http://www.Lohmander.com/mil/ORFAU1.pdf

Operations Research for Army Units

- Methodology for optimal combat decisions

Peter Lohmander
http://www.lohmander.com/
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CASE 1.

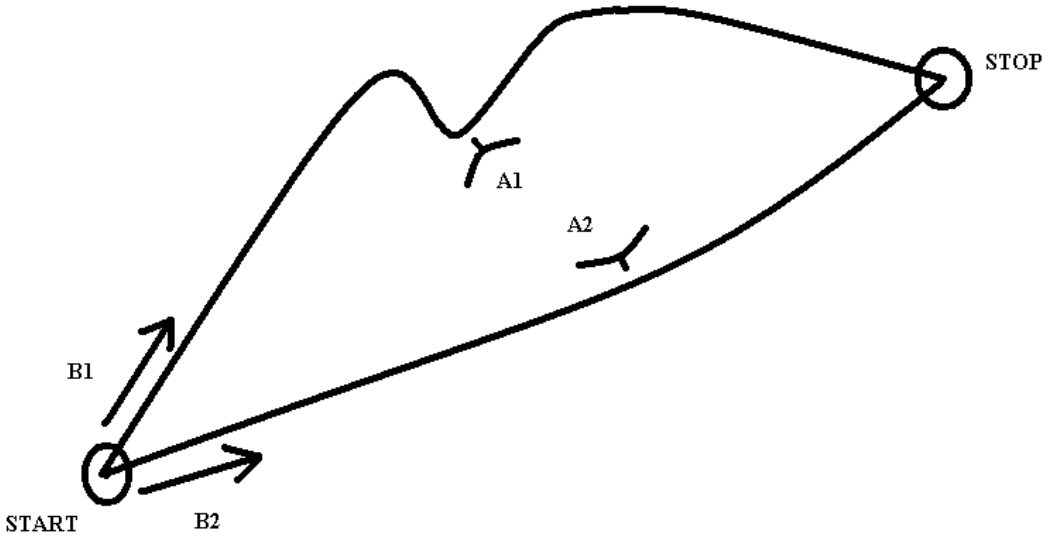

Figure 1.1.

		B1	***B2***
		Y1	**Y2**
A1	**X1**	**C11**	**0**
A2	**X2**	**0**	**C22**

		B1	***B2***
		Y1	**Y2**
A1	**X1**	**2**	**0**
A2	**X2**	**0**	**1**

$$\begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\max E$$

s.t.

$$E \le c_{11}x_1 + 0x_2 \quad (if\ B_1)$$

$$E \le 0x_1 + c_{22}x_2 \quad (if\ B_2)$$

$$1 = x_1 + x_2$$

$$0 \le x_1$$

$$0 \le x_2$$

$$x_2 = 1 - x_1$$

$$\max E$$

s.t.

$$E \le c_{11}x_1 \quad (if\ B_1)$$

$$E \le c_{22}(1-x_1) \quad (if\ B_2)$$

$$\max E$$

s.t.

$$E \le c_{11}x_1 \qquad (if\ B_1)$$

$$E \le c_{22} - c_{22}x_1 \quad (if\ B_2)$$

$$\max E$$

s.t.

$$E \le 2x_1 \qquad (if\ B_1)$$

$$E \le 1 - 1x_1 \qquad (if\ B_2)$$

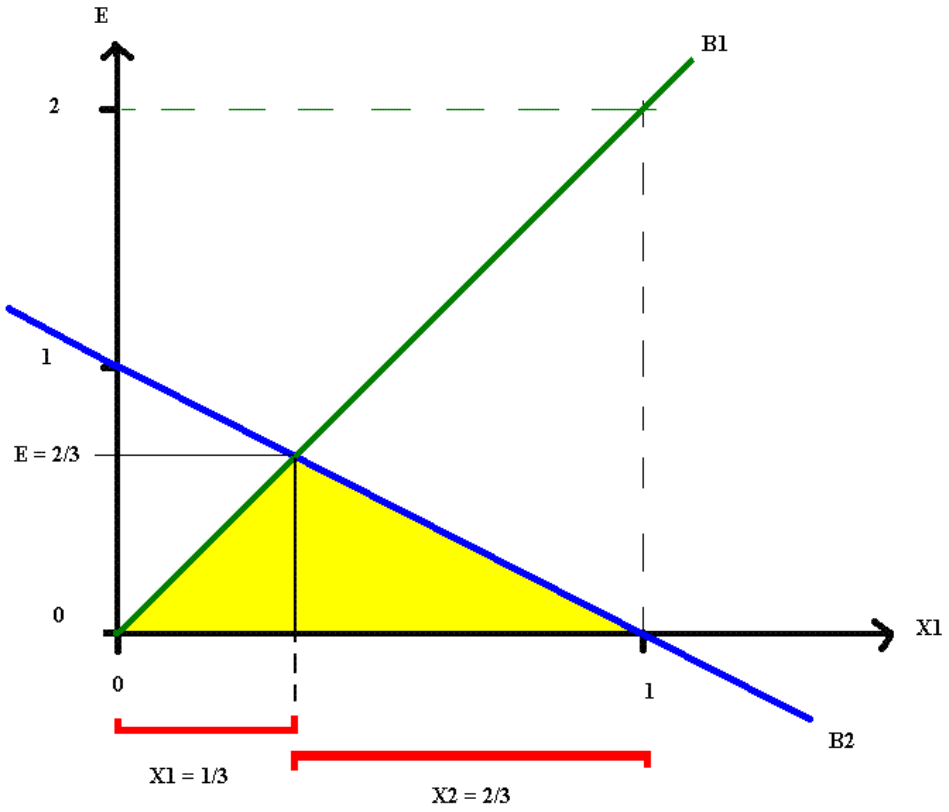

Figure 1.2.

$$2x_1 = 1 - x_1$$

$$3x_1 = 1$$

$$x_1 = \frac{1}{3} \approx 0.33$$

$$x_2 = 1 - x_1 = 1 - \frac{1}{3} = \frac{2}{3} \approx 0.67$$

$$E = 2x_1 = \frac{2}{3} \approx 0.67$$

$$\begin{bmatrix} c_{11} & 0 \\ 0 & c_{22} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\min E$$

s.t.

