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Optimal Forest Harvesting over Time in the Presence of Air Pollution and Growth Reduction

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This paper is an attempt to formalize and solve the optimal resource management problem which arises under the influence of temporally changing growth conditions. One analytical and one numerical optimization model are developed and used in the analysis of the most urgent application, namely optimal forest management under the influence of temporally distributed pollution and growth changes. The numerical model also calculates the harvest path of the first generation conditional on the derived optimal harvesting strategy. As opposed to most earlier studies in the field, the optimization is performed conditional on the initial age and density distribution. This strongly affects the qualitative and quantitative results. It is found that the optimal harvest year generally occurs earlier in the initial generation and later in the following generations when the growth declines.

The implications of different temporal growth parameter specifications are highlighted. The critical model assumptions are discussed in connection to the empirically observed pollution – growth relationships. New empirical investigations that will be needed as support to forest management optimization are suggested. Numerical results from optimizations with Pinus contorta serve as illustrations.

1. Introduction

1.1. Earlier Work in the Field

The solution of the interdisciplinary problem under investigation in this paper is of course dependent on previous work in many different scientific areas. One attempt to classify these areas and publications is the following:

a. Pollution and growth effects in forests (McLaughlin [26], Schotte [37], Kauppi [16], Kuusela [18], Ovaskainen [31], Nylander [30])
b. Pollution, acidification and general despositions (Eliassen et al. [7], Jolanki et al. [15], Kauppi et al. [17], Scott [38], Alcamo et al. [2, 3])
c. Pollution control costs and revenues in forestry and other resources (Andersson et al. [5], Adams [1], Schotte [37], Johansson et al. [13], United Nations [43], Silvander et al. [40], Stoklasa et al. [42])
d. Pollution control optimization and simulation
   - Pollution control cost minimization (Morrison et al. [27], Shaw [39])
   - General pollution control and monitoring optimization (Pinter [32], Pinter et al. [33, 34], Somlyody et al. [41], Silvander et al. [40])
   - Other applications of optimization to pollution control (Young et al. [44])
   - Pollution control simulation (Alcamo et al. [2, 4])

e. Pollution problems, general positive and normative analysis (Naturvårdsverket [28], Hettelingh et al. [9], Hordijk [10, 11], Jolankai et al. [15], Schotte [37], Scott [38], United Nations [43], Stoklasa et al. [42])
f. Resource control optimization models (Norstrom [29], Clark [6], Risvand [36], Johansson et al. [14], Lohmander [19–25])

1.2. The Purpose of the Paper

The ambition expressed by the content of this paper is the following:

a. The optimal resource management problem in forestry under the influence of environmental changes should be formalized via an analytical model based on general functions.

b. An applicable numerical method for the solution of the optimization problem should be designed.

c. The optimal solution should be derived for different sets of assumptions representing possible changes in the environment and the corresponding possible effects in the growth function parameters.

d. Finally, important remaining issues should be further investigated: From the optimization results, particularly the derived sensitivity of the optimal harvest program to the assumptions, it should be apparent that the relations between the environmental phenomena and the growth function parameters are critical to the optimal management solutions. These relations must in the next stage of analysis be more intensively investigated, most likely via extended empirical investigations.

2. Analysis

In this kind of analysis, where the ambition is to find the model assumptions that are critical to the derived optimal control, it is necessary to present the model in full detail. In the mathematical and the numerical appendix, the reader can follow all steps of the derivations. In this section, the results, represented by graphs and the discussion of the sensitivity to assumptions, will be the main ingredients.

2.1. General Model Properties and Qualitative Observations

In order to make the numerical analysis as general as possible, a widely applied functional form of stand density is used. It was originally suggested by Fridh and Nilsson [8] and has been approximated for a wide variety of species on very different sites. According to Fridh and Nilsson [8], the stand density V (m³/hectare), is a function of age t (years) given by (1).

$$V(t) = 1.6416 * A_{msy} t_{msy} [1 - 6.3582^{(-t/t_{msy})}]^{2.8967} . \quad (1)$$

t_{msy} denotes the rotation age which maximizes the sustainable yield and A_{msy} is the corresponding yield. The set of assumptions and growth function parameters utilized in the numerical optimizations and graphs are found in the text which belongs to Fig. 2. In the literature on forest growth effects caused by acidification, Kuusela [18] and Ovaskainen [31] have suggested that two kinds of growth change may exist: 1. general

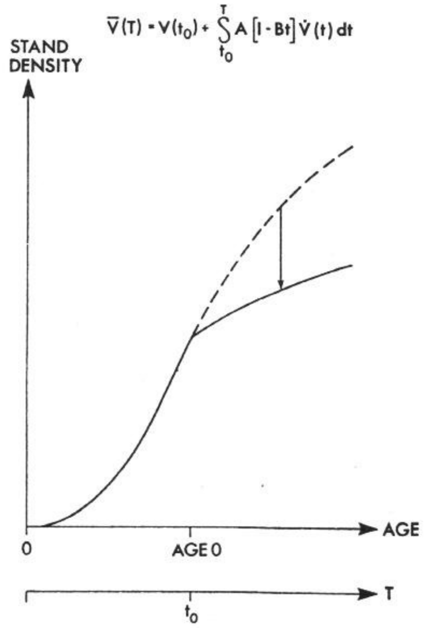

Fig. 1. The development of stand density before and after t_0, the time when a shift takes place in the environmental conditions (and a shift takes place in the growth parameter A). $V(t)$ denotes the "*undisturbed*" stand density function and $\bar{V}(t)$ is the "*disturbed*" stand density function. $0 < A < 1$, $B = 0$, $t_0 < t$. Note that $V(t)$ and $\bar{V}(t)$ are identical until the parameter change takes place at t_0.

growth reduction and 2. growth reduction which mainly affects the older trees. Hence, in order to formalize these hypoteses, we will in the rest of the analysis let the modified function $\bar{V}(T)$ represent the stand density.

$$\bar{V}(T) = V(t_0) + \int_{t_0}^{T} A(1 - Bt)\,\dot{V}(t)\,\mathrm{d}t. \tag{2}$$

In (2), t_0 denotes the stand age when the environmental change, for instance a new air pollution situation, takes place. After t_0, the new environmental state (pollution situation) is assumed to be constant. A is a general growth level parameter and B is the parameter which represents age dependent growth reduction. $\bar{V}(T)$ is identical to $V(T)$ if $A = 1$ and $B = 0$. In most graphs, the parameters A and B are replaced by APAR and BPAR in order make notation less ambiguous. Fig. 1 illustrates how $V(T)$ and $\bar{V}(T)$ are related.

