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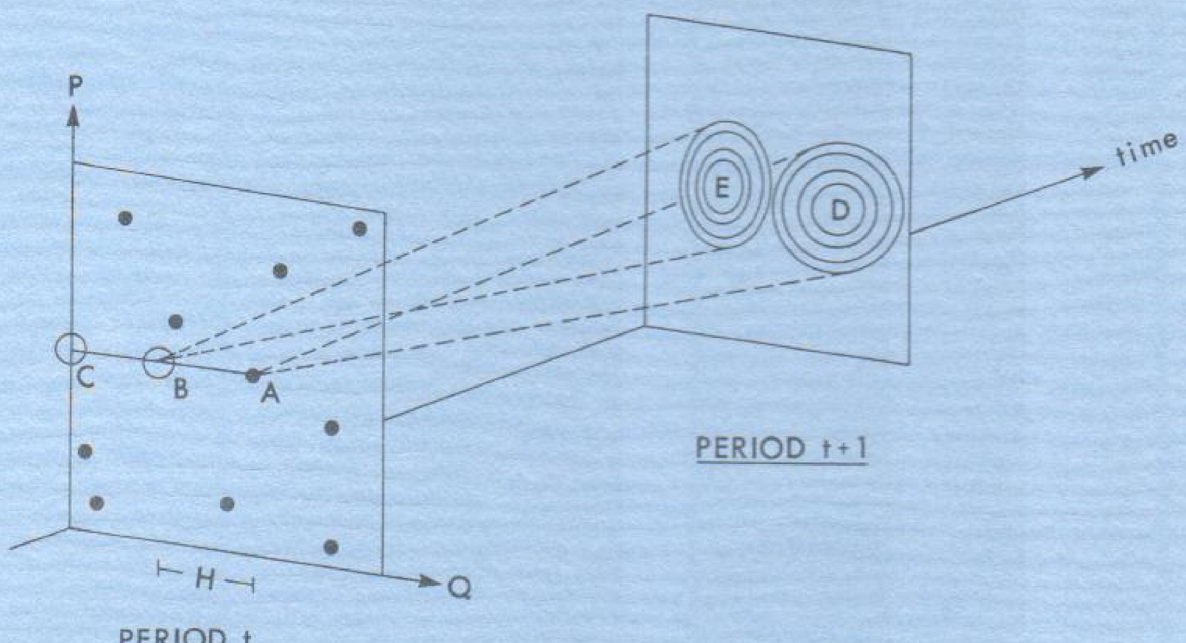
- a tool for adaptive economic forest management

by

Peter Lohmander



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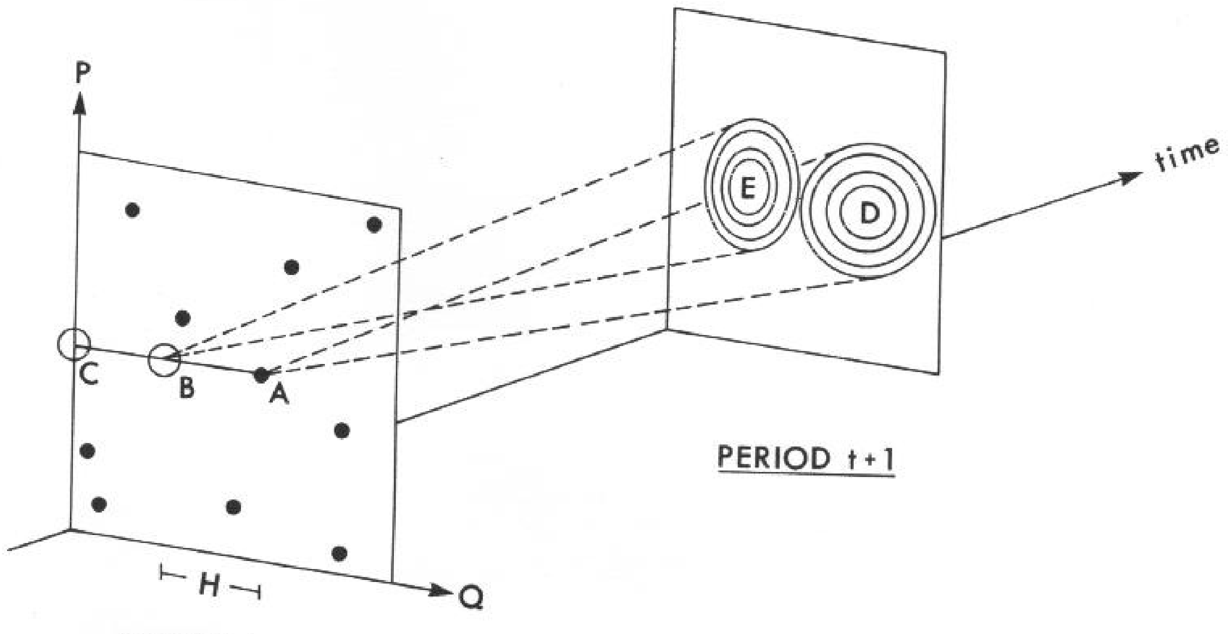

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– a tool for adaptive economic forest management

Abstract

A numerical method is suggested that can solve typical adaptive problems in the intertemporal management of forests and other natural resources. Time is treated as a discrete stage variable and the state space is continuous and multidimensional. Continuous approximations are made of the objective function via exactly determined multidimensional polynomials derived from different sets of objective function observations. A two dimensional third order example is discussed in detail. The initial set of observation sample coordinates is selected in a way that gives a high degree of state space representation and makes the coefficient matrix of the approximation equation system nonsingular. For each observation set, the coordinates of the individual observations are systematically changed in a way that keeps also the following equation system matrixes nonsingular. The standard deviations of the optimal decisions and the objective function values are determined and used to investigate the reliability of the derived solutions and the relevance and stability of the multidimensional polynomial objective function approximations.

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1. The need for adaptive decisions and optimization in an unpredictable world

This paper presents an adaptive optimization method which easily can be applied to typical economic harvest optimization problems in resource management. The method is discussed in connection to general problems of applied stochastic dynamic programming and approximation. A dynamic harvest problem with stochastic price and growth processes is analysed as an illustration. A computer program based on the suggested method is included in the appendix.

"Adaptive economic forest management" is not only included in the title of this paper but is also the name of a research project, ADECOFORM, at the Swedish University of Agricultural Sciences. This paper contains ideas and methodological foundations that find use within the ADECOFORM project and hopefully can be helpful also in other contexts.

The most important differences between adaptive optimization and other forms of optimization are the following: It is explicitly accepted that there are conditions in the environment that can not be perfectly predicted. (Environment here includes everything that is exogenous to the system which should be optimized). Furthermore, it is explicitly accepted that decisions can take place over time and that later decisions should be based on the best and latest information concerning the exogenous conditions. At this point the reader may wonder why not everyone makes use of adaptive optimization. Nevertheless, even though the ideas are simple and obvious, the main part of forest modelling is today based on methods that are not taking the principles of adaptive decision making and optimization into account. Of course, there may be several reasons for this. Some possible reasons will be listed below;

– The tradition of long term planning in forestry is based on the assumption that long term predictions of high precision are possible. Clearly, some phenomena, such as the growth of forests, have in the past been possible to predict with high precision (compared to other phenomena) over long periods of time. However, it is not obvious that long term forest planning is an objective in itself. Since the forest wood resource is one input among others in the production of sawn wood, pulp and paper, particle board etc., it is obvious that the economically optimal decisions in the forests are affected by the physical and economical conditions in the forest industry, the product markets and the labour market. These conditions outside the forest have never been possible to predict very well over long horizons. Some observations of this are:

– The labour productivity has increased very much during the latest decades, at least in swedish forestry. Hence, one can not argue that a constant long term harvest level will keep the level of employment on a constant level.

– The roundwood prices have varied very much over time. Hence, one can not argue that a long term constant harvest level is the most profitable or even the economically most "constant" or "stable" solution.

– The oil prices show considerable variation over time. Hence, since wood can also be used as a source of energy, the optimal use of the forest resource is most likely a function of the oil price level. Furthermore, since the forest industry is an energy consumer, the optimal solution to the resource allocation problem is far from trivial.

– The changes in the environmental conditions during the latest years have gained much attention and large projects are investigating the effects on the forests. Clearly, not even the forest growth, which earlier was assumed to be perfectly predictable, can motivate the use of nonadaptive optimization models.

Adaptive optimization gives us the opportunity to deal with a relevant stochastic multi stage description of the real world decision problems. The approach is more difficult than traditional deterministic optimization. The method presented in this paper hopefully simplifies the transition from traditional planning methods to the adaptive approach.

The adaptive optimizer will constantly follow the development of the exogenous state (the price level, the state of the natural resource, the environment etc.) and use the latest information in the decision process. This does not mean that the adaptive approach gives the same optimal solution as a traditional long term plan which is updated when new information appears. The adaptive approach also takes future options to adjust the decisions into account; The future level of flexibility is explicitly taken into consideration, since a flexible future situation increases the expected value function, the objective of optimization.

Let us turn away from the irrelevant deterministic modelling of yesterday and apply the adaptive methods of the future !