$$E \geq c_{11} y_1 + 0 y_2 \quad (if\ A_1)$$
$$E \geq 0 y_1 + c_{22} y_2 \quad (if\ A_2)$$
$$1 = y_1 + y_2$$
$$0 \leq y_1$$
$$0 \leq y_2$$

$$y_2 = 1 - y_1$$

$$\min E$$

s.t.

$$E \geq c_{11} y_1 \quad (if\ A_1)$$
$$E \geq c_{22}(1 - y_1) \quad (if\ A_2)$$

$$\min E$$

s.t.

$$E \geq c_{11} y_1 \quad (if\ A_1)$$
$$E \geq c_{22} - c_{22} y_1 \quad (if\ A_2)$$

$$\min E$$

s.t.

$$E \geq 2y_1 \qquad (if\ A_1)$$

$$E \geq 1 - y_1 \qquad (if\ A_2)$$

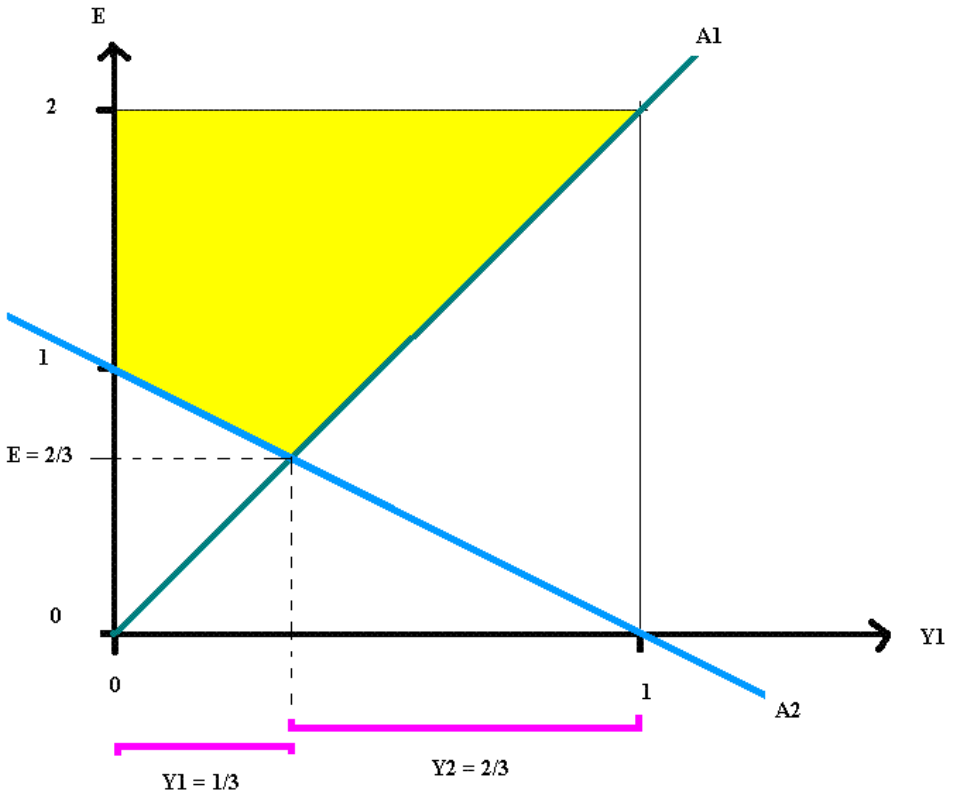

Figure 1.3.

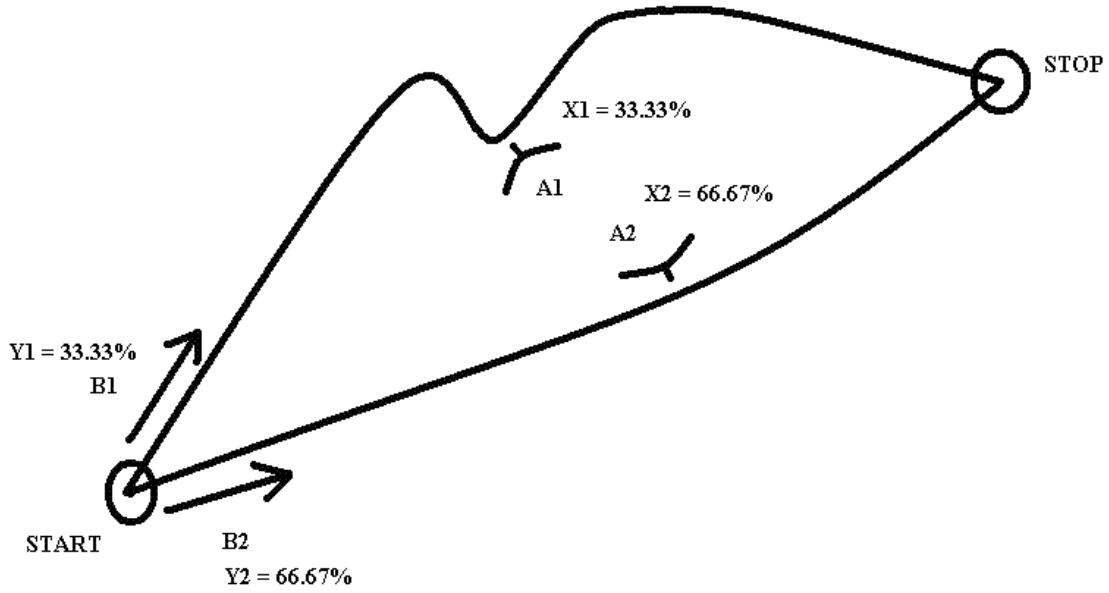

Figure 1.4.

CASE 2.