The mathematical appendix is devoted to the general function analysis. In M.1., a set of weak restrictions are placed on some of the parameters and in M.2. the general properties of the adjusted volume function are derived. The optimization problem is defined and the optimal solution is derived and discussed in M.3. From the comparative

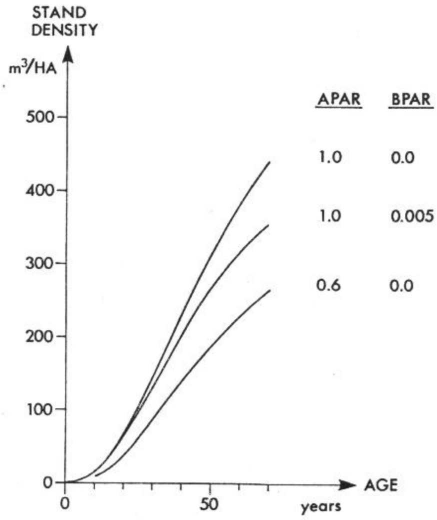

Fig. 2. The graph shows how the parameters A and B influence the stand density development. A general relative growth and stand density reduction is defined via A such that $0 < A(=APAR) < 1$. A relative growth reduction which is more severe in older stands is defined via B such that $0 < B(=BPAR)$. $A_{msy} = 6.4$, $t_{msy} = 60$, $t_0 = 1$. These parameters represent Pinus contorta when the dominating tree height at the age of 50 years equals 20 meters ($H50 = 20$ m), the number of stems per hectare is 1500 and no thinnings are undertaken. The empirical production data is presented by Hägglund [12].

statics analysis in M.4. and M.5. it is found that the optimal harvest age in the initially existing forest generation decreases, is unchanged or increases after an age independent reduction of the relative growth (via the parameter A). Five different special cases are defined. It is shown that the optimal direction of policy change can be determined in that special cases. The important points obtained here are the following:

a. The optimal direction and magnitude of policy change (harvest year) when the growth is reduced is ambiguous in the general function case. It is necessary to restrict the analysis to more specific cases in order to derive unambiguous harvest year guidelines.

b. Numerical optimization is strongly suggested if relevant questions should be analyzed.

In appendix M.6., the optimal change of the harvest year is investigated when the growth reduction takes place according to the parameter B (the relative growth decreases more in the old stands than in the young stands). Again, the general result is ambiguous and motivates numerical specifications. Some special cases with unambiguous conclusions are defined.

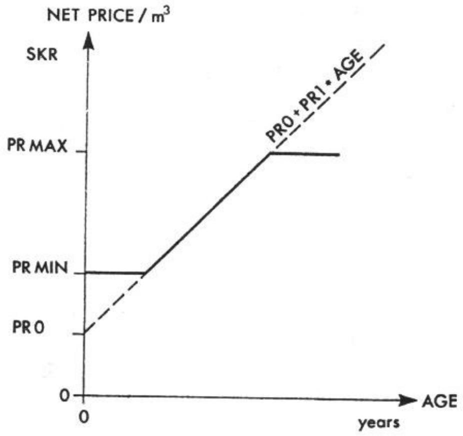

Fig. 3. The net price per m³ is described as a linear but kinked function of stand age. The four parameters *PR* 0, *PR* 1, *PRMIN* and *PR MAX* define the function. In the numerical examples presented by graphs in this paper, *PR MIN* and *PR MAX* are given values that make them irrelevant to the derived results. The option to use the parameters may be useful in other applications and is hence included in the computer code.

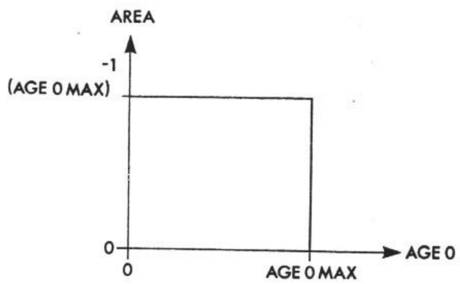

Fig. 4. In the optimal harvest path illustrations, it is assumed that the initial stand age (just before the change in environmental conditions takes place) is evenly distributed according to the graph. Hence, the relative area occupied by stands in a particular age interval is determined by the maximum initial age, *AGE* 0 *MAX*, which is a parameter.

The reader should be aware that the wood prices are assumed to be functions of age but not of the total harvest volume. Hence, in this respect, the analysis is partial and relevant only to operations that are marginal compared to the total wood market. On the other hand, since ambiguous results appear already in this partial analysis, we may be convinced that ambiguous results will appear also in more general specifications.

3. Conclusions and Remaining Questions

This paper contains both analytical and numerical modelling and derivations. Of course, unambiguous results extracted from general function models are of more interest than particular numerical results. However, as shown in this paper, even very simple "*general function*" representations of the investigated forest management optimization problem give ambiguous guidelines. The forest manager needs to get answers to questions of the kind: "– Should I harvest my stand earlier or later in the presence of growth reduction caused by environmental factors ? – How many years earlier (or later) should I harvest ?" It is always dangerous to draw conclusions from particular numerical model results. Furthermore, any model is just a model of reality. However, in order to extract unambiguous results, the model has to be restricted with respect to parameter choice. The undertaken numerical management optimizations have shown the sensitivity of the optimal harvest program to the parameter assumptions. Of course, a derived sensitivity

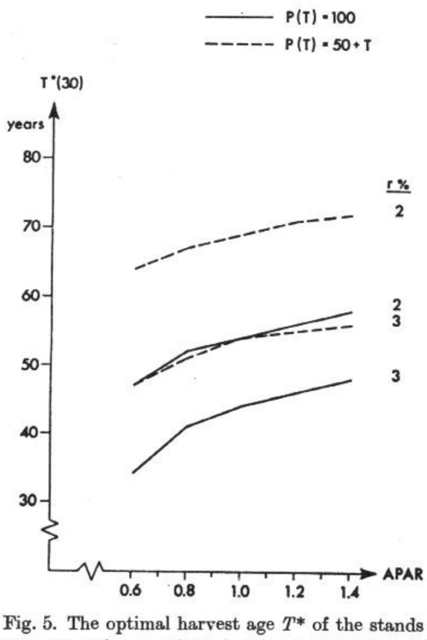

Fig. 5. The optimal harvest age T^* of the stands that are 30 years old when the growth parameter change takes place in A. The optimal harvest age is an increasing function of A, and a decreasing function of the rate of interest. The price function is also quite important in the optimization. Note that a *considerable* reduction of the growth parameter A from 1.0 (undisturbed growth) to 0.6 (40% growth reduction) implies only a 5 year reduction of the optimal harvest age (when $r = 2\%$ and $P(T) = 50 + T$). A "*small*" change in the rate of interest r from 2% to 3% will influence the harvest age much more, namely 15 years. If we replace one of the illustrated price functions by the other, the optimal harvest age will also change with 15 years.

$B(=BPAR) = 0$, $LANDV = 1000A$.