2. The optimization problem and numerical issues

The economic stochastic dynamic harvest optimization problem is presented in (1).

$$\max_{H_t} \phi_t(H_t;\ Q_t, P_t) \tag{1}$$

$$\text{s.t. } H_t \in \bar{H}_t$$

The objective function $\phi_t(.)$ is determined via backward recursion. Let T denote the final period, the "horizon", in the problem. ϕ_t, in an arbitrary period such that $0 \leq t < T$, can be expressed as:

$$\phi_t = \pi_t(H_t;Q_t,P_t) + W_{t+1}(H_t,Q_t,P_t) \tag{2}$$

π_t denotes the present value of the profit from harvest H_t in period t and W_{t+1} is the expected present value (in the sence of Bellman (1)) of the profits in future periods conditional on optimal future decisions. W_{t+1} is a function of the harvest level and the state before harvest in period t. The most general functional form of this relation is expressed in the equation. W_{t+1} is expressed in more detail in (4).
ϕ_T may be defined as:

$$\phi_T = \pi_T(H_T;Q_T,P_T) \tag{3}$$

In every period t such that $0 \leq t < T$, we have:

$$W_{t+1}(H_t,Q_t,P_t) = \int\int \phi^*_{t+1}(H^*_{t+1};Q_{t+1},P_{t+1})\, f(Q_{t+1},P_{t+1} | H_t,Q_t,P_t)\, dQ_{t+1} dP_{t+1} \tag{4}$$

ϕ^* and H^* denote optimal objective function value and optimal harvest decision respectively. $f(.|.)$ is the two dimensional probability density function of the initial state in period t+1 conditional on the initial state and decision in period t. $f(.|.)$ is derived from the stochastic price and growth processes, some of which may be controlled, relevant to the specific application. An example will be discussed as an illustration in this paper. Figure 1 shows an example of a discrete time continuous state stochastic dynamic optimization problem.

The state space contains the two dimensions P and Q which are treated as continuous. We let Q denote the size of the natural resource and P is the exogenous price. In period t, the state can be any combination of P and Q. Let us assume that the state in period t is A. Then, if no control, harvest, is undertaken in period t, the state will take some position close to D in period t+1. The circles surrounding D denote the iso probability density graphs of the transition function.

If we select to control the state in period t, a harvest, H, is undertaken that moves the state to position B. If no restrictions are present, the harvest may move the system to any position along the line between A and C. Of course, if a harvest is undertaken, this also affects the probability distribution of the state in period t+1. In the graph, the state will take some position close to E in the next period. Note in particular that the shape of the probability density function surrounding E is different from the shape of the function surrounding D. In this kind of application to resource modelling, this is a typical result if the stochastic deviations of the growth process are stock size dependent.

In order to optimize the control at time t of the system, the level of H, the ideal method would be the following if no calculations were time and cost consuming:

For each possible initial state in period t and level of the control H, we calculate the sum of the present value of the instant economic profit at time t as a function of H and the expected value of all future profits conditional on the control level H. The optimal objective value function in period t is determined this way.

The expected values of all future profits are derived via the transition probability distributions, one distribution for each control level H. The optimal objective value function in period t+1 is known when the decision is optimized in period t. The decisions and objective function in every period in the problem are functions of the objective functions in the following periods.

Some practical problems that instantly appear if not very restrictive assumptions are placed on the problem are the following:

a. We can not investigate every possible combination of the state variables P and Q if these are treated as continuous variables since the number is infinite.

b. We can not investigate every possible level of the control, H, for each combination of P and Q, even if the number of P and Q observations is finite. The number of calculations would be infinite anyway.

One simple way to overcome the problems a. and b. is to introduce a discrete and bounded state space, a discrete and bounded control decision and a discrete transition probability matrix. This has also been done by for instance Lohmander (8). Some of the more severe problems with this approach are the following:

c. The number of calculations increases very rapidly when the number of possible state levels for each dimension increases and in particular when the number of state space dimensions increases.

d. The size of the required internal computer memory increases very rapidly with the number of possible state levels and dimensions.

e. Problems often appear "close to the bounds" of the state space because transition probability functions based on discretized versions of common continuous probability density functions (such as the normal distribution) usually give transitions to positions outside the bounds with positive probability. This makes it necessary to modify the assumptions concerning the stochastic behaviour of the system close to the "artificial" bounds. Hence, the solution of the optimization may be of little relevance to the real world problem close to the state space bounds. Furthermore, when the number of time periods is high and the investigated processes are nonstationary, the "bound effects" discussed may severely affect the derived optimal solution also in parts of the state space that are far away from the bounds. Nevertheless, when the processes are in fact stationary and the number of dimensions is small, models with discrete state space may work well.

In this paper, a compromize will be made, that may give desireable properties to the optimization model in particular contexts to be discussed.

The basic approach used in the model is the following:

f. The objective value function, the expected present value of instant and future profits conditional on optimal controls, is approximated by a multidimensional polynomial of relatively low order in every time period. Hence, many of the problems associated with a discrete state space can be avoided. Furthermore, the number of observations needed in order to determine the approximating polynomial increases rather slowly with the number of state space dimensions.

g. The transition probability density function is approximated by a multidimensional discrete probability density function.

h. The control, harvest, level is optimized via a selection of discrete alternatives. This makes it possible to use nonconcave profit functions. However, if the properties of the profit function are known, and it is known that the optimal control is unique, then the approach can easily be modified. The control level could for instance be optimized via the Newton–Raphson method applied to an estimate of the derivative of the objective function with respect to the control. Linear or nonlinear programing could also be used in subroutines associated with each control level. Compare Lohmander (11)

3. Determination of the multidimensional approximating polynomial

The multidimensional polynomial approximation of the objective function can of course be made in different ways. One well known approach is to use the ordinary least squares method. Then the sum of the squares of the deviations from the estimated polynomial is minimized. If specific assumptions concerning the distribution of the residuals etc. are met, it is well known that detailed and sometimes useful statistical information can be obtained concerning the distribution of the errors associated with the coefficients and the predictions of the estimated polynomial. The standard assumptions in regression analysis include normally distributed residuals without heteroscedasticity. In the approximation problems under discussion in this paper, such assumptions do generally not hold.

Within the optimization program, a function which is approximated by the polynomial typically has the following properties:

a. The function is generally not several times continuously differentiable. Frequently the function is several times differentiable in some regions and kinked or linear in other regions. An observation made via discrete state space models is that these problems are worse in the time periods close to the time horizon, the final period of consideration in the optimization. The objective function has been observed to become "more smooth" as the time index decreases. This is of course an effect of positive probabilities of transitions to different states in later periods.

b. The observations made of the function to be approximated do not have measurement errors that can be assumed to be normally distributed. Within the algorithm, the function values are determined exactly. Maybe there are some errors because of numerical problems in the derivations. These can however not be assumed to be normally distributed in the general case.

Figure 7. illustrates how a simplified one dimensional version of the objective function (a concave curve) may be approximated by "polynomials", linear functions, of too low order. Clearly, one should try at least a second order polynomial in the illustration in order to capture the shape of the true function, the curve. However, in real world applications, the true shape of the function is not known. Maybe the true function is a polynomial of order 4 in some regions and linear in other regions. Then, the order of the approximating polynomial must be determined without this information.

Let us turn to Figure 8. Assume that observations (calculations) are made of the objective function (Y) in the points X_1, X_2, X_3 and X_4. The Y calculations will give the results Y_1, Y_2, Y_3 and Y_4 respectively. Ordinary least squares, OLS, minimization of the sum of squares of the residuals via a linear function gives function C. An other approach is to determine the linear equation that satisfies the observations in two points, X_1 and X_3. Then we obtain the function A. If we select the observations X_2 and X_4, we end up with equation B. The graph shows that all three equations A, B and C approximate the true function. All suggested linear functions give the correct solution for two different X values. We know the X values that make the functions A and B identical to the true function values but we do not know the X values that make the function C correct. Hence, we know in what region we can expect the functions A and B to be good approximations. This is a strong advantage of an exactly determined polynomial compared to a polynomial determined via a least squares method.

Furthermore, the traditional assumption in regression analysis of identically distributed residuals is clearly not satisfied. This is partly a consequence of the fact that the order of the approximating polynomial is not the same as the order of the true function. Hence, the "absolute prediction errors" are zero for two different X values for each approximating function. The "absolute prediction errors" are (increasing) nonlinear functions of the distances to these X values. Hence, we can not assume that the standard deviation of the estimate, which is standard information from most regression analysis software, can be used to determine the expected difference between the approximating function C and the true function. The residuals are highly dependent on the X values in a way which is generally nonlinear and unknown to the investigator.

One dangerous way to use different approximations of the function is the following: If the two approximating equations A and B give the same function value for a particular X value, one may get the impression that the error of the approximation is zero for that X value. This is however completely wrong, which is shown in Figure 7. When the X value is X_5, both A and B give the function value Y_6, which is a strong underestimation of the true function value Y_5.

This section can be concluded the following way: When the sum of residual squares is minimized, we do not necessarily obtain more useful polynomial approximations than when the polynomials are exactly determined. Furthermore, exact polynomial determination has the advantage over least squares solutions that the X values where the polynomial is identical to the true function are exactly known. Finally, exact determination of a polynomial is computationally less effort consuming than regression analysis.

4. The multi dimensional pattern of objective function observation coordinates

Let us initially discuss a low order problem. Assume that we want to determine the following two dimensional polynomial exactly:

$$Z(X, Y) = c_1 + c_2X + c_3Y + c_4XY \tag{5}$$

A possible pattern of objective function observation coordinates is suggested in Figure 10. (The dimensions X and Y have been given the application specific names Q and P in Figure 10.)