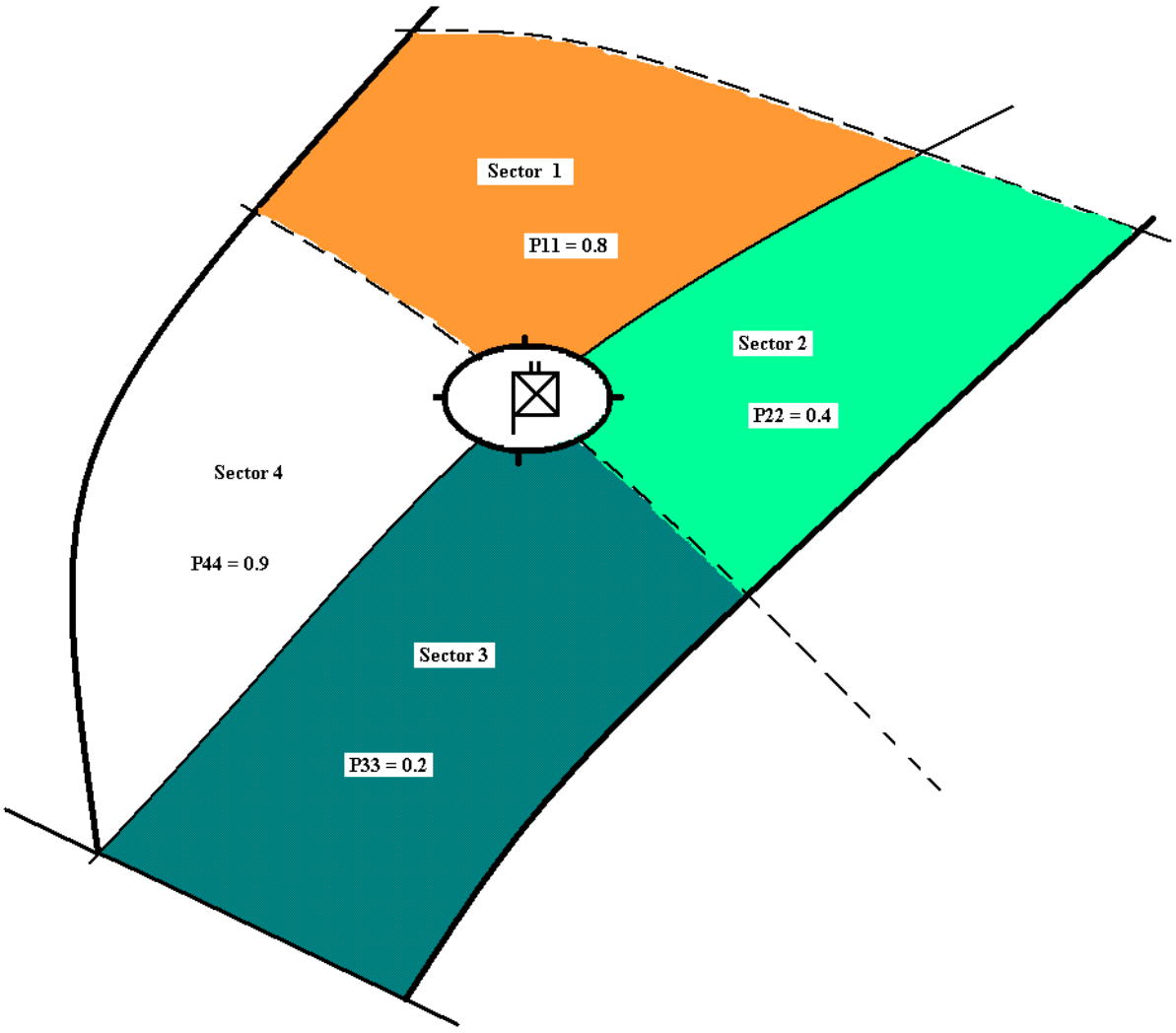

Figure 2.1.

$$\begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 0 \\ 0 & 0 & 0 & p_{44} \end{bmatrix} = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

$$\max \; E$$

s.t.

$$E \le p_{11}x_1 \qquad (if \;\; B_1)$$
$$E \le p_{22}x_2 \qquad (if \;\; B_2)$$
$$E \le p_{33}x_3 \qquad (if \;\; B_3)$$
$$E \le p_{44}x_4 \qquad (if \;\; B_4)$$
$$1 = x_1 + x_2 + x_3 + x_4$$
$$x_1 \ge 0;\; x_2 \ge 0;\; x_3 \ge 0;\; x_4 \ge 0$$

Assumption: All x's are strictly positive. (The usual case.)

$$E = p_{11}x_1$$
$$E = p_{22}x_2$$
$$E = p_{33}x_3$$
$$E = p_{44}x_4$$

$$E = p_{11}x_1 = p_{22}x_2 = p_{33}x_3 = p_{44}x_4$$

$$x_1 = \frac{E}{p_{11}}$$

$$x_2 = \frac{E}{p_{22}}$$

$$x_3 = \frac{E}{p_{33}}$$

$$x_4 = \frac{E}{p_{44}}$$

$$x_1 + x_2 + x_3 + x_4 = 1$$

$$\frac{E}{p_{11}} + \frac{E}{p_{22}} + \frac{E}{p_{33}} + \frac{E}{p_{44}} = 1$$

$$\frac{1}{p_{11}} + \frac{1}{p_{22}} + \frac{1}{p_{33}} + \frac{1}{p_{44}} = \frac{1}{E}$$

$$\frac{p_{22}p_{33}p_{44} + p_{11}p_{33}p_{44} + p_{11}p_{22}p_{44} + p_{11}p_{22}p_{33}}{p_{11}p_{22}p_{33}p_{44}} = \frac{1}{E}$$

$$\frac{0.4 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.2}{0.8 \bullet 0.4 \bullet 0.2 \bullet 0.9} = \frac{1}{E}$$

$$E = \frac{0.8 \bullet 0.4 \bullet 0.2 \bullet 0.9}{0.4 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.2}$$