(from comparative statics analysis) is also a particular numerical result. However, since all parameters have been varied in the neighbourhood of the typical values, it is claimed that the quantitative results obtained and discussed in connection to the figures are of the correct order of magnitude in typical and relevant applications.

Through this methodology, it has been found that the optimal harvest program is very sensitive to economic parameters such as the rate of interest and the "*quality growth*" (the derivative of the net price with respect to age). The optimal control, namely the *harvest year*, is not very sensitive to possible growth reductions. A change in the rate of interest from 2% to 3% influences the optimal harvest year 3 times more than a 40% growth reduction in a typical forest stand! (Compare Fig. 5). The long run *harvest*

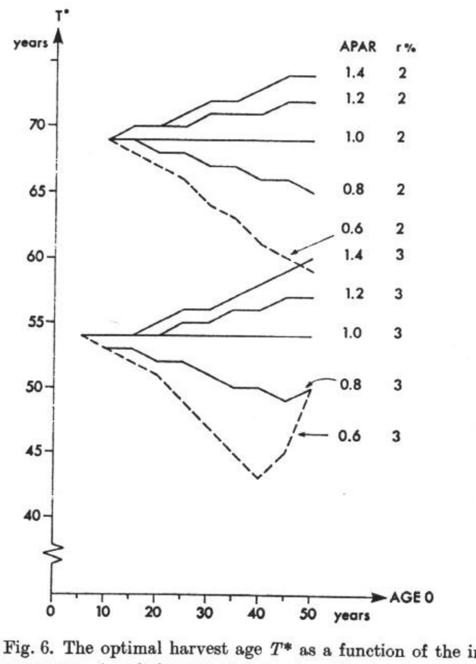

Fig. 6. The optimal harvest age T^* as a function of the initial age ($AGE\,0$), the growth parameter A and the rate of interest r. Clearly, the optimal harvest age is a decreasing function of the rate of interest r and an increasing function of the parameter A. Furthermore, the sensitivity of the optimal harvest age to the parameter A is an increasing function of the initial age (the age when the parameter shift takes place). The only exception is that no stand can be harvested before it has reached the initial age. Hence, the two bottom graphs have particular kinks.

$P(T) = 50 + T$, $B = 0$, $LANDV = 1000A$.

level on the other hand is quite sensitive to a 40% growth reduction and decreases with approximately 40% (Note that the long run harvest level is also a function of the harvest age, which is endogenous and changes.).

The main conclusions are the following:

a. *Modelling, optimization and empirical growth estimations*

1. An analytical optimization model for the forest management problem under the influence of temporally distributed growth parameter changes has been constructed and transformed to a numerical model.

2. It is possible to determine the optimal harvest policy change in the forest enterprise in the presence of environmental changes such as acidification. However, this requires that the relationship between the introduced growth function parameters and the environmental situation is known. According to reported empirical investigations, unambiguous relations between the forest growth and the acidity have not yet been possible to find (compare McLaughlin (1985)). Clearly, new empirical studies are needed in order to quantify the growth consequences of pollution and environmental changes in general.

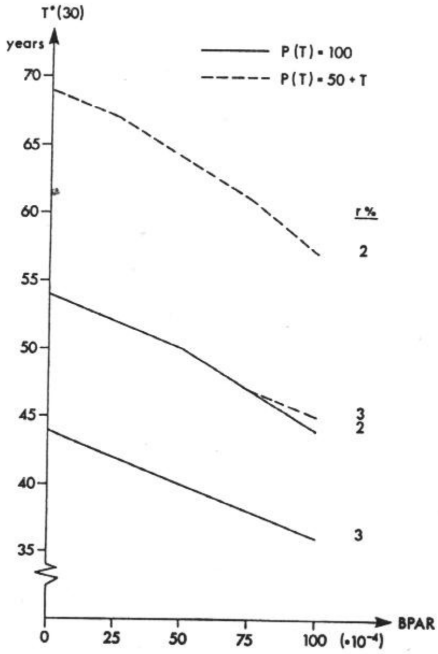

Fig. 7. The optimal harvest age, T^*, of the stands that are 30 years old when the growth function changes. T^* is a decreasing function of the parameter B and the rate of interest. $A = 1$, $LANDV = 1000(1\text{–}50B)$.

b. *Model observations and implications*

1. The optimal control of the established forest, the optimal harvest year, occurs earlier if the growth is reduced via the suggested parameter changes. This is shown in the Figs. 5 and 7.

2. The change in the optimal harvest year is greater in the stands that are old when the environmental change takes place than in the young stands. The reason is the following: If the stand has grown for a long time before the growth reduction (proportional to the undisturbed growth, via parameter A, or age dependent, via parameter B) takes place, it represents a large capital. Then, if it should be optimal not to harvest the stand directly, the interest gained (via the value growth) on the capital (the present value of the stand and the forest land) must be large. Obviously, a growth reduction (via the

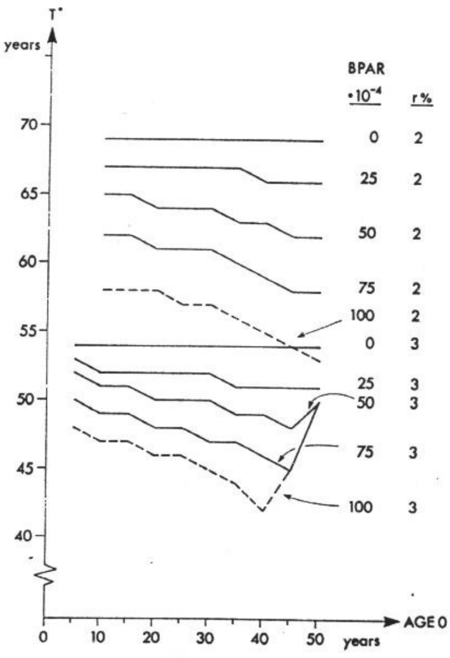

Fig. 8. The optimal harvest age, T^*, as a function of the initial age ($AGE\,0$), the growth function parameter B and the rate of interest. Clearly, T^*, is a decreasing function of the interest rate and B. The only exception is that no stand can be harvested before *"the initial age"*, which is shown in the three bottom graphs.

$A = 1$, $LANDV = 1000(1\text{–}50\,B)$, $P(T) = 50 + T$.

parameter A or B) implies that the optimal harvest year occurs earlier than otherwise. If the stand is very young when the growth conditions become worse, the present value of the stand and the land (the capital) will not reach the same high level at the particular age which is the optimal harvest age in the stand with higher initial age. Thus, in the presence of growth reductions, the optimal harvest age of the stands with low initial age is higher than the optimal harvest age of the stands with high initial age. This is illustrated in the Figs. 6 and 8.