The function value will be observed (calculated) in 4 positions, namely:

(X_1, Y_1), (X_1, Y_2), (X_2, Y_1) and (X_2, Y_2). The corresponding function values are denoted by Z_1, Z_2, Z_3 and Z_4.

The linear equation system which determines the polynomial becomes Dc = Z, or more explicitly:

$$\begin{bmatrix} 1 & X_1 & Y_1 & X_1Y_1 \\ 1 & X_1 & Y_2 & X_1Y_2 \\ 1 & X_2 & Y_1 & X_2Y_1 \\ 1 & X_2 & Y_2 & X_2Y_2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \\ Z_4 \end{bmatrix} \tag{6}$$

We have to show that the matrix D in the equation system is nonsingular. Compare Chiang (2) page 110. We will investigate if |D| is different from zero. If this is the case, then the solution of the system exists and will be $\bar{c} = D^{-1}Z$.

Laplace expansion of |D| along column 1 gives:

$$|D| = +(1)\begin{vmatrix} X_1 & Y_2 & X_1Y_2 \\ X_2 & Y_1 & X_2Y_1 \\ X_2 & Y_2 & X_2Y_2 \end{vmatrix} - (1)\begin{vmatrix} X_1 & Y_1 & X_1Y_1 \\ X_2 & Y_1 & X_2Y_1 \\ X_2 & Y_2 & X_2Y_2 \end{vmatrix} + \tag{7}$$

$$+(1)\begin{vmatrix} X_1 & Y_1 & X_1Y_1 \\ X_1 & Y_2 & X_1Y_2 \\ X_2 & Y_2 & X_2Y_2 \end{vmatrix} - (1)\begin{vmatrix} X_1 & Y_1 & X_1Y_1 \\ X_1 & Y_2 & X_1Y_2 \\ X_2 & Y_1 & X_2Y_1 \end{vmatrix}$$

It can be shown that |D| can be expressed as:

$$\begin{aligned} |D| = \; & -4X_1X_2Y_1Y_2 + 2X_1X_2Y_2Y_2 + 2X_1X_2Y_1Y_1 \\ & + 2X_1X_1Y_1Y_2 - X_1X_1Y_2Y_2 - X_1X_1Y_1Y_1 \\ & + 2X_2X_2Y_1Y_2 - X_2X_2Y_2Y_2 - X_2X_2Y_1Y_1 \end{aligned} \tag{8}$$

If we rewrite the expression, we have:

$$|D| = -\left[X_1^2 - 2X_1X_2 + X_2^2 \right] * \left[Y_1^2 - 2Y_1Y_2 + Y_2^2 \right] \tag{9}$$

This can be simplified as:

$$|D| = -(X_1 - X_2)^2 (Y_1 - Y_2)^2 \qquad (10)$$

Hence, it is obvious that $|D| < 0$ for all X_1, X_2, Y_1 and Y_2 such that X_1 is different from X_2 and Y_1 is different from Y_2. The matrix D is nonsingular and an approximating exactly determined polynomial will be found.

Now, we will turn to the general two dimensional second order polynomial. In order to simplify notation, we still use Z as the function value.:

$$Z(X,Y) = c_1 + c_2X + c_3Y + c_4X^2 + c_5XY + c_6Y^2 \qquad (11)$$

, We will need six observations in order to determine the six coefficients exactly. Let us look at two very similar patterns of objective function observation coordinates. One pattern will result in a solution and the other will not. Pattern A is shown in figure 11 and pattern B is found in figure 12. Again, we note that X and Y are used in this general section but that Q replaces X and P replaces Y in the application oriented graphs. It will be assumed that X_1, X_2 and X_3 are different from each other and that Y_1, Y_2 and Y_3 also take different values.

Table 1.

Observation number	Coordinates pattern A	Coordinates pattern B
1	(X_1, Y_1)	(X_1, Y_1)
2	(X_2, Y_1)	(X_3, Y_1)
3	(X_3, Y_1)	(X_4, Y_1)
4	(X_1, Y_2)	(X_2, Y_2)
5	(X_2, Y_2)	(X_1, Y_3)
6	(X_3, Y_2)	(X_3, Y_3)

The determinants of the coefficient matrixes according to the patterns A and B become $|D_A|$ and $|D_B|$ respectively:

$$|D_A| = \begin{vmatrix} 1 & X_1 & Y_1 & X_1X_1 & X_1Y_1 & Y_1Y_1 \\ 1 & X_2 & Y_1 & X_2X_2 & X_2Y_1 & Y_1Y_1 \\ 1 & X_3 & Y_1 & X_3X_3 & X_3Y_1 & Y_1Y_1 \\ 1 & X_1 & Y_2 & X_1X_1 & X_1Y_2 & Y_2Y_2 \\ 1 & X_2 & Y_2 & X_2X_2 & X_2Y_2 & Y_2Y_2 \\ 1 & X_3 & Y_2 & X_3X_3 & X_3Y_2 & Y_2Y_2 \end{vmatrix} \qquad (12)$$

$$|D_B'| = \begin{vmatrix} 1 & X_1 & Y_1 & X_1X_1 & X_1Y_1 & Y_1Y_1 \\ 1 & X_3 & Y_1 & X_3X_3 & X_3Y_1 & Y_1Y_1 \\ 1 & X_4 & Y_1 & X_4X_4 & X_4Y_1 & Y_1Y_1 \\ 1 & X_2 & Y_2 & X_2X_2 & X_2Y_2 & Y_2Y_2 \\ 1 & X_1 & Y_3 & X_1X_1 & X_1Y_3 & Y_3Y_3 \\ 1 & X_3 & Y_3 & X_3X_3 & X_3Y_3 & Y_3Y_3 \end{vmatrix} \qquad (13)$$

Calculations show that $|D_B|$ is different from zero. D_B^{-1} exists and can be derived analytically. Hence, the function value observation coordinates can be chosen according to pattern B when we want to determine the general two dimensional second order polynomial exactly. Pattern A, on the other hand, should not be used. $|D_A|$ is equal to zero and D_A^{-1} can not be found. Thus, even if the two patterns A and B look very similar, pattern B must be selected because of these reasons. The general lesson seems to be that we should avoid simple geometric patterns. Investigations of other simple geometric patterns in two and three dimensions have resulted in the same conclusion.

The general two dimensional third order polynomial, which is used in the included numerical optimization model, contains 10 coefficients. Hence, 10 function value observation coordinates are needed. These 10 positions are plotted in Figure 2. Note that the positions do not reveal a simple geometric pattern. Furthermore, the ambition has been to select points that will give a high degree of state space representation within the region selected in Figure 2. In Figure 3 a possible way to systematically move the sample is shown. This method is used in the computer program and makes sure that the pattern gives solutions to all estimated polynomials.

The 10 selected coordinates in the initial sample are:

Table 2.

Observation	X_1	X_2
1	1	1
2	90	5
3	7	90
4	99	95
5	32	25
6	88	44
7	55	1
8	6	52
9	50	50
10	35	75

Maybe one should in the future optimize also the function value observation coordinates ?

5. A numerical algorithm

A numerical algorithm which solves the stochastic dynamic programming problem of this paper has been constructed and included in the numerical appendix. The computer code contains remarks that sequentially explain the numerical method. In the presented version, the model can handle a two dimensional continuous state space. The value function approximation is based on general two dimensional third order polynomials that are exactly determined through 10 observations. Clearly, through minor program modifications, problems of higher dimensionality could be solved as long as the approximation approach is relevant to the specific problem. In applications where the true shape of the objective function can be expected to deviate much from a polynomial, other approximation methods should of course be used.

The objective function is approximated by:

$$Z = c_1 + c_2X + c_3Y + c_4X^2 + c_5XY + c_6Y^2 + c_7X^3 + c_8X^2Y + c_9XY^2 + c_{10}Y^3 \tag{14}$$

6. Application to a natural resource control problem

6.1. Definition of the problem

The typical natural resource control problem that will be discussed as an application contains two stochastic processes. The computer code included in the appendix is constructed to deal with the process assumptions presented below. However, it could easily be modified to treat Markov processes with other functional forms. The price process, P, is a Markov process, exogenous to the decision maker. More specifically, it is assumed to be a first order autoregressive process of the form:

$$P_{t+1} = \alpha + \beta P_t + \epsilon_t^P \tag{15}$$

It is assumed that the stochastic component ϵ_t^P has zero autocorrelation and is normally distributed with mean 0 and standard deviation σ_P. This process assumption is based on empirical investigations and more details can be found in Lohmander (4) and Lohmander (6).

The volume process, Q, is a controlled Markov process of the following form:

$$Q_{t+1} = (Q_t - H_t) * (\gamma + \epsilon_t^Q) \tag{16}$$

H_t denotes harvest volume at time t and $(Q_t - H_t)$ is the size of the resource remaining after harvest in period t. $(\gamma - 1)$ is the expected relative growth. ϵ_t^Q is a stochastic component with zero autocorrelation. It is normally distributed with mean zero and standard deviation σ_Q. Note that the residuals of growth are heteroscedastic if the resource volume Q_t is treated as the independent variable. This is a reasonable assumption in many applications where growth is an increasing function of the initial volume.

