Lahmande Peter

SVERIGES LANTBRUKSUNIVERSITET
Institutionen för skogsskötsel

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Sveriges lantbruksuniverstitet
Institutionen för skogsskötsel
901 83 UMEÅ, Sweden
Tel: 090-16 62 74
Fax: 090-16 76 69

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Adaptive economic forest management

Lohmander, P., Assistant professor, Dept of Forest Economics, The Swedish University of Agricultural Sciences, S-901 83 Umeå, Sweden,

Not yet rejected hypotheses and results:

1. It is not possible to make reliable and detailed long term (10 years or more) predictions of the developments of the markets and the biologically relevant environment. Hence, we should regard the long term behaviour of the generalized environment as a stochastic process.

2. We can often increase the expected economic present value of the profit from forestry in the presence of stochastic economic and biological changes.

3. It is usually economically optimal to strongly modify the forest management methods in the presence of stochastic economic and biological changes.

4. It is important to create a flexible decision situation in later stages of forest planning. The optimal level of future flexibility is also an economic problem. Key concepts include: a) Multi species investments. b) Continuous observation of the market development and the changes in the biological environment. c) Continuous adaptive adjustment of the forest management decisions to the latest information.

Methods: Stochastic dynamic programming and related methods for multistage adaptive optimization.

Continuos harvesting with a nonlinear stock dependent growth function and stochastic prices: Optimization of the adaptive stock control function via a stochastic quasi-gradient method. *

Lohmander, P., Assistant professor, Dept of Forest Economics, The Swedish University of Agricultural Sciences, S-901 83 Umeå, Sweden,

* A detailed appendix may be obtained from the author.

Abstract
The continuous harvesting problem has gained much attention in the literature. Very detailed and instructive discussions of the qualitative properties of the optimal solutions in deterministic models have been presented. Highly detailed analytical models of the harvesting problem with stochastic prices and growth have also been presented where the main ambition has been to report the qualitative answers to the questions: - In what directions are the optimal control decisions changed in the presence of increasing risk in markets and/or growth ? This paper contains a growth model of a type often used in analytical deterministic control optimizations. Risk is introduced in the future price path. The harvest decisions which maximize the expected present value of all profits over time are made adaptively, conditional on the latest available price and stock level information.

In this paper, such harvest control functions are optimized via a stochastic quasi-gradient method. In principle, this method could be replaced by stochastic dynamic programming. Then, however, the numerically obtainable discrete state space resolution would restrict the quality of the results. In this model, the state and the control are continuous variables. The control function parameter optimizations are made for different combinations of exogenous parameter assumptions. The objective function is, within the gradient method optimization routine, sequentially estimated via large numbers of long horizon stochastic simulations of the complete adaptively controlled system. Finally, the optimized adaptive harvest function is used in connection to a large number of new stochastic price series. The expected present value of adaptive harvesting with the optimized function is compared to the expected present value which is obtained when a deterministic rule is used. The optimal harvest and stock levels are discussed and compared to the results obtained from deterministic optimal control models. It is found that considerable economic gains can be obtained when the optimized adaptive control function is used instead of a deterministic control rule. Even very low levels of stochastic price variation make it economically optimal to control the forest stock in a way which is very different from what you should do if you follow rules derived from deterministic models. The optimal control and stock levels change drastically over time.

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Introduction
Is there a practical way to find the optimal market adapted harvest policy in natural resource economics when the stock of the natural resource and the harvest level are described as continuous variables ?

This question will be discussed in this paper under the assumptions that the price of the natural resource is a stochastic process and that the growth of the natural resource follows a logistic function.

A preliminary and explorative analysis will be made as an illustration of continuous thinning in forestry.

Thinning and the literature

The problem of thinning in forestry has been studied from many perspectives and for a long time. We may view the thinning problem as the continuous forest control problem. We may, over time, control the forest via changing thinning levels. We may sequentially take new information into account in the decision problem. If we want to, we can harvest all of the trees in the forest stand. Such an activity is usually defined as a final felling or a clear cut. A forester would never call that a thinning. From a control theoretical point of view, it is just a special case of "thinning". One of the problems in forestry is to find a reliable growth function that can be applied to the relevant economic problem.

The age structure, the diameter, height and species distributions, make this complicated. Frequently, foresters want very detailed information about how the trees in the forest grow. Obviously all of the mentioned distributions also affect the local growth conditions and hence the growth of the trees in different parts of the stand. In some cases, a high stand density is desirable in order to improve the quality development. Unfortunately, in the end, it turns out that it is not possible to treat all of the interesting sub-phenomena within one model. We have to select the most interesting variables and try to control the most important things. That is the purpose of this paper. Some of the more recent papers in this field that should be mentioned are given below. It is impossible to discuss all of them here. The reader is encouraged to consult them for interesting ideas.

Thinning, emphasis on economics and decision theory

Brazee and Mendelsohn (1990), Gong (1992), Haight (1990), Kaya and Buongiorno (1987), Kaya and Buongiorno (1989), Lohmander (1987), (1988a), (1990a), (1990b), (1990c), Reed and Apaloo (1991), Solberg and Haight (1991), Teeter and Caulfield (1991), Valsta (1992)

Thinning, emphasis on silviculture and growth

Hagner (1990a), (1990b), (1992), Klensmeden (1984), Lindman (1984), Lundqvist (1989), Mielikäinen (1991), Nilsen (1988)

Growth functions and estimations

Carlsson (1991), Eriksson (1986), Huuri, Lähde and Huuri (1987), Somers and Farrar (1991), Valinger (1990)

Some other papers in fields of more partial relevance to the content of this paper are also included in the reference list. Some of the more methodology oriented publications will be discussed in the later sections in connection to the analysis.

The model and the optimization problem

The objective function is the expected present value of all present and future harvesting. In the discrete time version of the problem, the present value π may be written as in (1):

$$\pi = \sum_{t=t_0}^{\infty} e^{-rt} P_t h_t \qquad (1)$$

t is the time period, P is net price (price - variable costs) and h is harvest level. r denotes the rate of interest. Real prices and rates of interests will be used. Fixed costs are not explicitly treated in this paper since it will be assumed that such costs really are fixed and do not affect optimal behaviour. More details concerning numerical assumptions and bounds will be found below.