3. The optimal total harvest path is of course a function of the optimal harvest decisions (the optimal harvest *year* decisions), the initial age distribution and the growth changes. The Figs. 9 and 10 show that the qualitative properties of the environmentally affected optimal harvest path are almost the same according to the two suggested definitions (via parameter A and B). The harvest activities instantly increase but the long run harvest level is reduced.

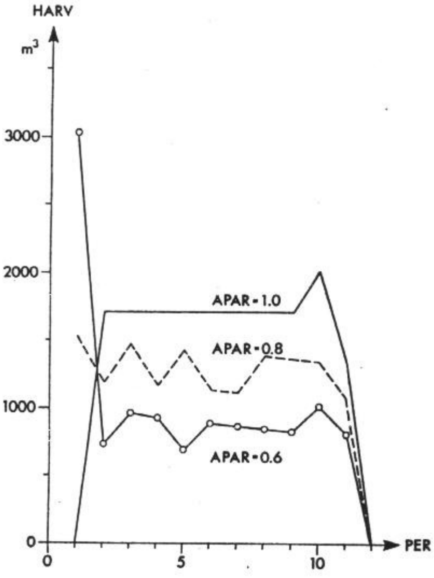

Fig. 9. The optimal harvest path of the first generation (does not include the stands that are results of reforestration undertaken after t_0). The initial harvest level is a decreasing function of the growth parameter A but the long run harvest level is an increasing function of A. PER denotes 5 year period (where period 1 starts at t_0) and $HARV$ is the total harvest during one period (if each one year age class of the initial forest occupies one hectare).

$B = 0$, $LANDV = 1000A$, $AGE\ 0\ MAX = 50$, $P(T) = 50 + T$, $r = 3\%$.

4. Fig. 11 shows that the present value of a forest is not proportional to the growth level. Clearly, even if the growth suddenly is reduced to zero, the forest at least represents the present value of the land and the stands that already (initially) exist. Fig. 12 corresponds to Fig. 11 but the relative growth reduction is defined as age dependent via parameter B.

Finally, we may ask the question:

– How should the management of the presently existing forests be modified in the new environmental situation?

The date when the growth change parameters suggested in this paper are known functions of the environmental state, then the answer will easily be obtained.

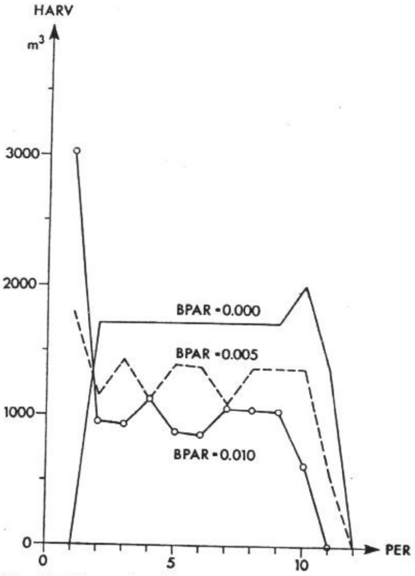

Fig. 10. The optimal harvest path of the first generation (does not include the stands that are results of reforestration undertaken after t_0). The initial harvest level is an increasing function of the growth parameter B but the long run harvest level is a decreasing function of B. PER denotes 5 year period (where period 1 starts at t_0) and $HARV$ is the total harvest during one period (if each one year class of the initial forest occupies one hectare). $A = 1$, $LANDV = 1000(1{-}50B)$, $AGE0MAX = 50$, $P(T) = 50 + T$, $r = 3\%$.

4. Mathematical Appendix

M1. Definitions

$V(t)$ denotes the volume per area unit (stand density) as a function of age (time interval since the plant investment took place), t, when the growth conditions have not been changed (disturbed). $\dot{V}(t) = V'(t) = \frac{\delta V(t)}{\delta t} > 0$, $V(0) = 0$. $\bar{V}(T)$ is the volume func-

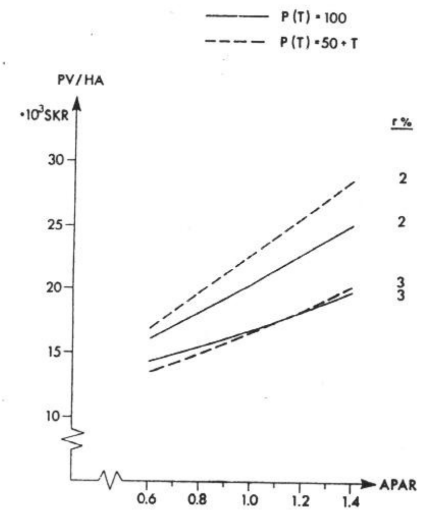

Fig. 11. Optimal present value per hectare of a forest with a uniform age distribution as a function of the growth parameter A. The present value is an increasing function of A and a decreasing function of the interest rate. Note that a change in the interest rate from 2% to 3% implies an economic loss of the same order as a 40% growth decline (A changes from 1.0 to 0.6).

$B = 0$, $LANDV = 1000\,A$, $AGE0MAX = 50$.

tion when the growth conditions have been changed (disturbed) during the time interval (t_0, T), $0 < t_0 < T$. $\bar{V}(T)$ is defined via $V(t)$ and the parameters A and B. In the analytical derivations of this mathematical appendix, we assume that A and B are constants in the time interval (t_0, ∞). In the computer program included in the numerical appendix, A and B can, after some modifications of the presented code, be defined as arbitrary functions of time. In that case, the objective function is generally not concave and many local optima may coexist. This is the reason why a grid search method is used in the computer program.

$$\bar{V}(T) = V(t_0) + \int_{t_0}^{T} A(1 - Bt)\,\dot{V}(t)\,dt$$

The net price per volume unit $P(t)$ is assumed to be a linear function of stand age. This function reflects the value of increasing dimensions, quality and decreasing unit harvesting cost of a typical forest stand with age. In the numerical analysis, $P(T)$ is bounded from above and from below by arbitrary constants.

$$P(T) = p_0 + p_1 T$$

L denotes the value of the land "*released*" after harvest per area unit if the optimal land use is selected after the harvest. Hence, L is affected by all available species options in possible future generations and other land uses. This value is assumed to be a linear

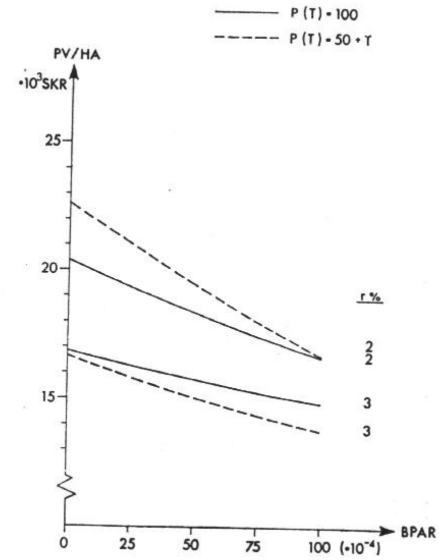

Fig. 12. Optimal present value per hectare of a forest with a uniform age distribution as a function of the growth parameter B. The present value is a decreasing function of B and of the interest rate.