In this application, we will assume that the present value of the profit is:

$$\pi_t = e^{-rt}(P_t H_t - \kappa H_t^3) \tag{17}$$

P_tH_t is the revenue and κH_t^3 is a nonlinear harvest cost function, where $\kappa > 0$. Note that the marginal harvest cost is an increasing function of the harvest level. This makes π_t strictly concave in H_t. A convex profit function may imply that "pulse harvesting" is the optimal intertemporal harvest pattern. Compare Clark(3), Lohmander(4), (6) and (8). The specific parameter values used in P, Q and π only serve as illustrations and are of little general interest. However, the reader may find them in the computer code included in the appendix.

In the illustration, we will consider a five period problem. The sensitivity of the optimal harvest decision in period t, H_t, will be investigated with respect to the price state, the volume state and the standard deviations of the stochastic components in the growth and price processes. The sensitivity of the optimal objective function value to the model parameters will also be studied.

6.2. The results of the resource example

All of the specific numerical assumptions made in the problem may be found in the computer program for adaptive optimization included in the numerical appendix. The assumptions of particular interest to the following discussion of results are on the other hand presented also in connection to the illustrating graphs.

The control of the system is the harvest level. The optimal harvest level and the optimal objective function value are dependent on the time period, the volume state and the price state. The computer program automatically presents a table of all of these relations. The graphs included in this section only represent a fraction of the totally available information. Figure 4 shows the optimal harvest level as a function of the price and the volume states in period 1 in a five period problem. The price process is assumed to be stochastic and the volume process is deterministic. It is found that the optimal harvest level is an increasing function of the price state and the volume state. These results seem reasonable from a general point of view. Since the price process is stationary, it is better to harvest during a good price period than to wait for even better prices since the price can be expected to approach the stationary equilibrium in the future.

There are two reasons why the optimal harvest volume should be an increasing function of the resource stock:

– The profit function π is increasing (in the interesting interval) and strictly concave in the harvest volume in every period. Hence it is "reasonable" (it can be shown that it is optimal in simple models of this kind), that more harvest quantity is distributed to each period as the stock increases.

– The profit function π in the future periods is strictly concave in the future harvest volumes. Hence, according to the Jensen inequality, the expected future profit is a decreasing function of the risk in the future volume (and more or less conditional harvest level) probability density function. The risk in the probability distribution function of the future volume is an increasing function of the volume after harvest in the initial period. Hence, one way to decrease the risk in the future (and hence to increase the expected present value of the enterprise) is to decrease the amount of the resource that is left for the future. Thus, we should increase the present harvest level as the initial volume increases.

Figure 5 shows how the expected present value is affected by the price and volume state in period 1 in a five period problem under the same conditions as those discussed in connection to figure 4. The objective function is an increasing function in both arguments. This should of course be expected since we get more and more profitable harvest options as the price and volume increase. Figure 6 illustrates how different assumptions concerning the stochastic components in the price and volume processes affect the state dependent optimal control, the optimal harvest as a function of price. The volume dimension is not shown in the graph since this would make the figure very difficult to interpret. It is found that the optimal harvest level is a decreasing function of the price variability and an increasing function of the volume variability. These results are dependent on several problem specific assumptions discussed in more detail in connection to the graph.

7. Discussion

The resource management application introduced in this paper is of course only one example of an infinite number of different applications. However, in this application, an adaptive management strategy and an objective function approximation are easily found. These have properties that seem reasonable in the specific application. Similar solutions are typical to similar adaptive optimization problems when other analytical or numerical methods are used. Compare the results reported in Lohmander (4), (5) and (8). Note, however, that a complete comparision of different methods generally is impossible. When a particular analytical or numerical method is used, the problem to be solved most often has to be modified or restricted in some sence in order to be solvable. For instance, when we use methods based on a discrete and bounded state space, the continuous functions of the application discussed in this paper must be restricted. Hence, we will never know if the solution obtained through the algorithm suggested in this paper is "better" than a solution from a restricted and discrete version of the problem. "Better" may here be defined with respect to similarity between the adaptive strategy derived through the algorithm and the adaptive strategy which really is optimal in the complete and in every sence unrestricted application. "Better" may also be defined with respect to similarity between the optimal objective function value and the optimal objective function value in the complete and in every sence unrestricted application. The author hopes, that the suggested method will be tested in a large number of resource management applications. Maybe it will be found that the method is able to capture the most essential properties of the functions in some of the applications. Then, the numerical algorithm is motivated. The complete and final numerical method will never be found.

8. References

(1) Bellman, R., Dynamic programming, Princeton University Press, Princeton, N.J., 1957

(2) Chiang, A.C., Fundamental methods of mathematical economics, McGraw–Hill, 2 ed., 1974

(3) Clark, C.W., Mathematical bioeconomics, Wiley, 1976

(4) Lohmander, P., The economics of forest management under risk, Swedish University of Agricultural Sciences, Dept. of Forest Economics, No. 79, 1987

(5) Lohmander, P., Continuous extraction under risk, International Institute for Applied Systems Analysis, System and Decision Sciences, WP–86–16, Mars 1986, and Systems Analysis – Modelling – Simulation, Vol. 5, issue 2, 1988

(6) Lohmander, P., Pulse extraction under risk and a numerical forestry application, International Institute for Applied Systems Analysis, System and Decision Sciences, WP–87–49, June 1987, and Systems Analysis – Modelling – Simulation, Vol. 5, issue 3, 1988

(7) Lohmander, P., Optimal resource control in continuous time without Hamiltonian functions, Systems Analysis – Modelling – Simulation, Vol. 6, issue 6, 1989

(8) Lohmander, P., A quantitative adaptive optimization model for resource harvesting in a stochastic environment, Systems Analysis – Modelling – Simulation, Vol. 7, issue 1, 1990

(9) Lohmander, P., editor, Economic planning of dynamic resource harvesting, Proceedings from the Economic Planning Group conference, Scandinavian Society of Forest Economics, Oslo, May 1988, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP–84, 1988

(10) Lohmander, P., editor, Scandinavian Forest Economics No. 31, Proceedings from the biennial meeting of the Scandinavian Society of Forest Economics, Visby, Sweden, 1989

(11) Lohmander, P., Stochastic dynamic programming with a linear programming subroutine: Application to adaptive planning and coordination in the forest industry enterprise, forthcoming in Systems Analysis - Modelling - Simulation

(12) Lohmander, P., Economic two stage multi species management in a stochastic environment: The case of selective thinning options and stochastic growth parameters, Swedish University of Agricultural Sciences, Dept. of Forest Economics, WP-112, 1990, forthcoming in Systems Analysis - Modelling - Simulation

Figure 1.

The stochastic dynamic optimization problem in period t. The state space is two dimensional. Q denotes quantity (volume) of the natural resource and P is price. Position A (one position of an infinite number of possibilities) happens to be the state before control, harvest, in period t. Q can be controlled via the harvest level in period t, H. Any level of H can be selected as long as the state after harvest, B, is found on the line between A and C. There are two different economic consequences of H: (a) the instant profit from H in period t and (b) the change in the expected present value of future profits. (a) is determined via a one dimensional profit function or via deterministic one period optimization and (b) is determined via the probability density function of the state in period t+1 conditional on H (D, E and an infinite number of similar probability density functions) and the state dependent optimal objective function in period t+1.

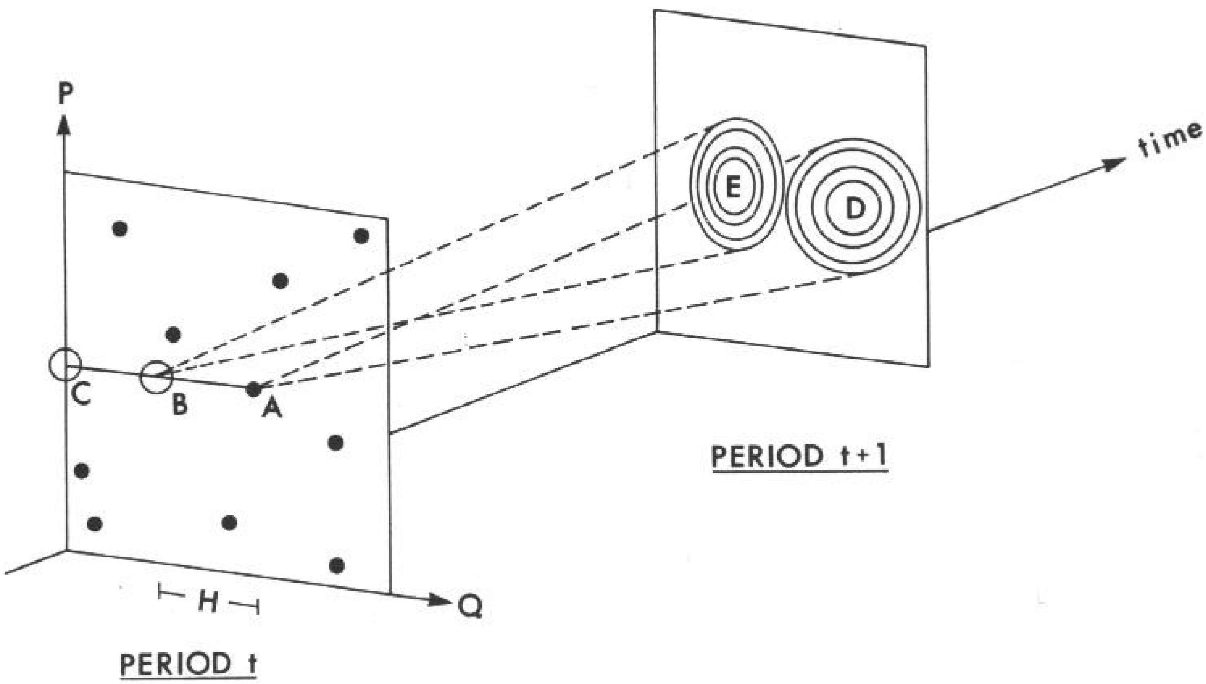


Figure 2.