A "classical" deterministic, continuous state, continuous time, natural resource model, different versions of which have been presented by Clark (1976), is shown in (2) and (3):

$$\max \pi = \int_{t_0}^{\infty} e^{-rt} R(h)\, dt \qquad (2)$$

$$s.t.\ \dot{x} = F(x) - h \qquad (3)$$

R(.) is the net revenue function, F(.) is the growth function (strictly concave) and x is the stock of the natural resource. In the analytical discussion of this paper, we will assume that R(.) is strictly concave in h in the neighbourhood of the optimal harvest level and that a unique optimum exists.

The biological growth function and the price function are two important components that deserve special attention.

The deterministic solution

Let us derive some important results from the continuous deterministic problem as a reference solution. An economic background and interpretation of the famous formulae of optimal control theory will be contained below.

In (4), we find the Hamiltonian (Hamiltonian function), H, of the optimization problem in (2) and (3). Detailed presentations of this equation are found in Clark (1976) and in other texts. H contains two components, the integrand of (2) and the right hand side of (3) multiplied by lambda, the dual variable of the problem, which may be interpreted as the shadow price of the resource. The harvest, the stock and the shadow price are all functions of time.

$$H=e^{-rt}R(h)+\lambda(F(x)-h) \qquad (4)$$

The optimization idea, which conceptually is similar to traditional Lagrangian optimization, is to derive the harvest level which maximizes H. In optimum, the first order derivative found in (5), should have the value zero.

$$\frac{\partial H}{\partial h}=e^{-rt}R'(h)-\lambda=0 \qquad (5)$$

We may now consider the similarity with Lagrange function optimization more in detail: The derivative (5), which is equal to zero, contains two components: The present value of the marginal (net) revenue of harvesting minus the marginal resource value, the shadow price. Via this equation, we instantly get one function of the shadow price (6).

$$\lambda=e^{-rt}R'(h) \qquad (6)$$

Note that (6) could be obtained directly from the economic insight that the marginal net revenue should be equal to the marginal cost of harvesting, which is equal to the shadow price of the resource. Equation (6) reveals that, in case harvesting is constant over time and the (net) revenue function is identical over time, then the shadow price (the value of a marginal unit of the resource) is a monotonically decreasing function of time as long as $r > 0$. This is reasonable from an economic point of view. We may use (6) to derive the more general result (7):

$$\frac{\partial\lambda}{\partial t}=e^{-rt}(-rR'(h)+R''(h)\dot{h}) \qquad (7)$$

In (7) we find that not only the rate of interest and time, but also the harvest path and the first two derivatives of the revenue function affect the time path of the shadow price.

In (8), the "adjoint equation" of optimal control theory is stated. (The shadow price is also sometimes denoted the adjoint variable.) This equation must be satisfied along the optimal path. Below, an economic interpretation of this equation will be given via the help of a two period example. (Disciplin specific interpretations of these equations can be found also in mechanics.)

$$\frac{\partial\lambda}{\partial t}=-\frac{\partial H}{\partial x} \qquad (8)$$

The adjoint equation and its economical interpretation

Imagine a two period harvest problem (9) and (10). We initially have X units of the resource available. We shall determine the optimal harvest vector x.

In period 0, the net price is P minus the price change, and in period 1, the price is P. We implicitly assume that the prices are real, discounted prices. In period 0, we may harvest one part of X and save one part for the future period 1. The amount that we have available for harvesting in period 1 is equal to the amount that we did not harvest in period 0 plus the amount of growth that the saved resource gave. This is found in (10).

$$\max\ \pi=(P-\Delta P)x_0+Px_1 \qquad (9)$$

$$s.t.\quad x_1=(X-x_0)+F(X-x_0) \qquad (10)$$

In fact, we may replace the two dimensional constrained optimization problem (9) and (10) by a one dimensional unconstrained problem (11).

$$\max\ \pi(x_0)=(P-\Delta P)x_0+P((X-x_0)+F(X-x_0)) \qquad (11)$$

The first order optimum condition of (11) is (12).

$$\frac{\partial \pi}{\partial x_0} = [P-\Delta P] + P[-1-F'(X-x_0)] = 0 \quad (12)$$

Denote the saved resource X (underlined) and rewrite (12) as (13).

$$-\Delta P-PF'(\underline{X})=0 \qquad (13)$$

From (13) we instantly get (14). This equation may be interpreted the following way:

When we, in period 0, have selected the optimal level of the stock saved for the future, this must mean that the marginal unit of harvesting in period 0 is exactly as profitable as the marginal unit of stock saved for harvesting in period 1. Let us assume that we can harvest one (marginal) cubic metre now, in period 0. Then we instantly get (P - price change) crowns more. We can also save the cubic metre and get the cubic metre plus growth in the next period. The profitability of doing this must be identical if we use the marginal resource unit optimally over time. Hence, if the marginal relative growth, F'(x), is positive, then the marginal relative net price change (discounted) must be negative.

$$F'(\underline{X}) = -\frac{\Delta P}{P} \qquad (14)$$

Another way to come to equation (14), using economic logic, is the following: Consider the marginal unit of the resource in a continuous time problem. The resource level is X(t). We want to harvest this over time in such a way that the real value of it, taking growth and real price changes into account, does not change over time. The value of the marginal unit, x, at time t is P(t)x(t).

The time derivative of the value is P'x + Px'. This time derivative must be equal to zero. If we divide the expression by Px, which must be greater than zero, we find that P'/P + x'/x must be equal to zero. Since x'/x is the relative growth of the marginal resource unit, we may replace it by F'(X). Hence, we may write P'/P + F'(X) = 0. This equation is the same as (14) if we go from continuous to discrete time, replacing time derivatives by time differences, and replace X by the corresponding discrete time variable, the underlined X.

Equation (14) may instantly be rewritten as (15).

$$-PF'(\underline{X}) = \Delta P \qquad (15)$$

If we replace the prices in (15) by shadow prices, we get (16).

$$-\lambda F'(\underline{X}) = \Delta\lambda \qquad (16)$$

Now, we consider time periods that become shorter and shorter. In the limit, time becomes continuous, and we can replace the differences by derivatives. Then, we have found the adjoint equation of optimal control theory (17).

$$-\frac{\partial H}{\partial x} = \frac{\partial \lambda}{\partial t} \qquad (17)$$

Note in particular that ways to find the important equations needed to solve the continous time optimal harvest control problem, (6) and (17), have now been reported that are based on economic logic only. The equations will be used to derive general and particular results in the following sections.