$A = 1$, $LANDV = 1000(1\text{–}50B)$, $AGE\ 0\ MAX = 50$.

function of the growth function parameters A and B. For obvious reasons, the *correct* functional form is in general not linear. This is just an approximation in this analysis. In any case, the magnitude of the contribution of L to the objective function is low (in relation to the value of the first harvest) in most applied problems. Hence, the linear approximation is justified.

$$L = k_0 + k_A A + k_B B \qquad (\text{Assumptions: } k_0 \gtreqless 0,\ k_A \geq 0,\ k_B \lessgtr 0)$$

M2. The properties of the adjusted volume function $\overline{V}(T)$

a. $[A = 1, B = 0] \Rightarrow [\overline{V}(T) = V(T)]$

b. $$\frac{\delta \overline{V}(T)}{\delta A} = \int_{t_0}^{T} (1 - Bt)\, \dot{V}(t)\, \mathrm{d}t$$

c. $$\left[\dot{V}(t) > 0,\ B < \frac{1}{T}\right] \Rightarrow \left[\frac{\delta \overline{V}(T)}{\delta A} > 0\right]$$

d. $$\frac{\delta \overline{V}(T)}{\delta B} = -A \int_{t_0}^{T} t \dot{V}(t)\, \mathrm{d}t$$

e. $$[\dot{V}(t) > 0,\ A > 0] \Rightarrow \left[\frac{\delta \bar{V}(T)}{\delta B} < 0\right]$$

f. $$\dot{\bar{V}}(T) = \bar{V}'(T) = \frac{\delta \bar{V}(T)}{\delta T} = A(1 - BT)\ \dot{V}(T)$$

g. $$\left[A > 0,\ B < \frac{1}{T}\right] \Rightarrow [\dot{\bar{V}}(T) = \bar{V}'(T) > 0]$$

M3. The optimization problem and the comparative statics analysis

The optimization problem (for each initial age class) is to maximize Π, the present value of the next harvest ($P(T)\ \bar{V}(T)$) and the value of the released land (L). The present value is determined at the time when the age of the stand is zero. Hence, the discounting time is set to T, the harvest age. The (continuous) rate of interest is denoted by r. The investment cost of the presently existing stand is a "*sunk cost*" (exogenous in the present optimal harvest decision problem) which has been excluded from the analysis. The value of the land (L) is in this analysis, as opposed to in most literature in the field, *not* defined as the present value of an infinite series of future generations identical to the first (already existing) one. In a changing environment, it is generally not optimal to keep the policy constant over time.

$$\max_T \Pi = (P(T)\ \bar{V}(T) + L)\ e^{-rT}$$

The first order optimum condition is:

$$\Pi_T = \underbrace{[P'(T)\ \bar{V}(T) + P(T)\ \bar{V}'(T) - r(P(T)\ \bar{V}(T) + L)]}_{=0}\ \underbrace{e^{-rT}}_{>0} = 0$$

Remark 1. When T, the harvest age, has been optimally chosen, we have:

$$[\Pi_T = 0] \Rightarrow \frac{P'(T)\ \bar{V}(T) + P(T)\ \bar{V}'(T)}{P(T)\ \bar{V}(T) + L} = r$$

One interpretation of the expression is that the value growth in the forest stand (the nominator) should give exactly the same rate of interest to the value of the forest stand + the occupied land (the denominator) as the best alternative investment (which gives the rate of interest r).

Explicit use of the function definitions gives (The expression below also defines the functions $f(.)$ and $g(.)$.):

$$\Pi_T = \underbrace{e^{-rT}}_{f > 0}\underbrace{\left[p_1 \bar{V}(T) + (p_0 + p_1 T) A(1 - BT)\ \dot{V}(T) - r\left[(p_0 + p_1 T)\ \bar{V}(T) + (k_0 + k_A A + k_B B)\right]\right]}_{g = 0}$$

M4. Determination of the sign of Π_{TA}

In the comparative statics analysis, we will need to know the sign of Π_{TA}.

$$\Pi_{TA} = \frac{\delta f}{\delta A} g + f \frac{\delta g}{\delta A}$$

Since $f > 0$ and $g = 0$, $\text{sgn}(\Pi_{TA}) = \text{sgn}\left(\frac{\delta g}{\delta A}\right)$.

$$\frac{\delta g}{\delta A} = p_1 \int_{t_0}^{T} (1 - Bt)\, \dot{V}(t)\, \mathrm{d}t + (p_0 + p_1 T)(1 - BT)\, \dot{V}(T)$$
$$- r\left[(p_0 + p_1 T) \int_{t_0}^{T} (1 - Bt)\, \dot{V}(t)\, \mathrm{d}t + k_A\right]$$

$$A\frac{\delta g}{\delta A} = p_1 A \int_{t_0}^{T} (1 - Bt)\, \dot{V}(t)\, \mathrm{d}t + (p_0 + p_1 T)\, A(1 - BT)\, \dot{V}(T)$$
$$-r\left[(p_0 + p_1 T)\, A \int_{t_0}^{T} (1 - Bt)\, \dot{V}(t)\, \mathrm{d}t + k_A A\right]$$

$$\psi = \left[\underset{>0}{A\frac{\delta g}{\delta A}} - \underset{=0}{g}\right] = -p_1 V(t_0) - r\left[-(p_0 + p_1 T)\, V(t_0) - k_0 - k_B B\right]$$

$$\psi = [r p_0 + p_1(rT - 1)]\, V(t_0) + r(k_0 + k_B B)$$

$$\text{sgn}(\Pi_{TA}) = \text{sgn}(\psi)$$

M5. The optimal harvest age and the parameter A

Differentiation of the first order optimum condition ($\Pi_T = 0$) with respect to T and A gives (we assume that $\Pi_{TT} < 0$ and that a unique optimum T^* already has been found):

$$\underset{<0}{\Pi_{TT}}\ \underset{\text{policy change}}{\mathrm{d}T^*} + \underset{\text{sgn}(\psi)}{\Pi_{TA}}\ \underset{\text{parameter change}}{\mathrm{d}A} = 0$$

Clearly, $\frac{\delta T^*}{\delta A} = -\frac{\Pi_{TA}}{\Pi_{TT}}$ and $\text{sgn}\left(\frac{\delta T^*}{\delta A}\right) = \text{sgn}(\psi)$.

