} \end{vmatrix}}{|D|} = \frac{-\dfrac{d^2\pi}{dV_1 dP_B}\dfrac{d^2\pi}{dt^2} + \dfrac{d^2\pi}{dt dP_B}\dfrac{d^2\pi}{dV_1 dt}}{|D|}$$

$$\frac{dt^*}{dP_B} = \frac{\begin{vmatrix} \dfrac{d^2\pi}{dV_1^2} & -\dfrac{d^2\pi}{dV_1 dP_B} \\ \dfrac{d^2\pi}{dt dV_1} & -\dfrac{d^2\pi}{dt dP_B} \end{vmatrix}}{|D|} = \frac{-\dfrac{d^2\pi}{dV_1^2}\dfrac{d^2\pi}{dt dP_B} + \dfrac{d^2\pi}{dt dV_1}\dfrac{d^2\pi}{dV_1 dP_B}}{|D|}$$

Special assumptions of relevance to the two dimensional comparative statics analysis:

$\frac{d^2\pi}{dtdV_1} = \varepsilon < 0$. $ABS(\varepsilon)$ **is very small. We have a unique maximum.** $|D| > 0$

As a consequence,

$$\frac{dV_1^*}{dP_B} = \frac{\begin{vmatrix} -\frac{d^2\pi}{dV_1 dP_B} & \frac{d^2\pi}{dV_1 dt} \\ -\frac{d^2\pi}{dt dP_B} & \frac{d^2\pi}{dt^2} \end{vmatrix}}{|D|} = \frac{-\frac{d^2\pi}{dV_1 dP_B}\frac{d^2\pi}{dt^2} + \frac{d^2\pi}{dt dP_B}\varepsilon}{|D|} \approx \frac{-\frac{d^2\pi}{dV_1 dP_B}\frac{d^2\pi}{dt^2}}{|D|}$$

$$\frac{dt^*}{dP_B} = \frac{\begin{vmatrix} \frac{d^2\pi}{dV_1^2} & -\frac{d^2\pi}{dV_1 dP_B} \\ \frac{d^2\pi}{dt dV_1} & -\frac{d^2\pi}{dt dP_B} \end{vmatrix}}{|D|} = \frac{-\frac{d^2\pi}{dV_1^2}\frac{d^2\pi}{dt dP_B} + \varepsilon\frac{d^2\pi}{dV_1 dP_B}}{|D|} \approx \frac{-\frac{d^2\pi}{dV_1^2}\frac{d^2\pi}{dt dP_B}}{|D|}$$

$$\text{sgn}\left(\frac{dV_1^*}{dP_B}\right)=\text{sgn}\left(\frac{d^2\pi}{dV_1 dP_B}\right)$$

$$\text{sgn}\left(\frac{dt^*}{dP_B}\right)=\text{sgn}\left(\frac{d^2\pi}{dt dP_B}\right)$$

<u>Conclusions from the one and two dimensional comparative statics analyses:</u>

If the initial value of V_1 is lower than V_{MSY} and the time interval t is sufficiently short, then V_1 and t increase if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is lower than V_{MAX} and the time interval t is sufficiently long, then V_1 and t may be unchanged or decrease if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is equal to V_{MSY} and the time interval t is very short, then V_1 and t are not changed if P_B (P_{B_1} and/or P_{B_2}) increase(s).

If the initial value of V_1 is higher than V_{MSY} and/or the time interval t is sufficiently long, then V_1 and t decrease if P_B (P_{B_1} and/or P_{B_2}) increase(s).

Workshop References

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