Qualitative results from the deterministic model

Now, we shall make use of the adjoint equation and derive some general results from the deterministic continuous model. The derivative in (18) is derived from (4).

$$\frac{\partial H}{\partial x} = \lambda F'(x) \qquad (18)$$

The adjoint equation (17), (6) and (7), give us (19).

$$-e^{-rt}R'(h)F'(x) = e^{-rt}[-rR'(h)+R''(h)\dot{h}] \qquad (19)$$

(19) may be rewritten as (20).

$$-R'(h)F'(x) = -rR'(h)+R''(h)\dot{h} \qquad (20)$$

From (20), we get (21). We find that the time derivative of the optimal harvest level is an explicit function of the difference between the rate of interest and the marginal growth rate.

The ratio of the first and second order derivatives of the net revenue function is strictly negative if we assume a strictly concave function with positive marginal revenue.

$$\dot{h} = [r - F'(x)] \frac{R'(h)}{R''(h)} \qquad (21)$$

Mostly, there is a stock level which makes (21) take the value zero. This stock level is called an equilibrium stock level. When the rate of interest is equal to the marginal relative growth, we have reached the equilibrium.

If the stock level is lower than the equilibrium (the marginal relative growth is higher than the rate of interest), then harvesting is initially low (lower than the growth) and increases over time as the stock increases to the equilibrium.

If the stock level is higher than the equilibrium (the marginal relative growth is lower than the rate of interest), then harvesting is initially high (higher than the growth) and decreases over time as the stock decreases to the equilibrium.

If R(.) becomes more linear, R''(.) approaches zero. Then the changes in the harvest level over time become more dramatic since the absolute value of the denominator in the RHS of (21) decreases. In the limit, we approach the famous bang-bang policy mentioned later in this paper.

General properties and functions

Now, we turn to the (stochastic) discrete time model. In order not to mix the concepts within the paper, the stock in the discrete time model at time t is denoted by Q(t). Growth in this model is denoted by G. We assume that growth from period t until period t+1 is a density (stock) dependent Markov function, G = G(Q(t)). Hence, in the absence of harvesting, the forward stock difference is:

$$G(t) = G(Q(t)) = \Delta Q(t) = Q(t+1) - Q(t) \qquad (22)$$

The harvest level at time t, h(t), is a function of the price level and the stock level in the same time period. h(t) = h(P(t),Q(t)). In the general case, when harvesting may take strictly positive values, we have:

$$\Delta Q(t) = G(Q(t)) - h(P(t), Q(t)) \qquad (23)$$

Price, P, is assumed to be an exogenous stochastic Markov process. In the general case, we have:

$$\Delta P(t) = \Delta P(P(t), \epsilon(t)) \qquad (24)$$

where ε is a series of random "errors" with some distribution and autocorrelation zero. In the more restricted first order autoregressive, AR1, case, we have:

$$P(t+1) = \alpha + \beta P(t) + \epsilon(t) \qquad (25)$$

We assume that e is a series of normally distributed errors with mean zero and autocorrelation zero. If $a > 0$ and $0 < ß < 1$, which we assume, then the process P is stationary with mean $\mu = a/(1-ß)$. It can be shown that the autocorrelation function of P declines geometrically in this case. The autocorrelation function approaches zero from above as the number of lags increases from zero. Hence, when we work with a time scale where each time step is sufficiently large, we may use the approximation:

$$P(t) = \mu + \underline{\epsilon}(t) \qquad (26)$$

The underlined epsilons denote a series of normally distributed errors with mean zero, standard deviation σ and autocorrelation zero. This kind of price process will be assumed in the numerical calculations of this paper. Of course, more complicated process assumptions could be made. This would however not serve the purpose of the study. It would make the calculations less easy to follow and require more assumptions. Furthermore, more complicated assumptions are only interesting if there is some empirical data that supports them. The same price process assumptions are also used by Lohmander (1983), (1987), (1988b), Haight (1990), Braze and Mendelsohn (1990) and others.

More specific growth assumptions

As a growth function, we start by, in the lack of more detailed relevant understanding, using a discrete approximation of the logistic function. The logistic function is usually presented in continuous time. When X(t) is the stock level and there is no harvesting:

$$\dot{x} = \frac{\partial x}{\partial t} = \theta x\left(1-\frac{x}{K}\right) \qquad (27)$$

θ and K denote the "intrinsic growth rate" and the "carrying capacity", respectively. For a detailed presentation of this function and its use, compare Clark (1976). Introducing a new constant,Γ, (27) may be written as (28).

$$\dot{x} = \theta x - \Gamma x^2 \qquad (28)$$

We have to find a quadratic function with two parameters. We may determine this equation of growth using two assumptions:

- The maximum growth per hectare occurs when the stand density is Q° (= 200) cubic metres per hectare.

- The maximum sustainable growth is 4 cubic metres per hectare and year. The maximum sustainable growth per hectare during a five year period is denoted G°.

Now we go back to our discrete approximation of the logistic function and the problem relevant variables. We add a density independent small "disturbance", Ω, reflecting the fact that growth will not stop for ever if all of the stock is harvested. If Q = 0 at a particular point in time, seeds and plants from neighbour stands will finally invade the area. Furthermore, we have the option of artificial regeneration.

$$G = \Omega + \theta Q - \Gamma Q^2 \qquad (29)$$

The first order condition of growth maximization is $\theta - 2\Gamma Q = 0$, or, if we use the more explicit notation:

$$\theta\left(1-\frac{2}{K}Q\right) = 0 \qquad (30)$$

Assuming that $\theta > 0$, which is quite reasonable, we find that K = 2Q°. Note in particular that Ω does not influence the relationship between the carrying capacity and the growth maximizing stock level. Making use of this finding, we have the following equation:

$$G^{\circ} = \Omega + \theta Q^{\circ} - \frac{\theta}{2Q^{\circ}}(Q^{\circ})^2 \qquad (31)$$

which can be simplified as

$$G^{\circ} = \Omega + \frac{\theta}{2}Q^{\circ} \qquad (32)$$

Now, we may select a combination of θ and Ω that is consistent with our "empirical facts" Q° and G°. Since the disturbance $\Omega \approx 0$, we have:

$$\theta \approx 2\frac{G^{\circ}}{Q^{\circ}} \qquad (33)$$

Now, we remember that Q° = 200 (cubic metres per hectare) and that the growth is 4 cubic metres per hectare and year. Each period is assumed to represent five years, which gives G° = 20. The resulting parameters are K = 400 and θ = 0.2. Γ = (θ/K) = 0.0005. We assume that the disturbance Ω = 0.3. This gives us the growth equation

$$G = 0.3 + 0.2Q - 0.0005Q^2 \qquad (34)$$

We find that the maximum sustainable growth, via this equation including the disturbance, is 4.06 cubic metres per year when the stand density is 200 cubic metres per hectare. This is a satisfactory representation of our "empirical" assumptions.