Remark 2. Inspection of the function $\psi(r, p_0, p_1, T, V(t_0), k_0, k_B, B)$ shows that $\text{sgn}(\psi)$, and hence $\text{sgn}\left(\frac{\delta T^*}{\delta A}\right)$ generally are ambiguous. However, some of the important special cases are:

a. $(r > 0,\ p_0 > 0,\ p_1 = 0,\ V(t_0) > 0,\ k_0 \geq 0,\ k_B = 0)$

Result: $\psi > 0$, $\frac{\delta T^*}{\delta A} > 0$. *Hence, if A decreases (the growth is reduced), then the presently existing stands should be harvested earlier.*

b. $(r > 0,\ p_0 > 0,\ p_1 > 0,\ V(t_0) > 0,\ k_0 \geq 0,\ k_B = 0,\ r > T^{-1})$

This set of assumptions may be relevant in many applied cases. (Compare the examples illustrated in figure 5.)

Result: $\psi > 0$, $\frac{\delta T^*}{\delta A} > 0$. *Hence, a growth reduction implies that the presently existing stands should be harvested earlier.*

c. $(V(t_0) = 0,\ k_0 = 0,\ k_B = 0)$

Result: $\psi = 0$, $\frac{\delta T^*}{\delta A} = 0$. Hence, if the land value is zero or proportional to A and the growth conditions change before the analysed forest stand has been established

($t_0 \leq 0$, $V(t_0) = 0$), then *the optimal harvest age is not a function of the growth reduction* represented by the decreasing value of A.

d. $(r > 0,\ V(t_0) = 0,\ k_0 < 0,\ k_B = 0)$

Result: $\psi < 0$, $\frac{\delta T^*}{\delta A} < 0$. *The optimal harvest age increases if the growth is reduced* as represented by a reduction of the growth function parameter A.

e. $(r > 0,\ V(t_0) = 0,\ k_0 > 0,\ k_B = 0)$

Result: $\psi < 0$, $\frac{\delta T^*}{\delta A} < 0$. *The optimal harvest age decreases if the growth is reduced* as represented by a reduction of the growth function parameter A.

M6. The optimal harvest age, a restricted case and the parameter B

In order to simplify the derivations, we restrict the attention to the special case when

$$V(t) = LN(t)$$

$$\dot{V}(t) = \frac{1}{t}$$

$$\overline{V}(T) = V(t_0) + \int_{t_0}^{T} A(1 - Bt)(1/t)\,\mathrm{d}t$$

In this case, we can express $\overline{V}(T)$ explicitly as:

$$\overline{V}(T) = V(t_0) + A\int_{t_0}^{T}\left(\frac{1}{t} - B\right)\mathrm{d}t$$

$$\overline{V}(T) = V(t_0) + A[LN(T) - LN(t_0) - B(T - t_0)]$$

Now, we make use of the explicit form of $\overline{V}(T)$ and write $g(.)$ as:

$$g = p_1\,[V(t_0) + A[LN(T) - LN(t_0) - B(T - t_0)]] + (p_0 + p_1T)\,A\left[\frac{1}{T} - B\right]$$
$$-r\,[(p_0 + p_1T)\,[V(t_0) + A\,[LN(T) - LN(t_0) - B(T - t_0)]]$$
$$+ k_0 + k_AA + k_BB]$$

This version of g may be used to determine the optimal value of T via the Newton Raphson method. ($g = 0$ when T is the optimal harvest age.) In the comparative static analysis, we will need to know the sign of $\frac{\delta g}{\delta B}$:

$$\frac{\delta g}{\delta B} = -p_1A(T - t_0) + (p_0 + p_1T)\,A[r(T - t_0) - 1] - rk_B$$

I—Δt—I

I— $\gtrless 0$ —I I— > 0 —I I— ≥ 0 —I

I— < 0 for small r and Δt —I

I—small—I

I— < 0 in most cases —I

Clearly, the sign of $\frac{\delta g}{\delta B}$ is in general ambiguous. The second order condition for a maximum is that $\Pi_{TT} < 0$.

$$\Pi_{TT} = \frac{\delta f}{\delta T} g + f \frac{\delta g}{\delta T}.$$

Since $f > 0$ and $g = 0$, it is obvious that $\operatorname{sgn}(\Pi_{TT}) = \operatorname{sgn}\left(\frac{\delta g}{\delta T}\right)$. However, the sign of $\frac{\delta g}{\delta T}$ is a complecated function of all parameters and will not be discussed here. We simply assume that $\Pi_{TT} < 0$ and that a unique optimum T^* has been found.
Differentiation of the first order optimum condition ($\Pi_T = 0$) with respect to T and B gives:

$$\underset{<0}{\Pi_{TT}} \;\; \underset{\substack{\text{policy}\\ \text{change}}}{\mathrm{d}T^*} \;\; + \;\; \underset{\operatorname{sgn}\left(\frac{\delta g}{\delta B}\right)}{\Pi_{TB}} \;\; \underset{\substack{\text{parameter}\\ \text{change}}}{\mathrm{d}B} \;\; = 0$$

Clearly, $\frac{\delta T^*}{\delta B} = - \frac{\Pi_{TB}}{\Pi_{TT}}$.

Remark 3. a. For small r and Δt (the "*initial*" age t_0 of the stand is high before the growth conditions become worse via the increasing value of B), in general $\frac{\delta g}{\delta B} < 0$ and hence $\frac{\delta T^*}{\delta B} < 0$.

This means that the presently existing old stands should be harvested earlier if the growth conditions become worse.

b. For high values of Δt (when the growth conditions become worse very early in the life of the trees and thus the "*initial*" age t_0 has a low value), it is possible that $\frac{\delta g}{\delta B} > 0$ and hence $\frac{\delta T^*}{\delta B} > 0$.

Then, the stands of the new generations or the young stands that exist when the growth conditions become worse, should be harvested later than the optimal harvest age in the undisturbed environment.