The initial sample of objective function observation coordinates when a general two dimensional third order polynomial is determined. The illustrated sample coordinates have the following properties:

(a) Different parts of the bounded state space ($0 < Q < 100$; $0 < P < 100$) are represented by observations. We may initially assume that the bounds are relevant to the application and that position (Q, P) = (50, 50) is close to the position where the optimally controlled system has high probability density. If, on the other hand, it turns out that the states of the controlled system have low probability density close to (50, 50), other bounds than 0 and 100 may be used. Ideally, the search for bounds should be based on iteration within the complete optimization problem.

(b) The number of observations, 10, is needed in order to exactly determine the general polynomial with 10 coefficients via 10 equations.

(c) The coefficient matrix of the equation system used in the polynomial determination will be nonsingular and a solution can be found. Some sample coordinates in the combination with some functional forms may imply singular coefficient matrixes. Compare the Figures 10, 11 and 12.

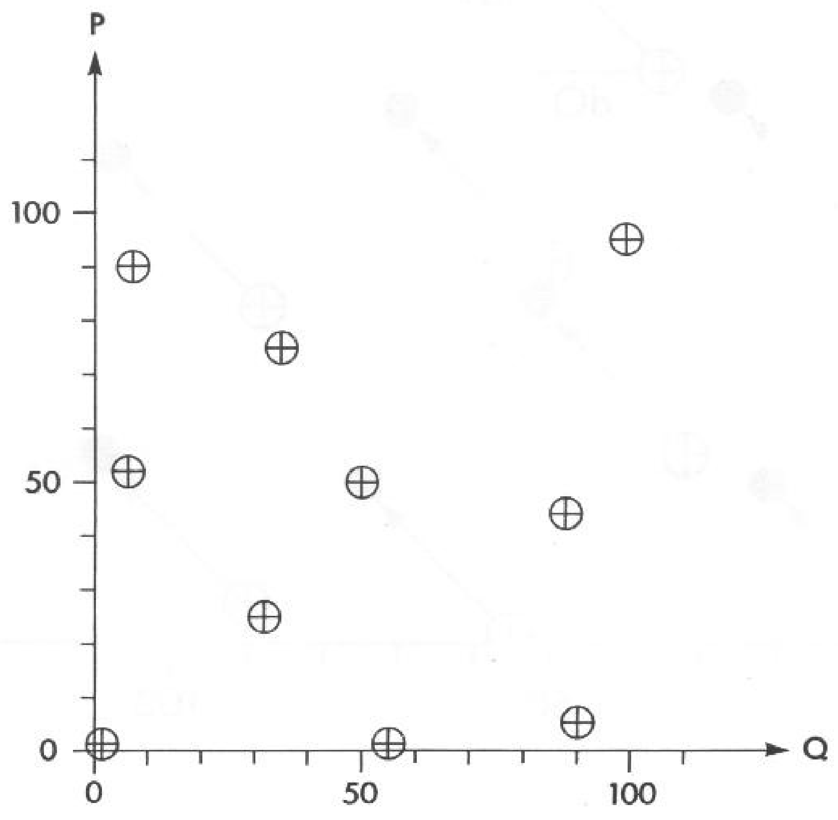


Figure 3.

The sample nr (N+1) of objective function observation coordinates is obtained through equal shifts (dQ, dP) of the coordinates of all observations in sample nr (N). This way, the important mathematical properties of the observation coordinates mentioned in connection to figure 2 are true in all observation coordinate samples.

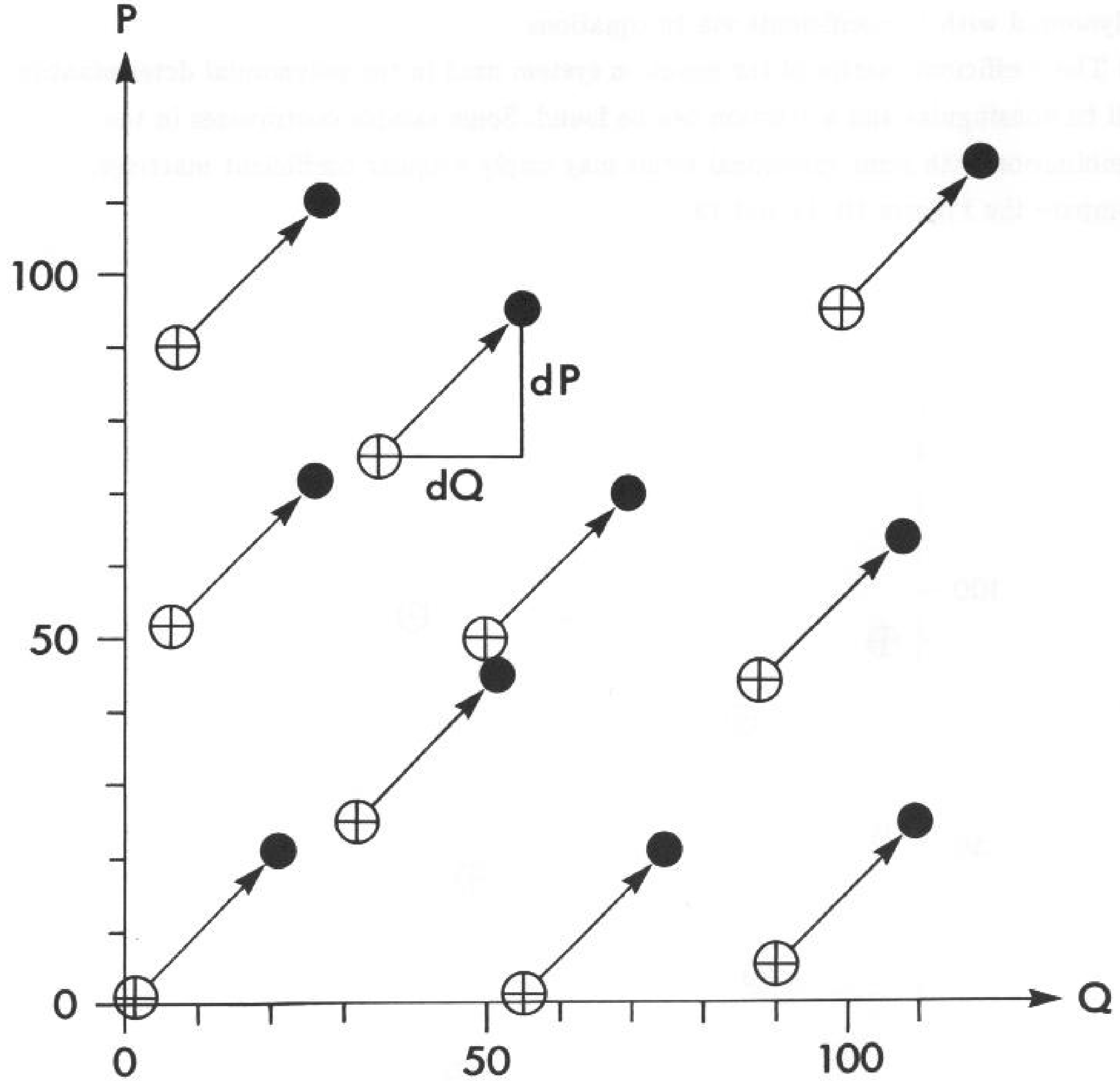


Figure 4.

The optimal harvest level, H^*, in period 1 as a function of price, P, and volume, Q. $\sigma_P = 30$, $\sigma_Q = 0$. H^* is an increasing function in both arguments.