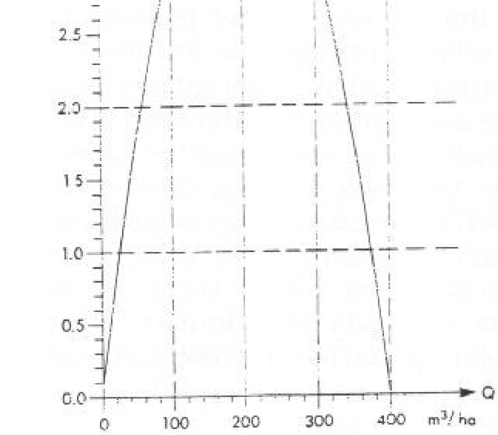

Figure 1. The growth function.

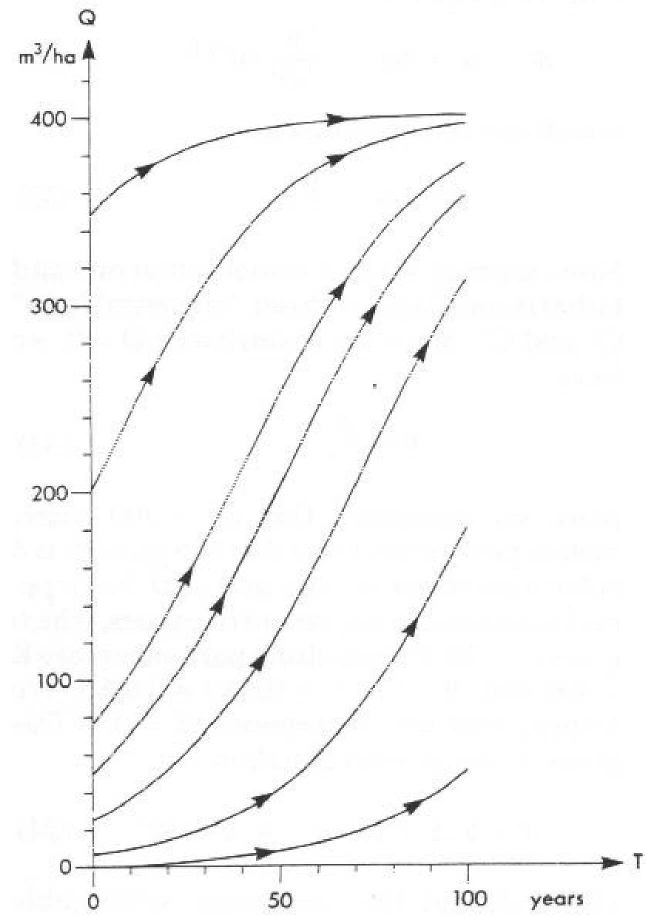

Figure 2. Stand density development without thinning according to the growth function in Figure 1.

More specific price assumptions

Each time period (which represents five years) we make a new price observation. The harvest decision in that period is based on that information. The price is used as a short notation for net price; price minus variable harvest costs per cubic metre. We may also assume that price is the price per cubic metre which the resource owner recieves from a buyer who is responsible for the harvest. There are empirical observations that support the assumption that the harvest cost per cubic metre is not very sensitive to the stand density. Compare Hagner (1990a), (1990b) and (1992). Hence, we may regard price (= net price) as completely exogenous. Of course, in cases where there are strong reasons to study the density effects in particular, special harvest cost studies should be performed and the signifficant results included in the analysis.

The price is assumed to be normally distributed with mean μ and standard deviation σ in each period. The autocorrelation is zero. In the numerical analysis, $\mu = 100$ and o takes different values.

Since we assume that the decision maker maximizes the expected present value of all harvests now and in the future, there is really no need to investigate the sensitivity of optimal harvesting to μ in this problem. (The expected present value is however an increasing function of μ. It is proportional to μ as long as the relative standard deviation, σ/μ, is held constant.) It can be shown that changes in μ leave the optimal harvest program unchanged as long as the relative standard deviation, σ/μ, is held constant. The effects of increasing standard deviation on optimal harvesting that are found in the numerical analysis are functions of changes in the relative standard deviation, σ/μ. Hence, we may interpret the numerical results more generally than it may seem at first glance. For example, when o is given the value 30 and μ is 100 in the calculations, the harvest consequences are relevant for every problem in the class where the relative standard deviation takes the value 30%.

The adaptive harvest function

One can determine the qualitative properties of optimal adaptive harvest functions via analytical stochastic dynamic optimization in several general cases. This has been done by Lohmander (1987) and (1988a). It is also possible to use stochastic dynamic programming with discrete stock state and harvest control spaces in order to determine optimal adaptive harvest functions numerically. Then, however, the dimensionality problem generally restricts the resolution of the results strongly. Some examples of discrete stochastic dynamic optimal harvesting models are Lohmander (1987) and (1990a).

It is sometimes possible to handle continuous stock and harvest controls, via polynomial approximations of the objective function, in the context of stochastic dynamic programming. One example of this is Lohmander (1990d).

In this paper, however, a completely different approach will be used: First the functional form of the adaptive harvest function will be discussed and determined. Then, the parameters of the harvest function are optimized. The objective function is approximated via a large number of stochastic simulations of the complete system over a 300 year period.

The adaptive harvest function parameters that optimize the objective function are determined through the gradient method. Here, the optimal step length is determined via the robust bisection method. More details concerning the principles of the optimization methods is found in Råde & Westergren (1988). In all investigated cases, the solutions have been found to converge to stationary points, local optima. The initial parameter guesses have not affected the solutions. There is no reason to believe that these optima are not also global optima. The structure of the problem is such that one unique optimum should be expected even if a formal analytical proof of this is practically impossible or at least extremely page consuming because of the complexity of the model. Proofs of uniqueness in slightly different models of a similar type has been reported by Lohmander (1987) and (1988a).

An intuitive discussion will here be presented which supports a particular kind of adaptive harvest function. From deterministic resource economics, presented in for instance Clark (1976), it is well known that the optimal harvest policy can belong to qualitatively different classes. If the objective function is linear (which is the case if the net price is exogenous), the optimal harvest policy is a "bang-bang" policy. One should make the resource approach the optimal stock as rapidly as possible. Initially, if we start at stocks that are lower than optimal, we will experience a period during which no harvest takes place. When the optimal stock is reached, on the other hand, the harvest will be identical to the growth at that stock, a constant, for ever.