5. Numerical Appendix

5.1. The Optimization Program

```
10 LPRINT" "
20 LPRINT CHR$(15)
30 LPRINT CHR$(14);"COMPUTER CODE FOR OPTIMIZATION OF THE HARVEST"
40 LPRINT CHR$(14);"PATH FROM AN EXOGENOUS INITIAL AGE DISTRIBUTION"
50 LPRINT CHR$(14);"AFFECTED BY TEMPORAL GROWTH PARAMETER CHANGES"
60 LPRINT CHR$(14);"Lohmander Peter 88-08-22"
70 LPRINT"Swedish University of Agricultural Sciences"
80 LPRINT"Dept. of Forest Economics"
90 LPRINT"S-901 83 UMEÅ, SWEDEN"
```

```
100 DIM A(200),B(200),AGEMAT(200),VOL(200),PRICE(200),HARV(40)
110 INPUT"THE MAXIMUM SUSTAINALBE YIELD WITH UNDISTURBED GROWTH
IS ? ",AMSY
120 INPUT"THE ROTATION AGE WHICH GIVES MAXIMUM SUSTAINABLE YIELD
IS ? ",TMSY
130 INPUT"THE GROWTH PARAMETER APAR IS ? ",APAR
140 INPUT"THE GROWTH PARAMETER BPAR IS ? ",BPAR
150 INPUT"THE PRICE FUNCTION PARAMETER PPAR IS ? ",PPAR
160 INPUT"THE RATE OF INTEREST IS ? ",R
170 INPUT"THE VALUE PER HECTARE OF THE LAND RELEASED AFTER HARVEST
IS ? ",LANDV
180 INPUT"THE INITIAL AGE DISTRIBUTION PARAMETER AGE0MAX
IS ? ",AGE0MAX
190 INPUT"THE HIGHEST POSSIBLE ROTATION AGE UNDER CONSIDERATION
IS ? ",AGEHMAX
200 PERMAX = AGEHMAX/5 + 1
210 PERMAX = INT(PERMAX)
220 REM
230 REM ********** PRINTOUT OF CENTRAL PARAMETERS
240 REM
250 LPRINT" "
260 LPRINT CHR$(14);"AMSY = ";AMSY;" TMSY = ";TMSY;" APAR = ";APAR
270 LPRINT CHR$(14);"BPAR = ";BPAR;" RATE OF INTEREST = ";R
280 LPRINT CHR$(14);"LANDVALUE = ";LANDV;" AGE0MAX = ";AGE0MAX
290 LPRINT CHR$(14);"AGEHMAX = ";AGEHMAX;" PPAR = ";PPAR
300 FOR PERIOD = 1 TO 40
310 HARV(PERIOD)=0
320 NEXT PERIOD
330 REM
340 REM ********** PARAMETER PATH DEFINITIONS
350 REM
360 FOR T = 1 TO AGEHMAX
370 PRICE(T)=PPAR
380 A(T)=APAR
390 B(T)=BPAR
400 NEXT T
410 REM
420 REM ********** DEFINITION OF THE EXOGENOUS INITIAL AGE
DISTRIBUTION
430 REM
440 FOR T = 1 TO AGE0MAX
450 AGEMAT(T)=1/AGE0MAX
460 NEXT T
470 REM
480 REM ********** DEFINITION OF "UNDISTURBED" GROWTH FUNCTION
490 REM ********** (ACCORDING TO FRIDH AND NILSSON (1980)) AND
500 REM ********** THE PRESENT VALUE OBJECTIVE FUNCTION
510 REM
520 DEF FNV(T)=AMSY*TMSY*1.6416*(1-6.3582^(-T/TMSY))^2.8967
530 DEF FNVD(T)=FNV(T+1) - FNV(T)
540 DEF FNPVALUE(YEAR,VOLT,P) = EXP(-R*YEAR)*(P*VOLT + LANDV)
550 REM
```



```
560 REM
570 REM
580 REM
LOCAL
590 REM
600 REM
610 LTO
620 FOR
630 REM
630 REM
640 REM
650 REM
660 VOL
670 FOR
680 AGE
690 VDI
700 IF V
710 YM
720 VOL
730 NEX
740 REM
750 REM
760 REM
770 IF A
780 LPR
790 LPR
800 LPR
810 FOR
820 YY=
830 LPR
840 NEX
850 REM
860 REM
SEARCH
870 REM
EACH
880 REM
890 REM
900 IF A
910 LPR
920 LPR
(=AGE
930 LPR
940 YE.
950 OB.
960 YE.
970 AGI
980 IF
990 IF
1000 YI
1010 IF
1020 AC
```

```
560 REM ********** FOR EACH INITIAL AGE CLASS, THE VOLUME PATH IS
570 REM ********** DERIVED. THEN, THE OPTIMAL HARVEST PERIOD IS
580 REM ********** DETERMINED VIA GRID SEARCH (BECAUSE MANY
LOCAL
590 REM ********** OPTIMA MAY COEXIST).
600 REM
610 LTOT = 0
620 FOR AGE 0 = 1 TO AGE 0 MAX
630 REM
630 REM
640 REM ********** DERIVATION OF THE VOLUME PATH'S
650 REM
660 VOL(1)=FNV(AGE 0)
670 FOR YEAR= 2 TO AGEHMAX
680 AGE = AGE 0 + YEAR - 1
690 VDIFF = A(YEAR)*(1-(B(YEAR))*(AGE-1))*(FN VD(AGE-1))
700 IF VDIFF<0 THEN VDIFF = 0
710 YM = YEAR - 1
720 VOL(YEAR) = VOL(YM) + VDIFF
730 NEXT YEAR
740 REM
750 REM ********** PRINTOUT OF ONE VOLUME PATH
760 REM
770 IF AGE 0>1 THEN GOTO 900
780 LPRINT" "
790 LPRINT CHR$(14);"THE VOLUME PATH FOR AGE 0 = ";AGE 0
800 LPRINT CHR$(14);" YEAR VOLUME"
810 FOR Y = 1 TO PERMAX
820 YY=Y*5
830 LPRINT CHR$(14) USING"#######";YY;VOL(YY)
840 NEXT Y
850 REM
860 REM ********** DERIVATION OF OPTIMAL HARVEST YEAR VIA GRID
SEARCH
870 REM ********** AND SEQUENTIAL PRINTOUT OF THE RESULTS FOR
EACH
880 REM ********** INITIAL AGE
890 REM
900 IF AGE 0>1 THEN GOTO 940
910 LPRINT " "
920 LPRINT CHR$(14);"OPTIMAL POLICY FOR EACH INITIAL AGE CONDITION
(=AGE 0)"
930 LPRINT CHR$(14);" AGE 0 AGE* YEAR* VOL* PVALUE/HA"
940 YEAROPT = 0
950 OBJ = -1000000!
960 YEAR = 0
970 AGE = AGE 0 - 1
980 IF AGE 0<10 THEN AGE=10
990 IF AGE 0<10 THEN YEAR=10-AGE 0
1000 YEAR = YEAR + 1
1010 IF YEAR>AGEHMAX THEN GOTO 1090
1020 AGE = AGE + 1
```

```
1030 EV = FNPVALUE(YEAR,VOL(YEAR),PRICE(AGE))
1040 IF EV<OBJ THEN GOTO 1000
1050 OBJ = EV
1060 YEAROPT = YEAR
1070 AGEOPT = AGE
1080 GOTO 1000
1090 VOLOPT = VOL(YEAROPT)
1100 CCC = (YEAROPT - A)/5+1
1110 PERIOD = INT(CCC)
1120 HARV(PERIOD) = HARV(PERIOD) + VOLOPT
1130 LPRINT CHR$(14) USING"#######";AGE0;AGEOPT;YEAROPT;VOLOPT;OBJ
1140 LTOT = LTOT + OBJ * AGEMAT (AGE0)
1150 NEXT AGE0
1160 REM
1170 REM ********** PRINTOUT OF TOTAL PRESENT VALUE AND THE
OPTIMAL
1180 REM ********** HARVEST PATH FROM ALL PARTS OF THE INITIAL
AGE
1190 REM ********** DISTRIBUTION (THE FIRST GENERATION ONLY)
1200 REM
1210 LPRINT " "
1220 LPRINT CHR$(14);"TOTAL PRESENT VALUE PER HECTARE = ";LTOT
1230 LPRINT" "
1240 LPRINT CHR$(14);"OPTIMAL PATH HARVEST IN FIVE YEAR PERIODS"
1250 LPRINT CHR$(14);" PER HARV PER HARV PER HARV PER HARV"
1260 YMAX = (PERMAX - 1)/4 + 1
1270 YMAX = INT(YMAX)
1280 FOR Y = 1 TO YMAX
1290 Y1 = (Y-1)*4 + 1
1300 H1 = INT(HARV(Y1))
1310 Y2 = Y1 + 1
1320 H2 = INT(HARV(Y2))
1330 Y3 = Y2 + 1
1340 H3 = INT(HARV(Y3))
1350 Y4 = Y3 + 1
1360 H4 = INT(HARV(Y4))
1370 LPRINT CHR$(14) USING"######";Y1;H1;Y2;H2;Y3;H3;Y4;H4
1380 NEXT Y
1390 REM
1400 REM ********** GRAPHICAL PRINTOUT OF THE OPTIMAL TOTAL
HARVEST PATH
1410 REM
1420 GMAX = 0
1430 FOR Y = 1 TO YMAX
1440 EV=INT(HARV(Y))
1450 IF GMAX<EV THEN GMAX = EV
1460 NEXT Y
1470 YGR = 20000
1480 IF GMAX<10000 THEN YGR = 10000
1490 IF GMAX< 9000 THEN YGR = 9000
1500 IF GMAX< 8000 THEN YGR = 8000
1510 IF GMAX< 7000 THEN YGR = 7000
```



```
1520 IF
1530 IF
1540 IF
1550 IF
1560 IF
1570 IF
1580 LI
1590 LI
1600 LI
PATH"
1610 LI
number
1620 LI
HARVI
1630 LI
1640 LI
1650 LI
1660 LI
80++
1670 F
1680 H
1690 H
1700 L
1710 N
```