$$E \approx 0.10140845$$

$$E \approx 10\%$$

$$x_1 = \frac{E}{p_{11}} = \frac{E}{0.8} \approx 12.68\%$$

$$x_2 = \frac{E}{p_{22}} = \frac{E}{0.4} \approx 25.35\%$$

$$x_3 = \frac{E}{p_{33}} = \frac{E}{0.2} \approx 50.70\%$$

$$x_4 = \frac{E}{p_{44}} = \frac{E}{0.9} \approx 11.27\%$$

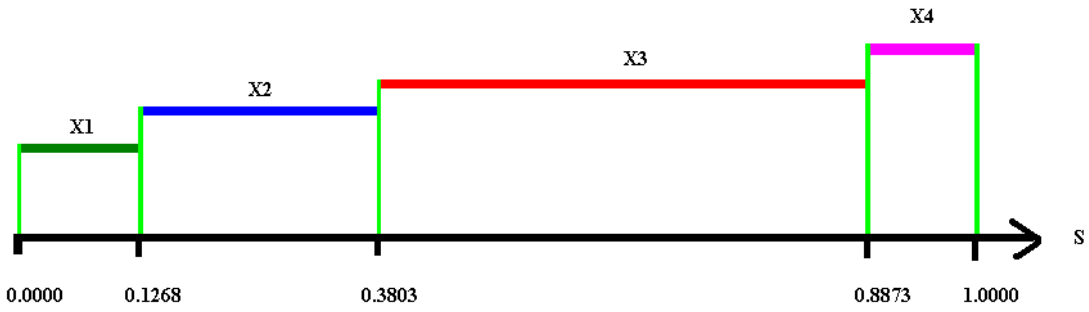

Figure 2.2.

$$\begin{bmatrix} p_{11} & 0 & 0 & 0 \\ 0 & p_{22} & 0 & 0 \\ 0 & 0 & p_{33} & 0 \\ 0 & 0 & 0 & p_{44} \end{bmatrix} = \begin{bmatrix} 0.8 & 0 & 0 & 0 \\ 0 & 0.4 & 0 & 0 \\ 0 & 0 & 0.2 & 0 \\ 0 & 0 & 0 & 0.9 \end{bmatrix}$$

$\min \; E$

s.t.

$$E \geq p_{11} y_1 \qquad (if \; A_1)$$
$$E \geq p_{22} y_2 \qquad (if \; A_2)$$
$$E \geq p_{33} y_3 \qquad (if \; A_3)$$
$$E \geq p_{44} y_4 \qquad (if \; A_4)$$
$$1 = y_1 + y_2 + y_3 + y_4$$
$$y_1 \geq 0;\; y_2 \geq 0;\; y_3 \geq 0;\; y_4 \geq 0$$

Assumption: All y's are strictly positive. (The usual case.)

$$E = p_{11} y_1$$
$$E = p_{22} y_2$$
$$E = p_{33} y_3$$
$$E = p_{44} y_4$$

$$E = p_{11} y_1 = p_{22} y_2 = p_{33} y_3 = p_{44} y_4$$

$$y_1 = \frac{E}{p_{11}} = x_1$$

$$y_2 = \frac{E}{p_{22}} = x_2$$

$$y_3 = \frac{E}{p_{33}} = x_3$$

$$y_4 = \frac{E}{p_{44}} = x_4$$

$$y_1 + y_2 + y_3 + y_4 = 1$$

$$\frac{E}{p_{11}} + \frac{E}{p_{22}} + \frac{E}{p_{33}} + \frac{E}{p_{44}} = 1$$

$$\frac{1}{p_{11}} + \frac{1}{p_{22}} + \frac{1}{p_{33}} + \frac{1}{p_{44}} = \frac{1}{E}$$

$$\frac{p_{22}p_{33}p_{44} + p_{11}p_{33}p_{44} + p_{11}p_{22}p_{44} + p_{11}p_{22}p_{33}}{p_{11}p_{22}p_{33}p_{44}} = \frac{1}{E}$$

$$\frac{0.4 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.2}{0.8 \bullet 0.4 \bullet 0.2 \bullet 0.9} = \frac{1}{E}$$

$$E = \frac{0.8 \bullet 0.4 \bullet 0.2 \bullet 0.9}{0.4 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.2 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.9 + 0.8 \bullet 0.4 \bullet 0.2}$$

$$E \approx 0.10140845$$

$$E \approx 10\%$$

$$y_1 = \frac{E}{p_{11}} = \frac{E}{0.8} \approx 12.68\%$$

$$y_2 = \frac{E}{p_{22}} = \frac{E}{0.4} \approx 25.35\%$$

$$y_3 = \frac{E}{p_{33}} = \frac{E}{0.2} \approx 50.70\%$$

$$y_4 = \frac{E}{p_{44}} = \frac{E}{0.9} \approx 11.27\%$$

$$y_1 = x_1$$

$$y_2 = x_2$$

$$y_3 = x_3$$

$$y_4 = x_4$$

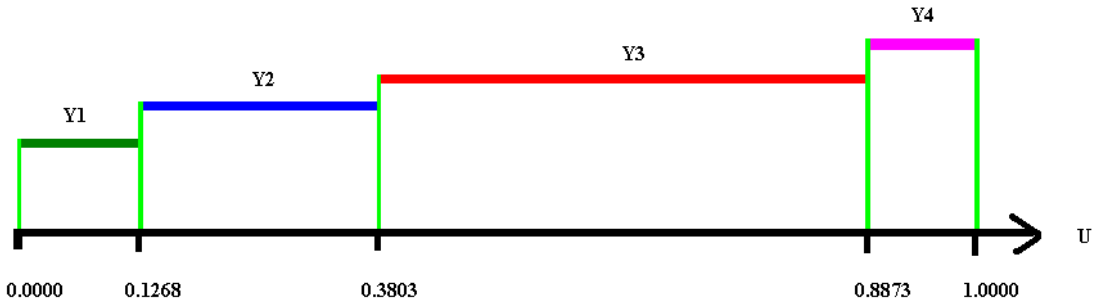

Figure 2.3.