If the objective function is strictly concave, which is often implicitly assumed without much concern in economic models, the approach to the equilibrium is more smooth, from above or from below, depending on the initial stock level. If, which is often highly relevant in resource economics, there are economies of scale (the objective function is strictly convex), then pulse extraction of the resource can often be shown to be optimal. This means that harvesting is a periodic activity and that the natural resource is allowed to change stock level considerably between the harvest sessions. In fact, with a time and space scale, where each period is sufficiently short and each unit area under harvesting is sufficiently small, it is obvious that pulse extraction is the only possible alternative. The forest worker can not work everywhere in the forest at the same time, slowly and partly harvesting every tree. The fisherman can not cover every part of the ocean simultaneously and always with his efforts. Of course, these observations are results of economies of scale. If the time periods are longer and the spatial resolution is lower, on the other hand, it is likely that the harvest activity will look continuous and smooth.

What is common in the above findings, however, is that the stock should sooner or later be adjusted towards some optimal level or at least to some interval. In other words, the harvest level should be an increasing function of the stock.

The deterministic resource models tell us little about the effects of price changes on harvesting. Some special assumptions have however been made in the literature and the results derived. Here, I will argue, with theoretical support from other analytical stochastic resource models, that optimal harvesting should be an increasing function of price as long as the price process is stationary. Compare Lohmander (1987), (1988a) and (1990a).

If price is stationary and if we observe that the present price is higher than usual, we should harvest more than usual, since the expected price in the future is lower than what we observe just now. On the other hand, maybe we should harvest less than in a deterministic situation even if the price is higher than usual! The reason in this case is that we can wait many periods for sufficiently high prices. We may want a price that is much higher than the average in order to find it economically interesting to harvest (and take away later harvesting options) in a particular time period.

In any case, we find that the optimal harvest level is an increasing function of price in the same period.

Let us suggest a functional form of U, the relative harvest level, which is simple and robust and makes it possible to estimate the suggested effects.

$$U = a + bP + cQ \qquad (35)$$

(More precisely, U follows equation (35) as long as $0 < U < 1$. U is bounded from below and from above. In the following discussion, this assumption is implicit in order to save space.) P is price and Q is stock level. a, b and c are parameters such that $a < 0$, $0 < b$, $0 < c$. The harvest level is thus

$$h(t) = U(t)Q(t) = aQ(t) + bP(t)Q(t) + c[Q(t)]^2 \qquad (36)$$

Why is the relative harvest level U and not the harvest level, h, selected to be a linear and increasing function of P and Q? This is partly a compromise. First, it makes it possible to determine the synergy effects of P and Q since "bPQ" appears in the harvest function (36). Second, it makes it possible to determine if a "smooth" approach to some more or less stable equilibrium should be tried via the quadratic term cQ^2 in the expression of h. Third, we can be convinced that harvesting will stay feasible, $h \leq Q$. Finally, the function is robust and simple. If there are more parameters in the function, it is possible that the solution to the parameter optimization problem may not converge or may need a very large number of experiments in order to give reliable parameter estimates.

Optimization of the adaptive harvest function parameters

When the parameters of the adaptive relative harvest volume function should be optimized, we may experience numerical problems if the original form is used, namely:

$$U = a + bP + cQ \qquad (37)$$

The reason is that we need to calculate approximative values of the first order partial derivatives of U with respect to a, b and c within the gradient method. These derivatives are used when the optimal directions of parameter combination changes are determined.

When we change one of the parameters a, b or c, this generally affects the first order derivatives of U with respect to the other parameters strongly in this equation.

One common way to take the second order derivatives into consideration, improving the speed of convergence, is the well known Newton-Raphson method. This, however, only works well if the objective function is strictly concave everywhere. In this application, in particular since we can only obtain approximative numerical values of the function and the derivatives, the Newton-Raphson method does not work well. Because of the approximation errors present already in the estimations of U, it is clear that first, and in particular second, order derivative approximations, will be based on several errors. In cases where the true objective function is strictly concave but almost linear in some regions, which is typical in these optimization problems, the approximation errors can make the local estimations of the partial second order derivatives get the wrong signs.

Hence, the function may seem convex and the solution diverges from the true optimum with, for instance, the Newton-Raphson method, or other methods based on second order derivative estimations. For this reason, a gradient method based solely on the first order derivative approximations is used in this problem. This method works well as long as the objective function is quasi-concave. Hence, the applicability of the approach covers a quite general class of problems: A strictly concave objective function is no longer necessary.

In this particular kind of problem, one obtains much faster convergence to the optimal parameter solution if some other parameters (a°, b°, c°) are used instead of (a, b, c) in the algorithm. The new parameters (a°, b°, c°) are then transformed back to the old parameters (a, b, c). It is important to define the new parameters in such a way that the mixed second order derivatives of the objective function with respect to these parameters are close to zero and that the old parameters can uniquely be determined from the new parameters. In other words, the new parameters should express relations that are not likely to affect each other very much.

What such new parameters should we select? Here is the list of new parameters used in the optimizations:

- Parameter (a°) is the price level which makes $U = a + bP + cQ = 0$ when $Q = 100$. In other words, a° is the lowest price that makes harvesting profitable if the stock level is 100 cubic metres per hectare.

- Parameter (b°) is slope of the "iso relative harvest line", $\delta P/\delta Q$, at the point (Q, P) = (100, a°).

- Parameter (c°) is the derivative of U, the relative harvest function, with respect to price, at stock and price combinations (Q, P) where optimal harvesting is strictly positive.

Scaling problems and other numerical issues exist that are not discussed here. In the specially constructed computer program, these questions are discussed within the algorithm in the form of remarks.

Finally, when the parameters (a°, b°, c°) have been optimized, the old parameters (a, b, c) are determined from three equations:

$$a = c^{\circ}(100b^{\circ} - a^{\circ}) \quad (38)$$

$$b = c^{\circ} \quad (39)$$

$$c = -b^{\circ}c^{\circ} \quad (40)$$

(Note that changes in parameter c° affect the values of a, b and c simultaneously. Changes in b° affect the values of a and c.) It should be stressed that convergence to local optima (that almost certainly are global) was obtained with this method in every tested case.

The parameter optimizations were performed for all combinations of the real rates of interest 0%, 1%, 2%, 3%, 4% and the relative standard deviations in the price process 5%, 10%, 15%, 20%, 25%, 30%, 40%, 50%. Hence, totally 40 parameter set optimizations where made.