6.2.

```
COMP
INITI
CHAN
AMSY
BPAR
LAND
AGEH
```

```
1520 IF GMAX< 6000 THEN YGR = 6000
1530 IF GMAX< 5000 THEN YGR = 5000
1540 IF GMAX< 4000 THEN YGR = 4000
1550 IF GMAX< 3000 THEN YGR = 3000
1560 IF GMAX< 2000 THEN YGR = 2000
1570 IF GMAX< 1000 THEN YGR = 1000
1580 LPRINT" "
1590 LPRINT" "
1600 LPRINT CHR$(14);"GRAPHICAL PRINTOUT OF THE OPTIMAL TOTAL HARVEST
PATH"
1610 LPRINT CHR$(14);"(each period represents five years, the line number is the period
number)"
1620 LPRINT CHR$(14);"MAXIMUM HARVEST = ";GMAX;". 100 PERCENT DENOTES
HARVEST = ";YGR
1630 LPRINT" "
1640 LPRINT" "
1650 LPRINT" "
1660 LPRINT CHR$(14); "1++10+++20+++30+++40+++50+++60+++70+++
80+++90++100 PERCENT"
1670 FOR Y = 1 TO PERMAX
1680 HEIGHT = (HARV(Y)/YGR)*50
1690 HE = INT(HEIGHT)
1700 LPRINT CHR$(14); STRING$(HE,"*")
1710 NEXT Y
```

6.2. A Numerical Example

COMPUTER CODE FOR OPTIMIZATION OF THE HARVEST PATH FROM AN EXOGENOUS INITIAL AGE DISTRIBUTION AFFECTED BY TEMPORAL GROWTH PARAMETER CHANGES
AMSY = 6.4 TMSY = 60 APAR = 1
BPAR = 0 RATE OF INTEREST = .03
LANDVALUE = 1000 AGE0MAX = 50
AGEHMAX = 70 PPAR = 100

THE VOLUME PATH FOR AGEO = 1

YEAR	VOLUME
5	2
10	13
15	35
20	67
25	104
30	146
35	189
40	232
45	274
50	314
55	350
60	384
65	414
70	442
75	0

OPTIMAL POLICY FOR EACH INITIAL AGE CONDITION (=AGE 0)

AGE 0	AGE *	YEAR *	VOL *	PVALUE/HA
1	45	44	266	7374
2	45	43	266	7598
3	45	42	266	7830
4	45	41	266	8068
5	45	40	266	8314
6	45	39	266	8567
7	45	38	266	8828
8	45	37	266	9097
9	45	36	266	9374
10	44	35	266	9659
11	44	34	266	9954
12	44	33	266	10257
13	44	32	266	10569
14	44	31	266	10891
15	44	30	266	11223
16	44	29	266	11564
17	44	28	266	11917
18	44	27	266	12279
19	44	26	266	12653
20	44	25	266	13039
21	44	24	266	13436
22	44	23	266	13845
23	44	22	266	14267
24	44	21	266	14701
25	44	20	266	15149
26	44	19	266	15610
27	44	18	266	16086
28	44	17	266	16576
29	44	16	266	17080
30	44	15	266	17601
31	44	14	266	18137
32	44	13	266	18689
33	44	12	266	19258
34	44	11	266	19845
35	44	10	266	20449
36	44	9	266	21072
37	44	8	266	21713
38	44	7	266	22375
39	44	6	266	23056
40	44	5	266	23758
41	44	4	266	24482
42	44	3	266	25227
43	44	2	266	25996
44	44	1	266	26787
45	45	1	274	27583
46	46	1	282	28369
47	47	1	290	29146
48	48	1	298	29914
49	49	1	306	30670
50	50	1	314	31416

TOTAL PRESENT VALUE PER HECTARE = 16826.96

OPTIMAL HARVEST PATH IN FIVE YEAR PERIODS

PER	HARV	PER	HARV	PER	HARV	PER	HARV
1	2829	2	1330	3	1330	4	1330
5	1330	6	1330	7	1330	8	1330
9	1330	10	0	11	0	12	0
13	0	14	0	15	0	16	0

GRAPHICAL PRINTOUT OF THE OPTIMAL TOTAL HARVEST PATH
each period represents five years

MAXIMUM HARVEST = 2829 . 100 PERCENT DENOTES HARVEST = 3000

```
1++10+++20+++30+++40+++50+++60+++70+++80+++90++100 PERCENT
*******************************************
**********************
**********************
**********************
**********************
**********************
**********************
**********************
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```

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