Test:

$$Z = x_1 y_1 p_{11} + x_2 y_2 p_{22} + x_3 y_3 p_{33} + x_4 y_4 p_{44}$$

$$Z \approx 0.10140845 \approx E$$

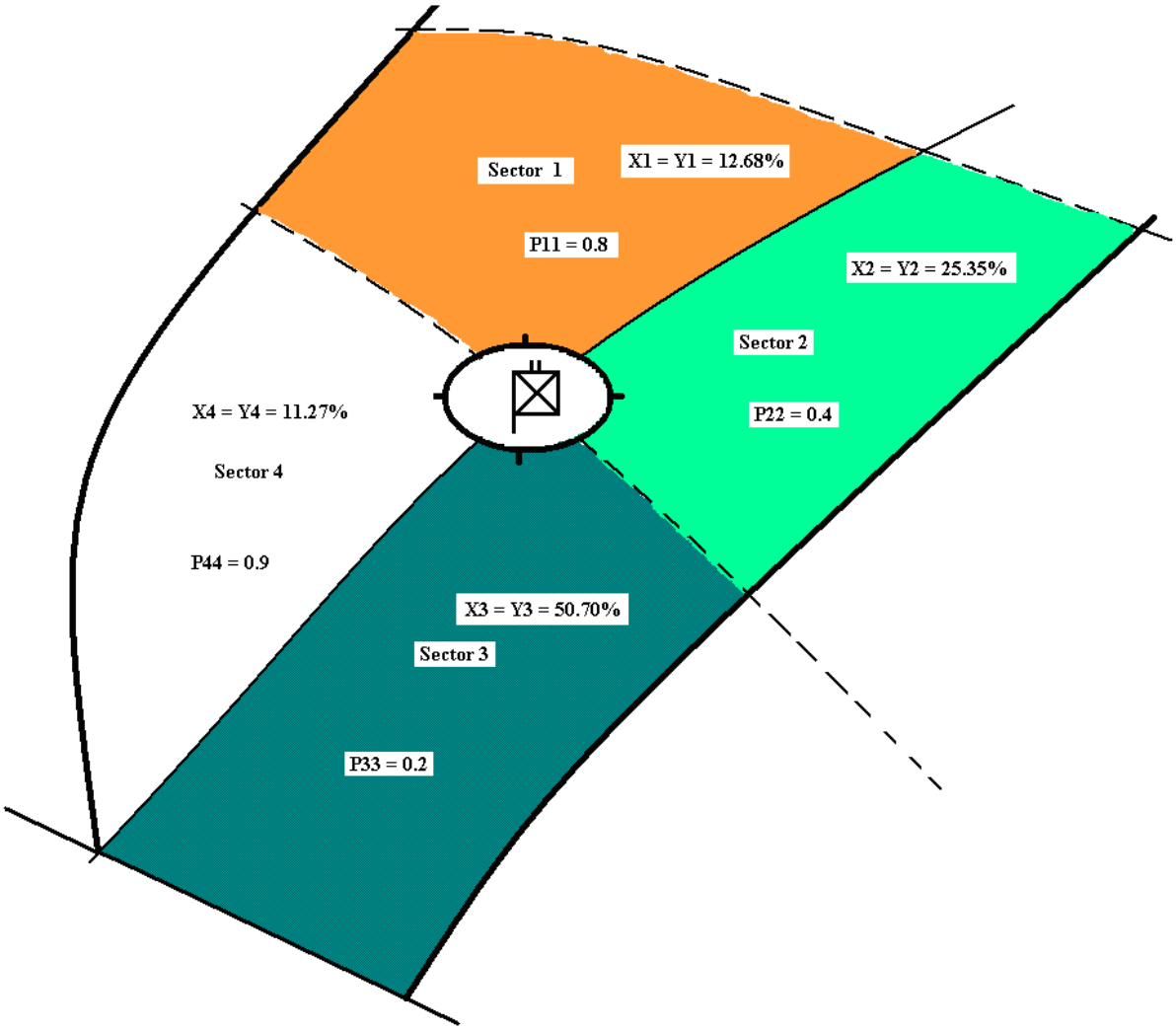
Sector 1
X1 = Y1 = 12.68%
P11 = 0.8
X2 = Y2 = 25.35%
Sector 2
P22 = 0.4
X4 = Y4 = 11.27%
Sector 4
P44 = 0.9
X3 = Y3 = 50.70%
Sector 3
P33 = 0.2

Figure 2.4.

Case 3.

Complete example (in Swedish) with general software:
http://www.lohmander.com/mil/2pzsg/Taktik_PL_1.htm