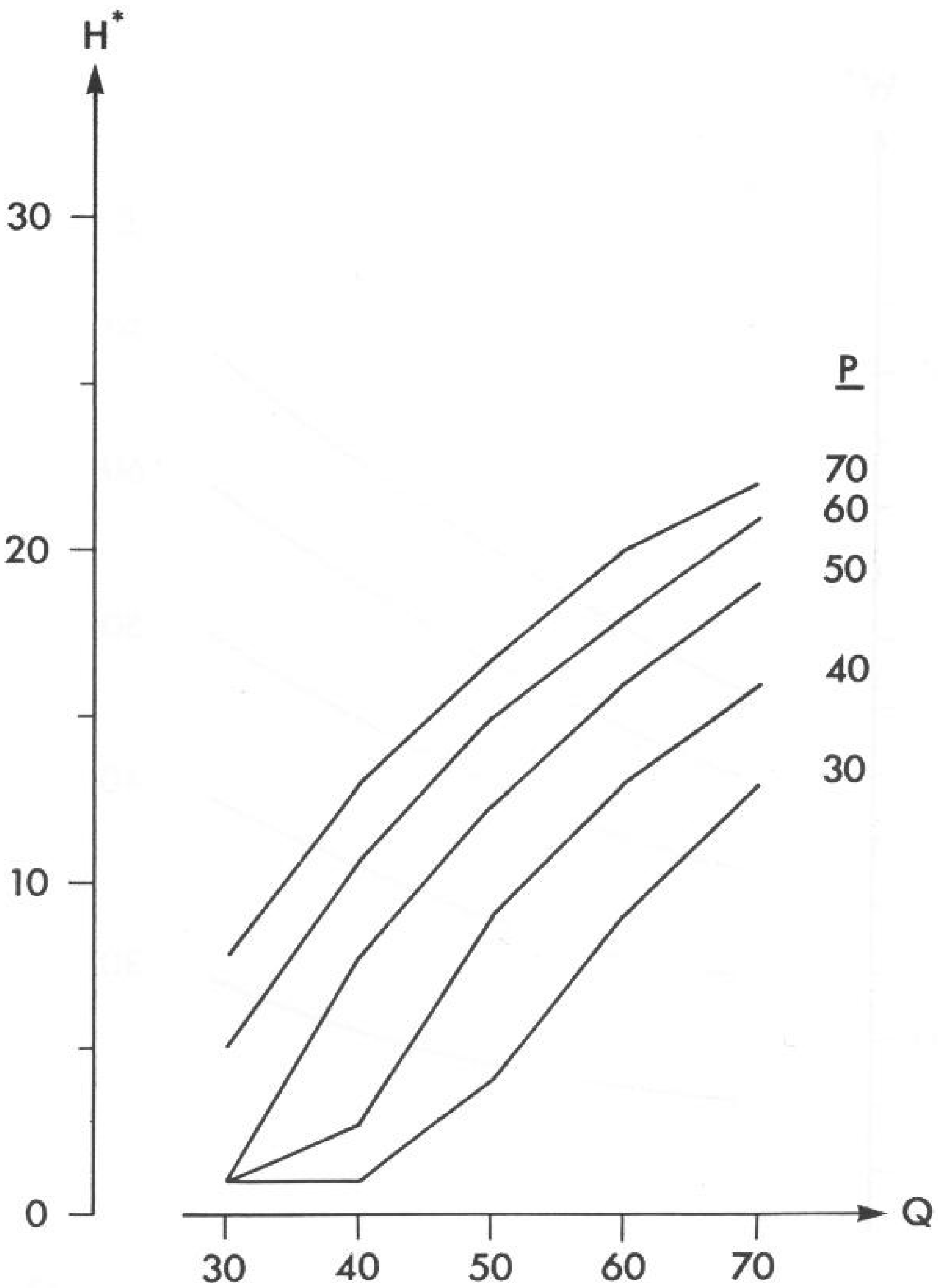


Figure 5.

The objective function value conditional on optimal decisions, W^*, in period 1 as a function of price, P, and volume, Q.

$\sigma_P = 30$, $\sigma_Q = 0$. W^* is an increasing function in both arguments.

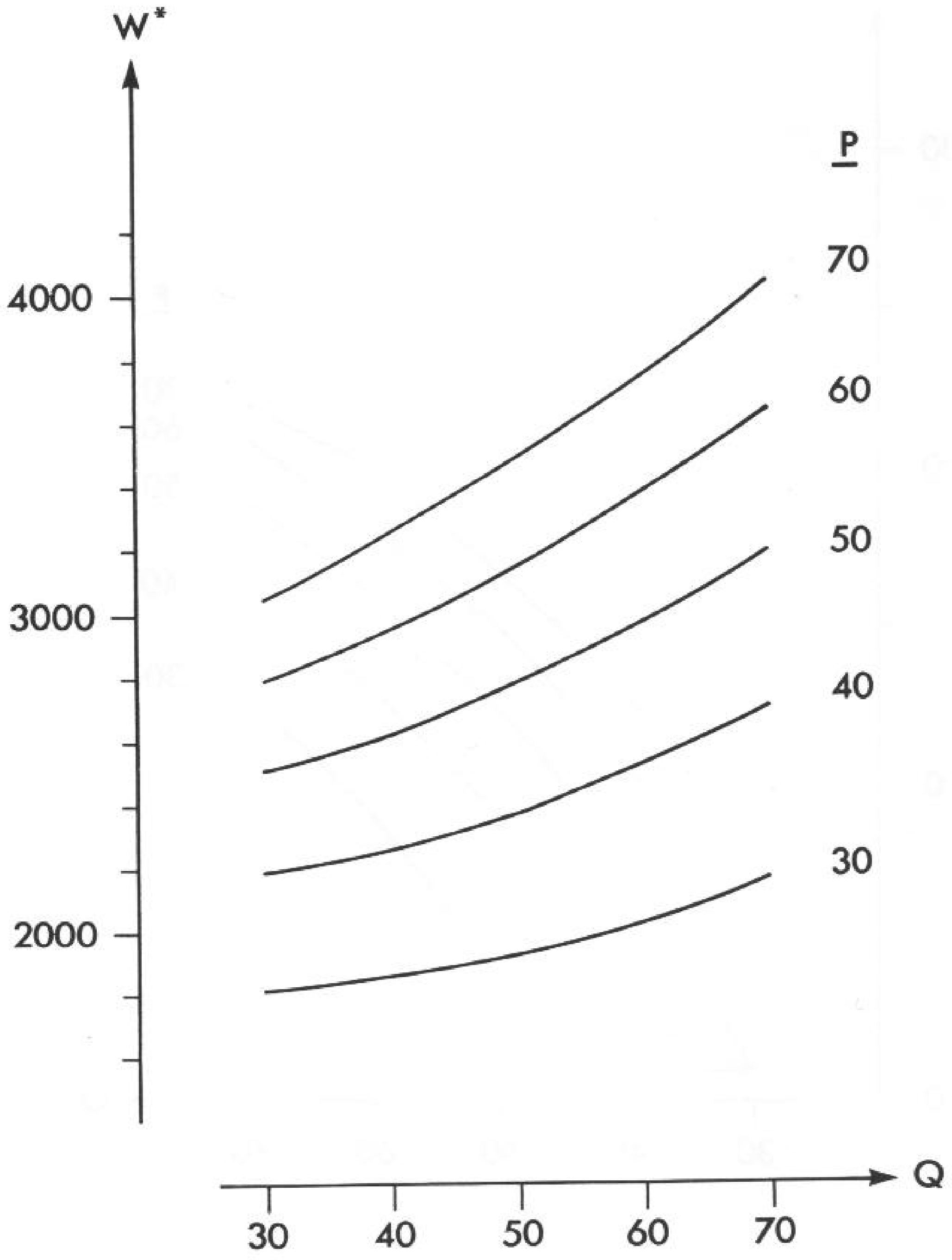


Figure 6.

The optimal harvest level, H^*, in period 1 as a function of price, P, and the standard deviations of the stochastic components of the stochastic price and growth processes, σ_P and σ_Q.

Case	σ_P	σ_Q
1	0	0
2	30	0
3	0	30

The optimal harvest level, H^*, is an increasing function of the price in all cases. H^* is a decreasing function of the price risk and an increasing function of the growth risk. The reasons for this are dependent on the specific model assumptions used. The signs and magnitudes of the the first, second and third order derivatives of the harvest profit function and of the growth function are of importance here. More details can be found in Lohmander (4), (5) and (8).

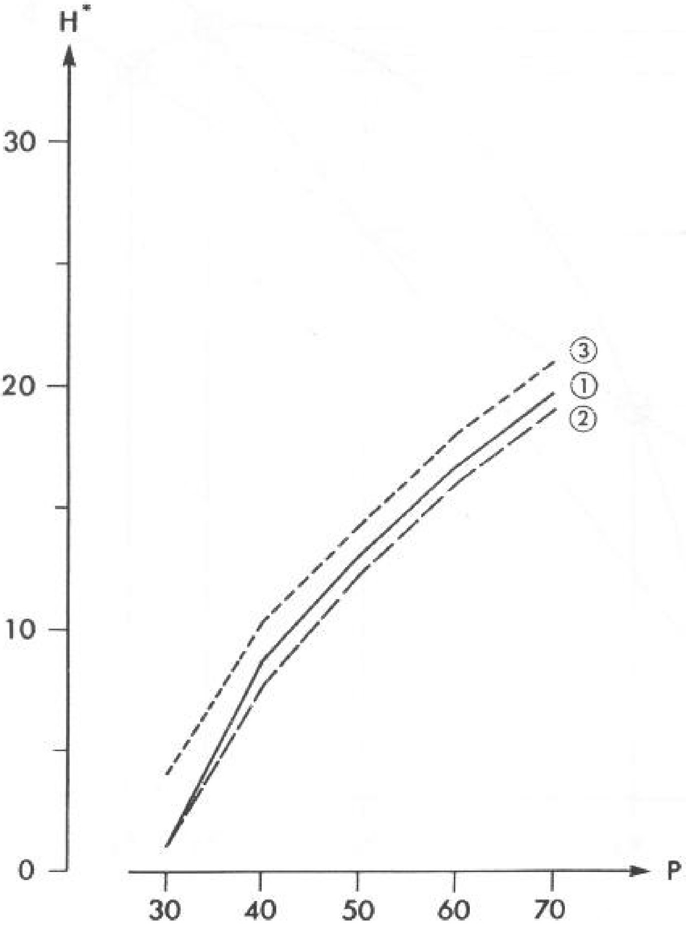


Figure 7.