Next, a multiple ordinary least squares regression analysis, OLS, was made of the new parameters as functions of the real rate of interest and the relative standard deviation of price. The following results were found:

$$a^{\circ} = 115.5 - 18.88r + 1.271s \quad (41)$$

$$b^{\circ} = -0.2303 - 0.002999s \quad (42)$$

$$c^{\circ} = 0.01397 + 0.00384r - 0.0001544s \quad (43)$$

r and s denote real rate of interest and relative price standard deviation respectively. The regression calculations gave the result that all coefficients of the estimated equations were statistically signifficant at the 95% confidence level. We should be aware that the residuals should not be expected to be normally distributed because of nonlinearities in the model generating the data. Hence, we should not take the signifficance information too seriously. Nevertheless, when the partial derivatives of the parameters with respect to the rate of interest and to the relative standard deviation are studied, we find reasonable economic results:

- When the rate of interest increases, the lowest price that motivates harvesting (when Q = 100) decreases. This decrease is about 19 percent of the mean price (18.88 SEK) per unit (%) of the rate of interest. This finding is consistent with traditional resource economics: When the rate of interest increases, we should decrease the stock level.

- When the relative standard deviation of the price process increases, the lowest price that motivates harvesting (when Q=100) increases. When the relative standard deviation increases with 1 unit (1 %), then this price increases by 1.3 % of the mean price (1.271 SEK). This is consistent with the literature on stochastic resource economics under the influence of stationary price processes (Compare Lohmander (1987) and (1988a).) Note in particular that this result, also reveals that the expected value of the stock increases as the standard deviation of the price process increases. This is why we should not accept to harvest at a particular price in the more risky environment. We should save the resource longer and let it grow to a higher stock level, waiting for even better future options!

- When the relative standard deviation of price increases, the slope of the "iso relative harvest level" line becomes a little steeper. (The derivative ≈ - 0.003)

- The derivative of optimal relative harvesting with respect to the rate of interest is positive and the derivative with respect to the relative price standard deviation is negative. Again, this is understandable: When the rate of interest increases, future profits become less essential. Hence, present harvesting should be more sensitive to the present price when the rate of interest increases. When the price standard deviation increases, very high prices become more probable. Then harvesting should be less price sensitive than otherwise. We should not harvest much if the price is not very high.

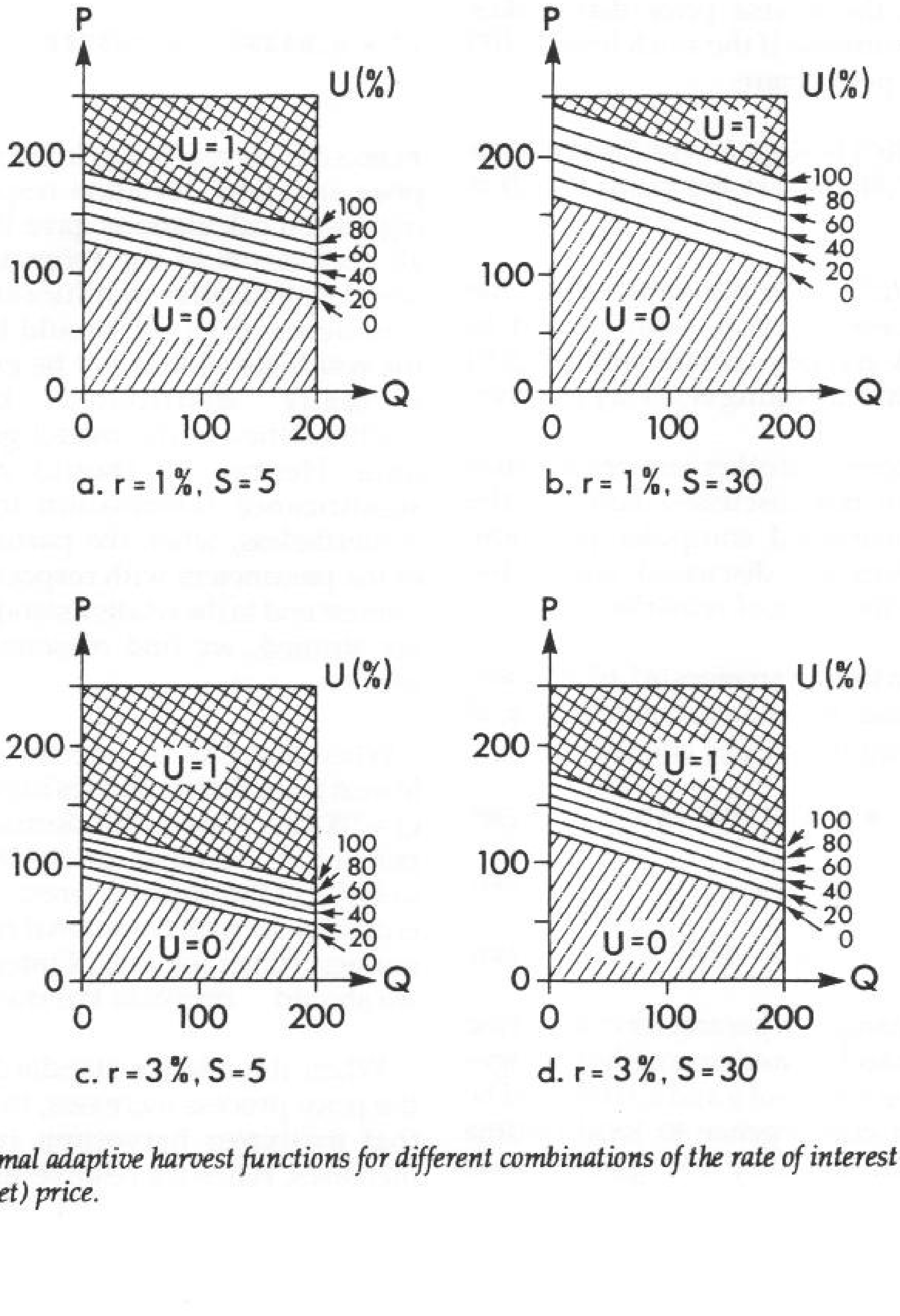

Figure 3. Optimal adaptive harvest functions for different combinations of the rate of interest and the standard deviation of (net) price.