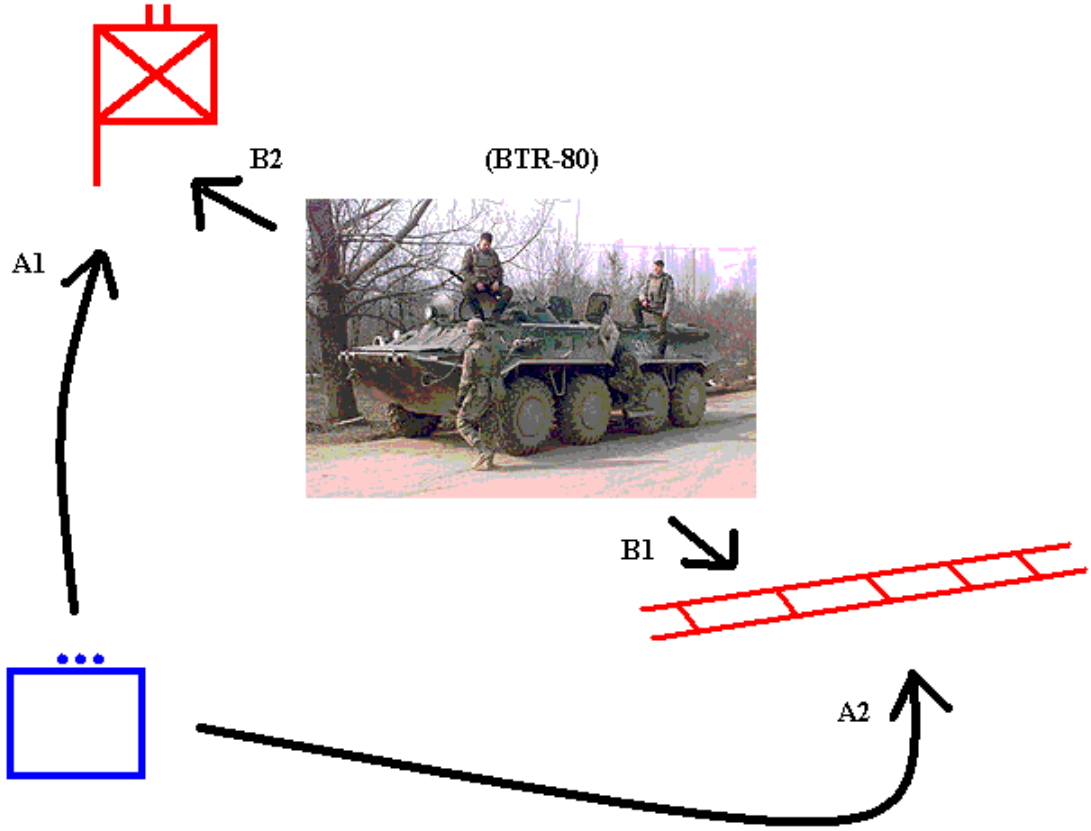

Figure 3.1.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} (9-1) & (1-2) \\ (5-6) & (5-3) \end{bmatrix} \begin{bmatrix} 8 & -1 \\ -1 & 2 \end{bmatrix}$$

$\max E$

s.t.

$$E \le 8x_1 - 1x_2$$
$$E \le -1x_1 + 2x_2$$
$$1 = x_1 + x_2$$
$$x_1 \ge 0;\ x_2 \ge 0$$

$$x_2 = 1 - x_1$$

$\max E$

s.t.

$$E \le 8x_1 - 1(1 - x_1)$$
$$E \le -1x_1 + 2(1 - x_1)$$

$\max E$

s.t.

$$E \le 9x_1 - 1$$
$$E \le 2 - 3x_1$$

$$E = 9x_1 - 1 = 2 - 3x_1$$

$$12x_1 = 3$$

$$x_1 = \frac{3}{12} = \frac{1}{4} = 0.25 = 25\%$$

$$x_2 = 1 - x_1 = 75\%$$

$$E = 9x_1 - 1 = \frac{9}{4} - \frac{4}{4} = \frac{5}{4} = 1.25$$

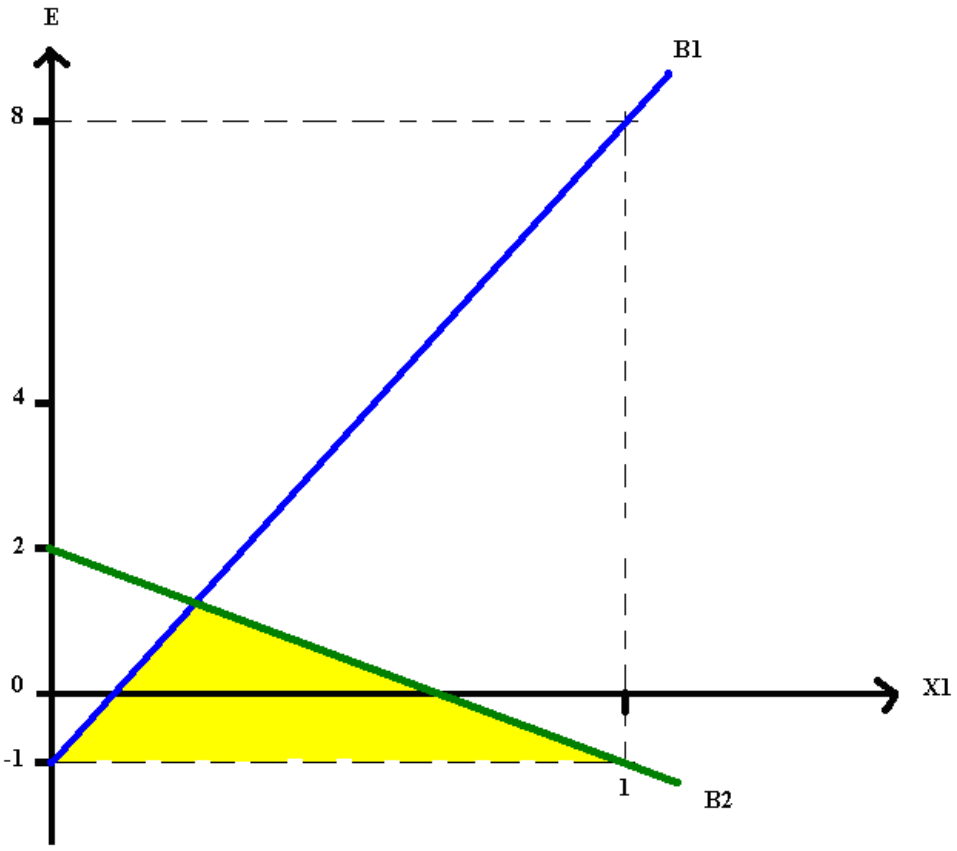

Figure 3.2.