An illustration of a problem which is present when a function, curve, is approximated by two polynomials of wrong (too low) order (linear functions). The observation coordinates X_1 and X_3 are used to determine approximation A. Approximation B is based on the observation coordinates X_2 and X_4. The investigator may believe that the selected degree (first degree) of the approximating polynomials is acceptable and that the approximations are very close to the true function value Y_5 for $X = X_5$ since both approximations give the same function value, Y_6. However, this is wrong. Y_5 is not equal to (or close to) Y_6. We only know that A gives the true function values in the coordinates X_1 and X_3 and that B gives the true values in X_2 and X_4. The discussion presented here is relevant also when high degree multidimensional polynomials are used as approximations. In real world problems, we seldom know the correct degree of the functions to be approximated.

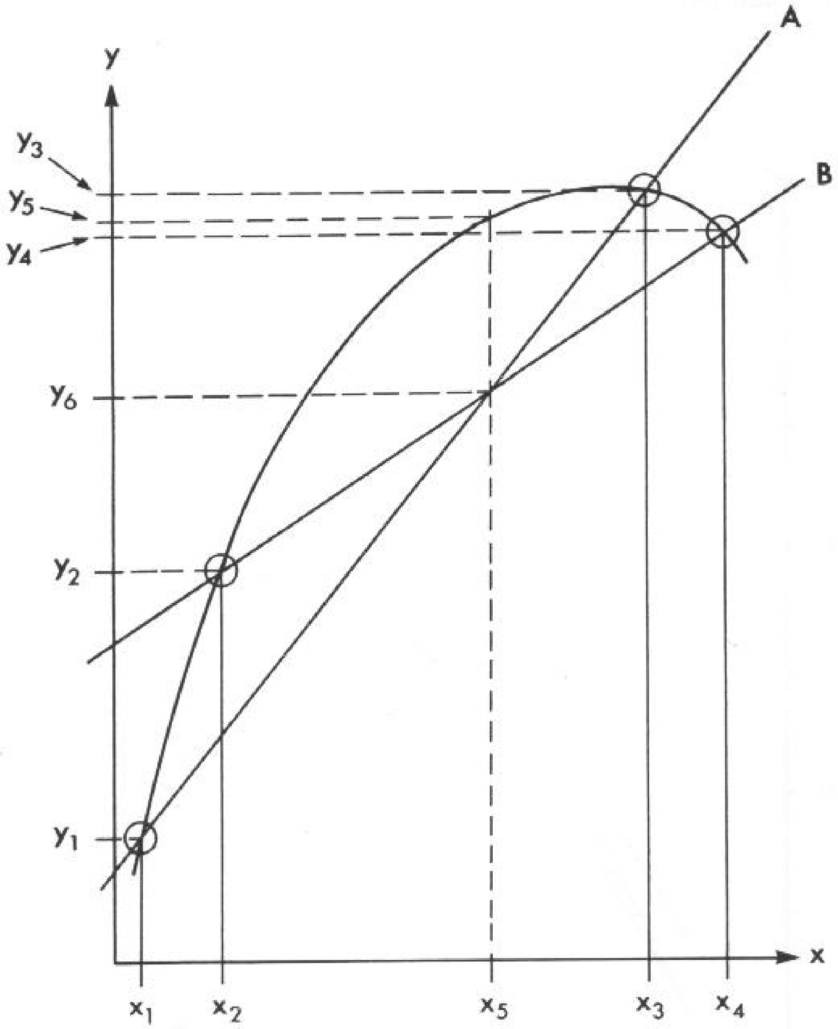


Figure 8.

The four observations X_1, X_2, X_3 and X_4 are used to produce two exactly determined approximations, A and B of the curve. The same observations are also used to produce approximation C via regression analysis (the ordinary least squares method). Here, some reasons are listed why C is not always better than A and B:

a. All suggested approximations imply deviations from the true function.

b. We know that A and B give exactly the true function values in two points each with known coordinates. We do not know if, or where, the regression equation C, gives exactly the true function values.

c. In the least squares method of regression analysis, the sum of squares of the residuals is minimized by the regression equation. Within the stochastic dynamic optimization problem, where the approximation is used, it is not always the case that the minimization of the squares of the residuals is the best way of approximation. Maybe one should minimize the sum of the absolute values of the residuals or the cubes of the absolute values of the residuals ? Maybe, it is more important to have approximating functions that represent the data in some specific part of the state space very well ? This could be solved via local approximations according to A and B or weighted regression analysis.

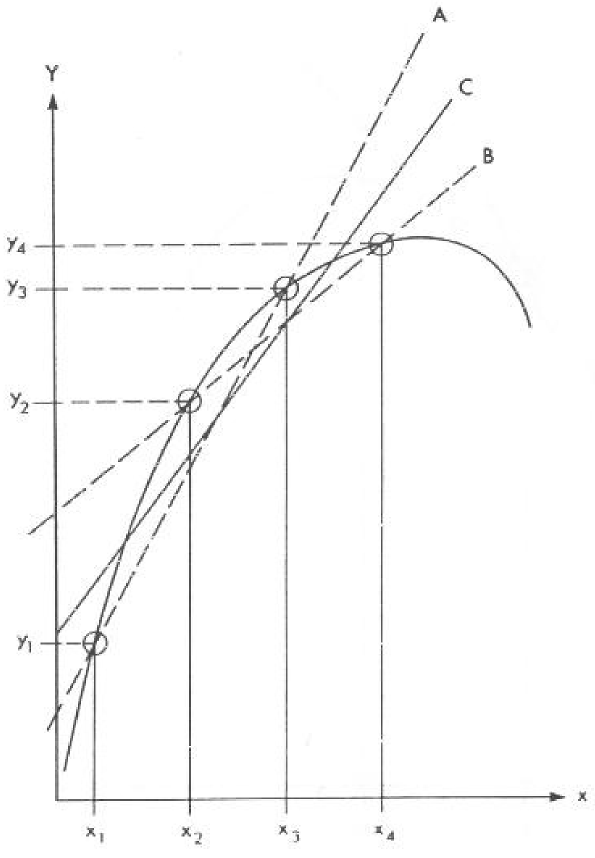


Figure 9.

It is important to be aware that an exactly determined approximation of a curve (linear approximations A and B in the illustration) is not a Taylor approximation. Hence, in the illustration, even if the true function is locally approximately linear, a linear approximation such as A or B will generally imply large deviations from the true function values also close to one of the coordinates where the approximation holds exactly. Compare this example: B holds exactly in X_4. Still, B is not a linear Taylor approximation of the curve (B is not a tangent to the curve in (X_4,Y_4).). Hence, for $X = X_5$, the curve and B give very different function values, namely Y_5 and Y_6. A linear Taylor function would have been a locally better approximation. The linear Taylor approximation would on the other hand have overestimated the function value everywhere except at X_4. Furthermore, since not all problem relevant functions are everywhere differentiable, the exactly determined polynomials represent a robust and important approximation method.

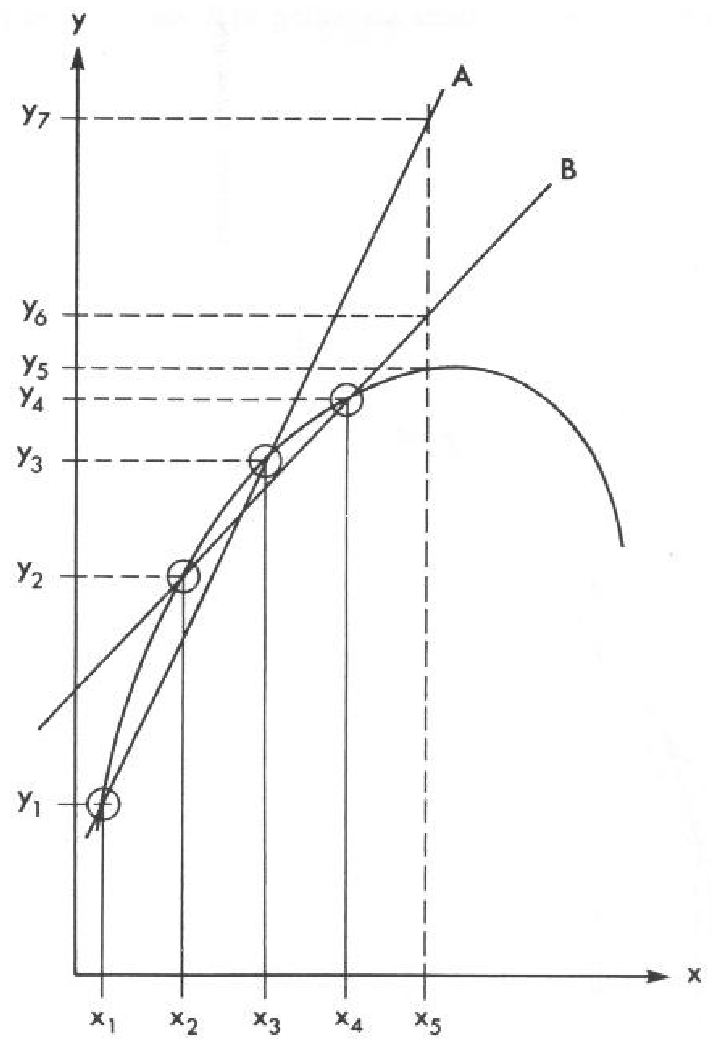


Figure 10.
The graph shows an observation coordinate pattern that may be used if the approximating two dimensional polynomial should be of the form:

$$Z(Q,P) = c_0 + c_Q Q + c_P P + c_{QP} QP$$

It can be shown that the coefficient matrix of the equation system used in the determination will be nonsingular and that a solution can be found as long as P_1 is different from P_2 and Q_1 is different from Q_2. Compare the main text.