Results

Below, the results from the analytical deterministic model will be discussed and compared to the results from the stochastic numerical model. Note in particular that the average optimal stock level derived from the stochastic optimization model is very similar to the optimal stock level in equilibrium, derived from the deterministic model. The degrees of stock and harvest variation in the stochastic model are however very high. Sometimes, the optimal "thinning" is to harvest all of the trees at once. Hence, we find that there is not always a stiff border between "pulse" and "continuous" harvesting in stochastic adaptive economic resource problems.

The deterministic equilibrium stock level

When we use one year long time periods, the empirical assumptions give us the growth equation

$$F(x) = 0.06 + 0.04x - 0.0001x^2 \quad (44)$$

The equilibrium stock level is determined by the condition that harvesting is constant,

$$\frac{\partial h}{\partial t} = 0 \quad (45)$$

This is fullfilled when

$$[F'(x^*) - r] = 0.04 - 0.0002x^* - r = 0 \quad (46)$$

From this equation, we may determine the equilibrium stock from the simple condition:

$$x^* = 200 - 5000r \quad (47)$$

Hence, we know that the equilibrium stock is 200 cubic metres per hectare when the rate of interest is 0. The equilibrium stock decreases with 50 cubic metres when the rate of interest increases with one percent.

Table 1. The equilibrium stock as a function of the rate of interest in the deterministic case

rate of interest	equilibrium stock
0	200
1	150
2	100
3	50
4	0

Optimal adaptive harvesting in the stochastic case

The results presented in this section are based on different assumptions concerning the rate of interest and the relative standard deviation of the net price. The optimized adaptive harvesting function is in each case determined via the equations reported in the earlier sections. In every case, the initial stock level is 50 cubic metres per hectare.

Figure 4. to Figure 9. include complete descriptions of the results obtained in the different cases. The quasi-random numbers used in the different cases are the same. Below each graph, the assumptions are printed. (r, S, SA, PV) denote (real rate of interest, relative standard deviation in the price process, "assumed" relative standard deviation in the price process, present value.) The "assumed" relative standard deviation is the value used to derive the optimal control function. Clearly, we can not always be sure that the controller knows the true value of the future standard deviation. In the analysis of this paper, we will however always assume that the controller makes the correct variability assumption. In Figure 4., 5. and 6., we find that the average stock level decreases dramatically with the rate of interest. This effect is almost identical to the effect which we found in Table 1. The present value is strongly negatively affected by the real rate of interest. Note that clear cuts take place also with this "thinning" model.

Figure 7. should be compared to Figure 4. In Figure 7., harvesting is constrained and in Figure 4. harvesting takes place according to the adaptive function and the revealed prices. The constraint implies that: 1/ - Harvesting takes place only if the stock level is above 150 cubic metres per hectare. 2/ - Then, the harvest level is such that the stock level after harvest is reduced to the value 150. We note that, even if the average stock level is almost the same as in Figure 4., the present value is much lower. The present value per hectare is reduced from SEK 26 263 to SEK 20 029 by the harvest constraint.

Figure 8. is based on a much lower relative price variation than Figure 4. We note that price variation and adaptive behaviour is

profitable: The present value in Figure 8. is SEK 20 036. The average stock level is still close to 150 cubic metres per hectare.

Figure 9. is the high net price variation case. Since the relative net price variation is higher than the average in areas where the variable harvesting and transportation costs are high, this figure represents such regions. The general result, that adaptive behaviour in the presence of high price variation is profitable, is emphasized.

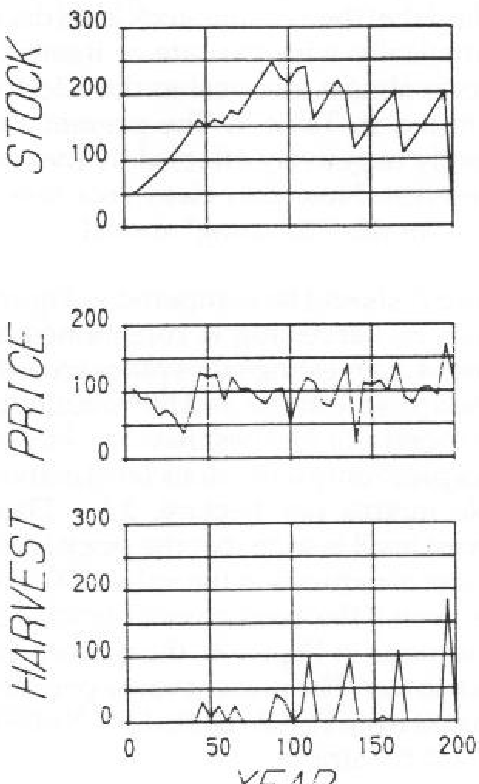

Figure 4. Stock level, net price and harvest level. Low rate of interest and medium price variation.

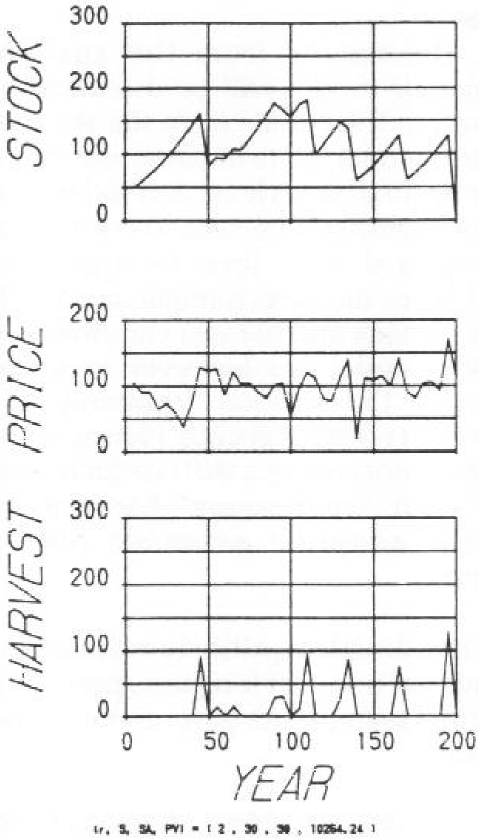

Figure 5.Stock level, net price and harvest level. Medium rate of interest and medium price variation

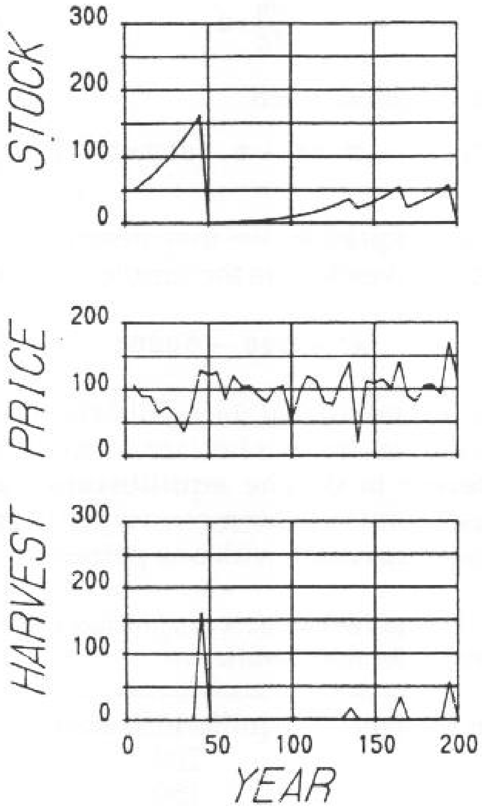

Figure 6. Stock level, net price and harvest level. High rate of interest and medium price variation.