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} 8 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\min E$$

s.t.

$$E \geq 8y_1 - 1y_2$$
$$E \geq -1y_1 + 2y_2$$
$$1 = y_1 + y_2$$
$$y_1 \geq 0;\ y_2 \geq 0$$

$$y_2 = 1 - y_1$$

$$\min E$$

s.t.

$$E \geq 8y_1 - 1(1 - y_1)$$
$$E \geq -1y_1 + 2(1 - y_1)$$

$$\min E$$

s.t.

$$E \geq 9y_1 - 1$$
$$E \geq 2 - 3y_1$$

$$E = 9y_1 - 1 = 2 - 3y_1$$

$$12y_1 = 3$$

$$y_1 = \frac{3}{12} = \frac{1}{4} = 0.25 = 25\%$$

$$y_2 = 1 - y_1 = 75\%$$

$$E = 9y_1 - 1 = \frac{9}{4} - \frac{1}{4} = \frac{5}{4} = 1.25$$

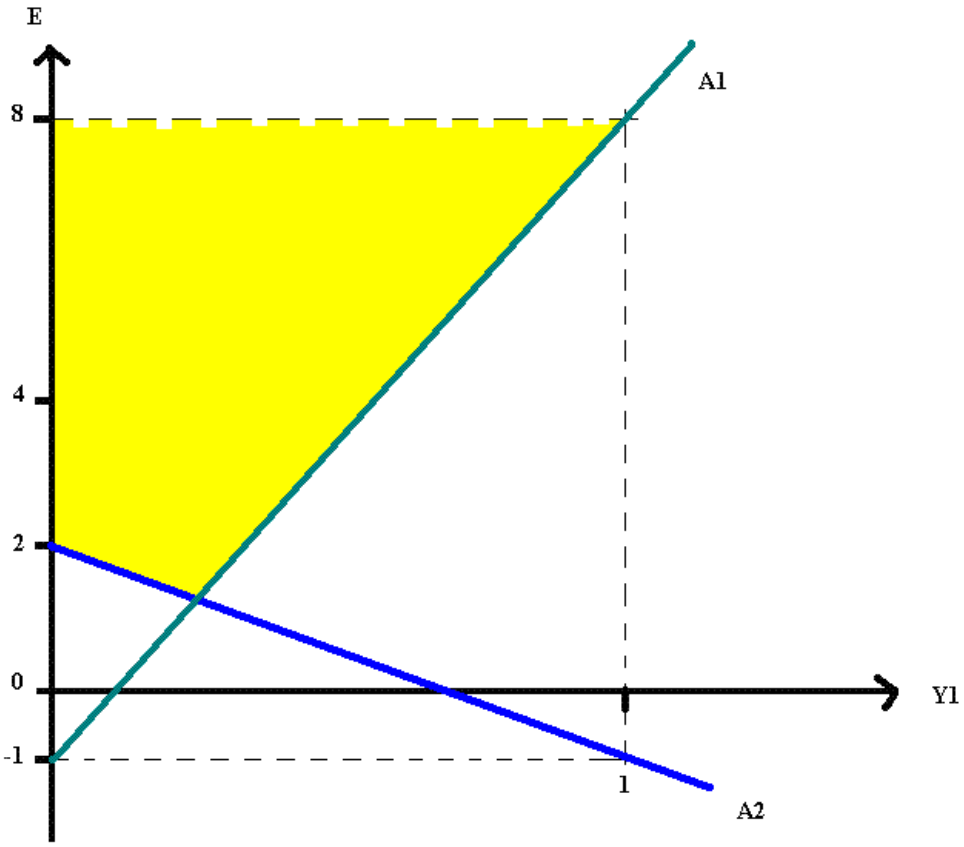

Figure 3.3.

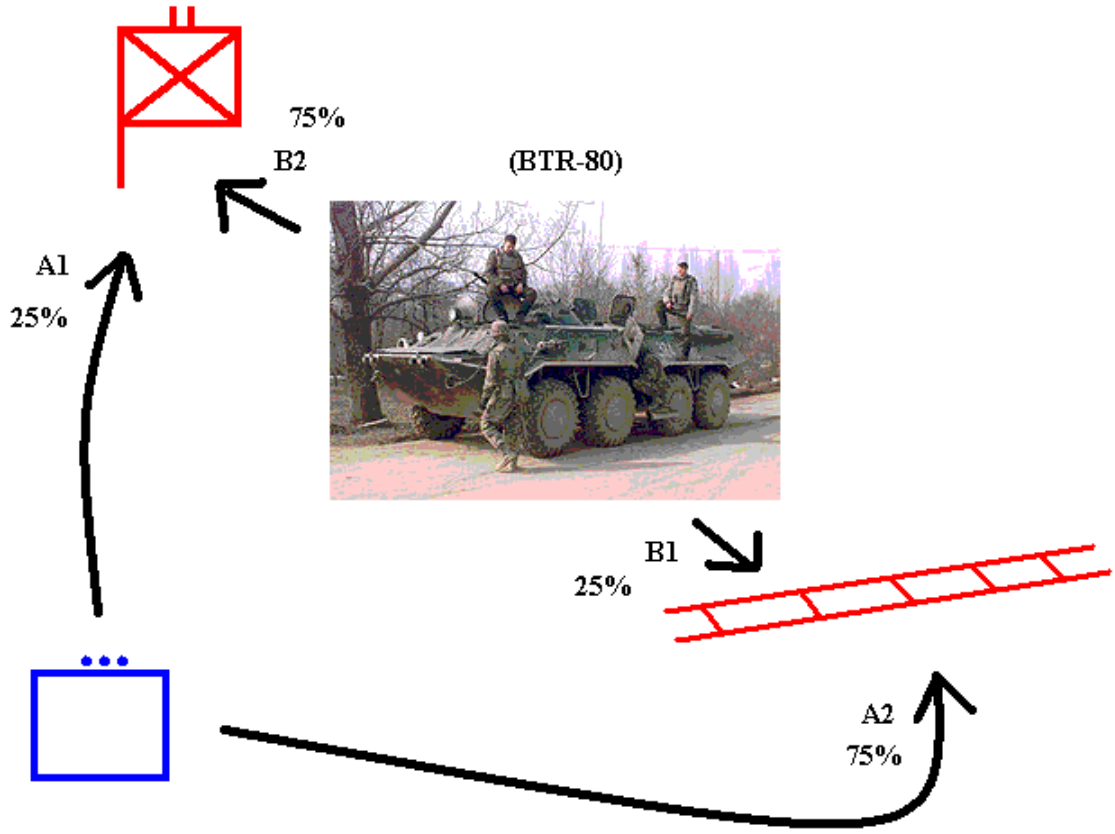

Figure 3.4.

Case 4.

Complete example (in Swedish) with general software:
http://www.lohmander.com/mil/ResFStri.html

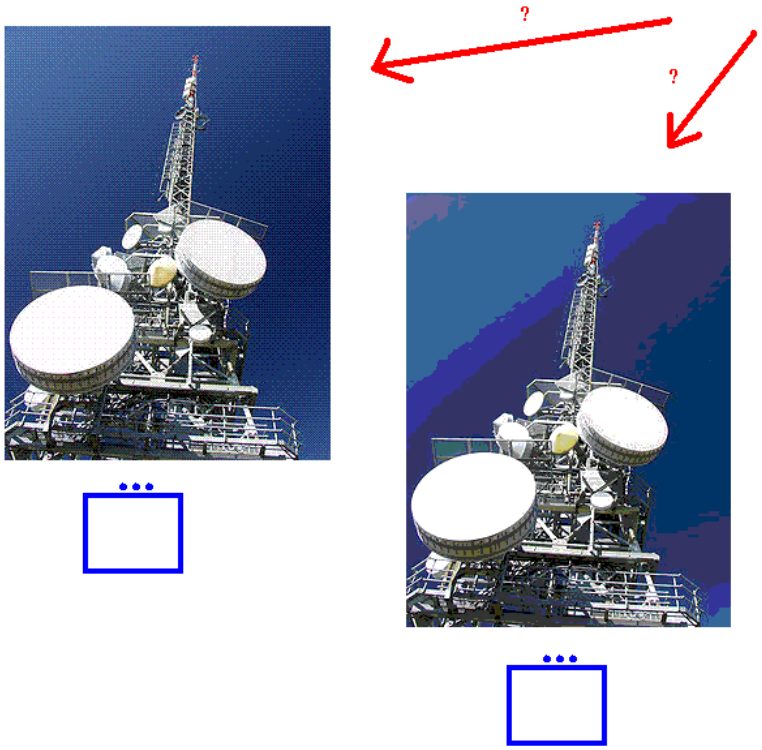

Figure 4.1.

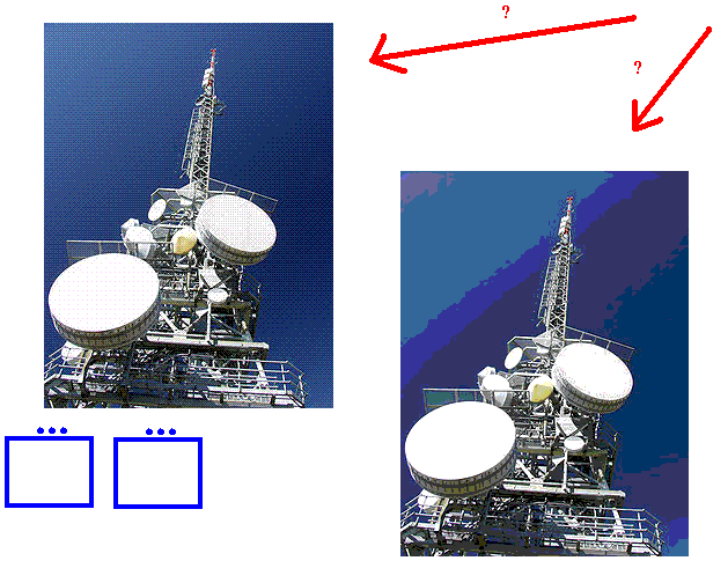

Figure 4.2.

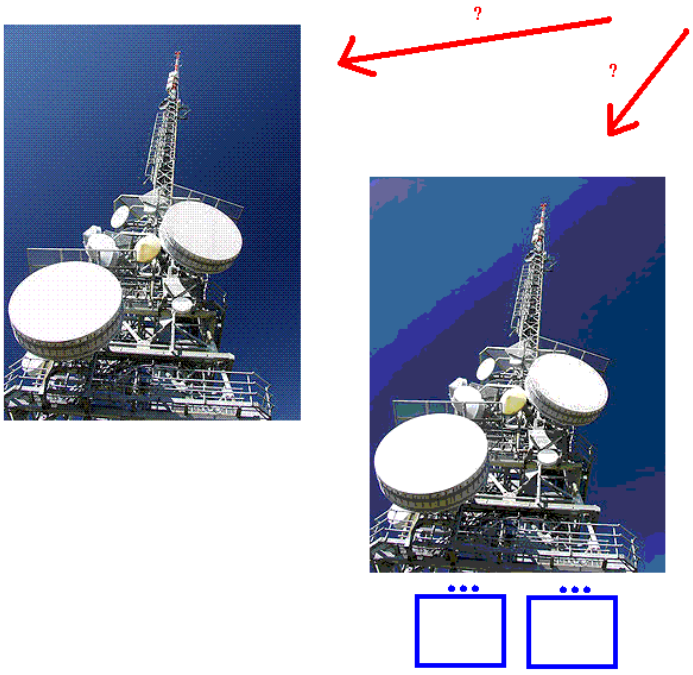

Figure 4.3.

Table 1.
Summary of results reported here:

http://www.lohmander.com/mil/ResFStri.html

(Value of one object)/(Value of one unit) = w	Probability that attack with one unit per object is optimal	Probability that a concentrated attack against only one object is optimal	Probability that the defence should use one unit per object	Probability that a concentrated defence of only one object is optimal
2	63%	37%	50%	50%
4	45%	55%	78%	22%
10	32%	68%	97%	3%

Table 2. (Swedish version of Table 1.)
Sammanfattning av resultat som rapporteras här:

http://www.lohmander.com/mil/ResFStri.html

(Värdet av ett objekt)/(värdet av en grupp) = w	Sannolikhet att A bör anfalla med en grupp mot varje objekt	Sannolikhet att A bör anfalla kraftsamlat med 2 grupper mot ett objekt	Sannolikhet att B bör skydda varje objekt med en grupp per objekt	Sannolikhet att B bör kraftsamla 2 grupper till skydd av ett objekt
2	63%	37%	50%	50%
4	45%	55%	78%	22%
10	32%	68%	97%	3%