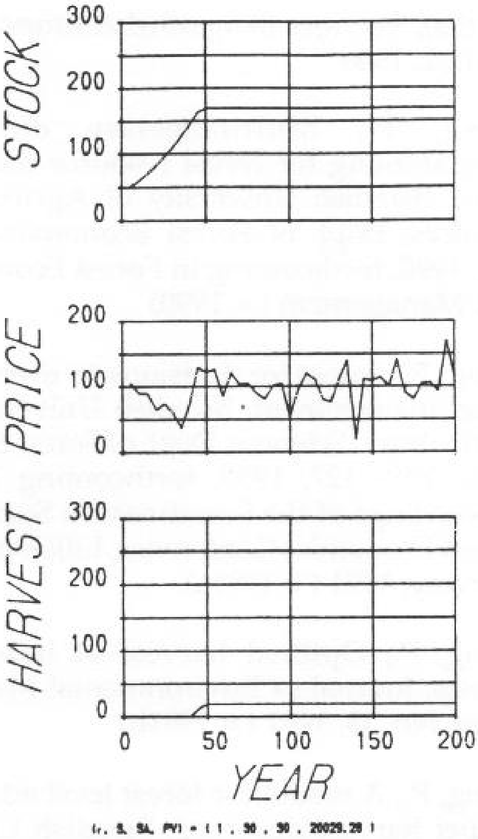

Figure 7. Stock level, net price and harvest level. Low rate of interest and medium price variation. Constrained and constant harvesting level.

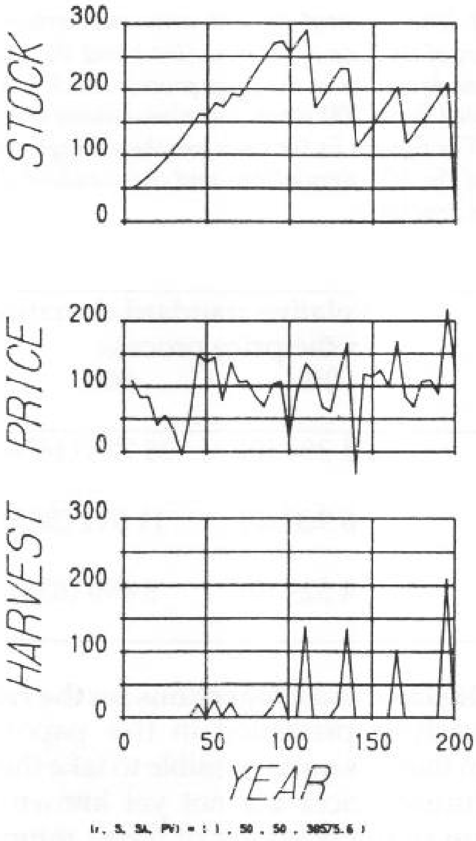

Figure 9. Stock level, net price and harvest level. Low rate of interest and high price variation.

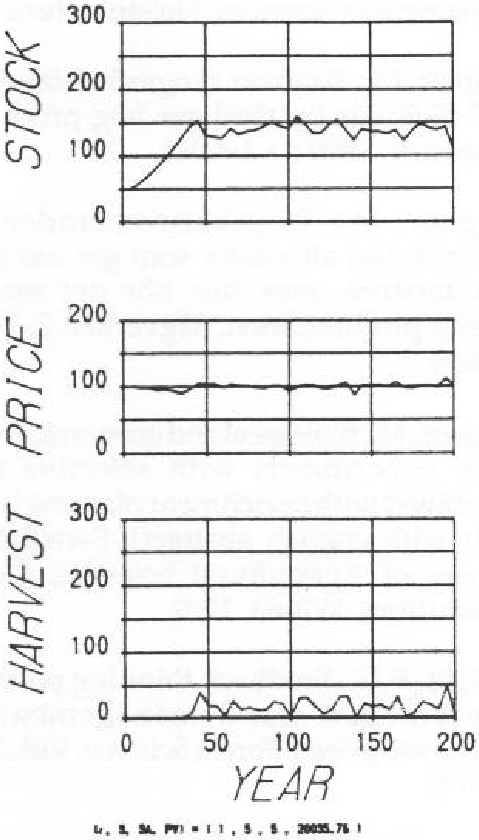

Figure 8. Stock level, net price and harvest level. Low rate of interest and low price variation.

In Table 2., the expected results from a large number of "stochastic price sequences" with adaptive harvesting are reported. The expected present value decreases rapidly with the rate of interest and increases much with the relative standard deviation in the price process. The standard deviations of the present values within each parameter group are surprisingly low. Hence, it is not very critical that a particular stochastic price sequence appears. The adaptive harvests take advantage of the good years when they appear and the forest volume can be distributed over time in an appropriate manner.

Table 2. The expected present value per hectare as a function of the rate of interest (Int.) and the relative standard deviation in the price process. Each figure is calculated from 100 stochastic simulations over 300 years. The figures in the table are the average present value of the 100 simulations and the standard deviation (in brackets)

Int.	Relative standard deviation in the price process 0%	30%
1%	21 294 (0)	28 726 (1656)
2%	6 960 (0)	11 011 (583)
3%	4 529 (0)	6 490 (633)

Conclusions and suggestions for the future
The analysis presented in this paper has shown that it is quite possible to take the fact that future prices are not yet known into account in the planning of forest thinnings (and clear cuts). The primary concern should be to optimize an adaptive harvest function. The harvest level should not be determined before we have observed the net price and the stock level. We can expect to gain considerable economic values from adaptive harvesting this way.

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