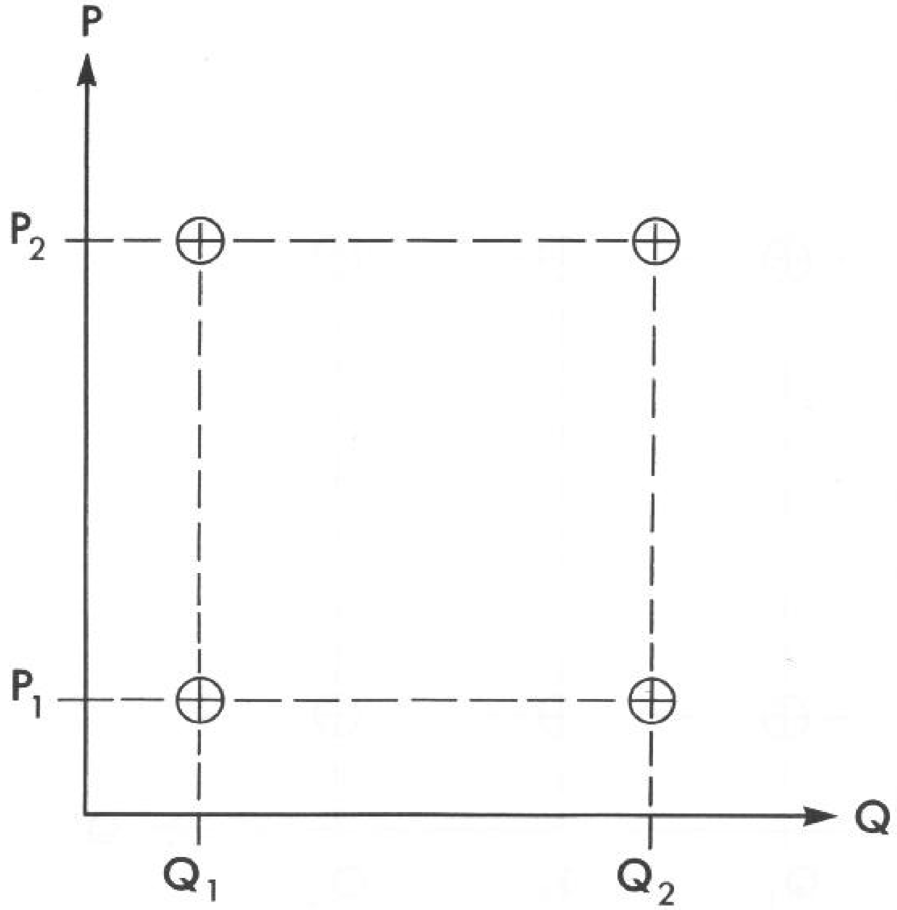


Figure 11.

The graph shows an observation coordinate pattern that may not be used if the approximating two dimensional polynomial should be of the form:

$$Z(Q,P) = c_0 + c_Q Q + c_P P + c_{QQ} QQ + c_{QP} QP + c_{PP} PP$$

It can be shown that the coefficient matrix of the equation system used in the determination will be singular and that the equation system will not give a unique solution. Compare figure 12.

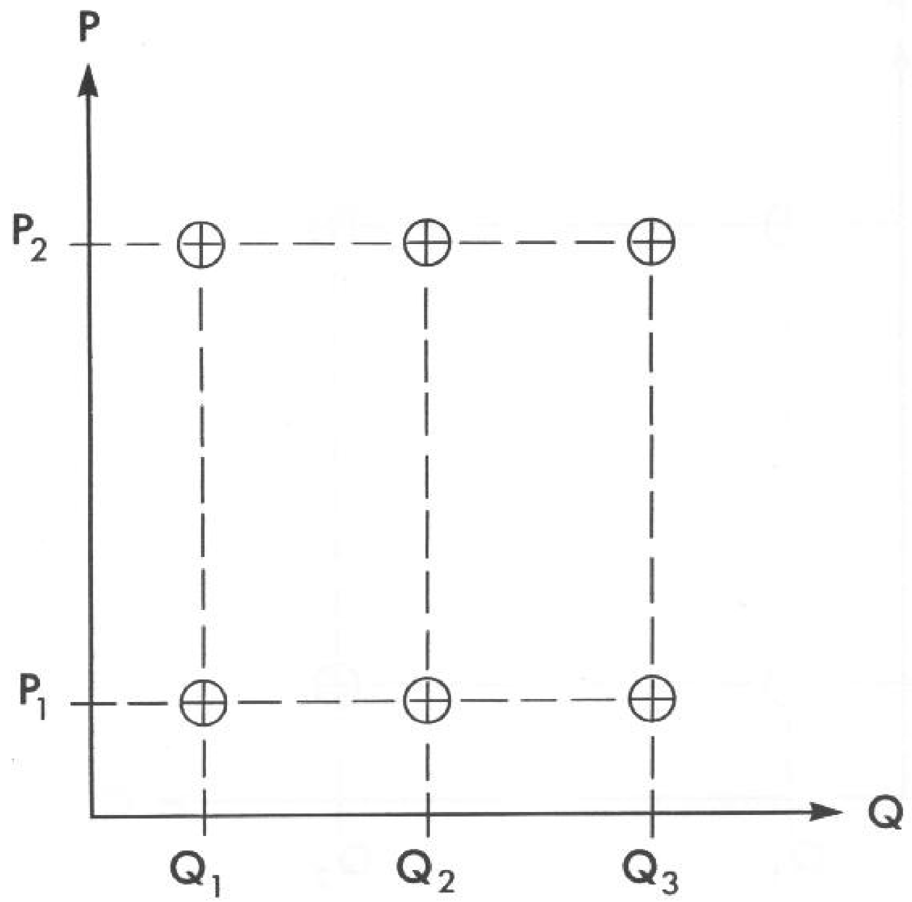
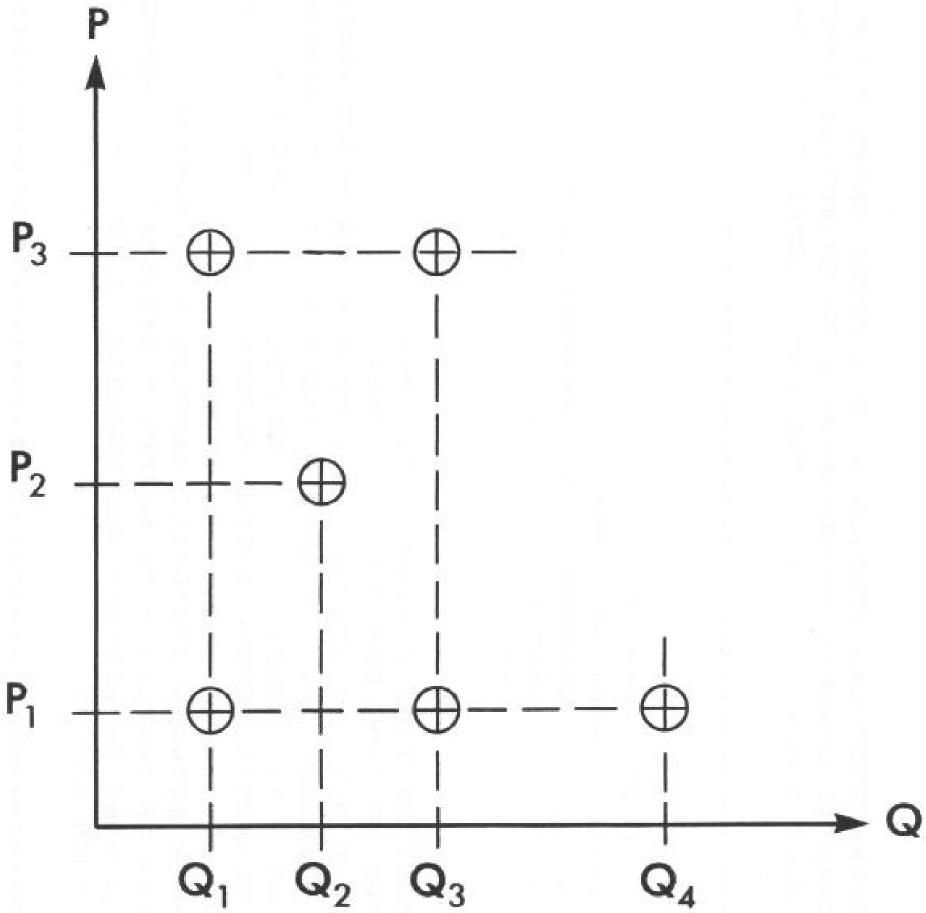


Figure 12.

This observation coordinate pattern may be used if the approximating two dimensional polynomial should be of the same form as presented in connection to figure 11.
It can be shown that the coefficient matrix of the equation system used in the determination will be nonsingular and that the equation system will give a unique solution.



Appendix

This appendix contains the computer code and an illustrative example, a part of which is included in the shape of the computer output list.

```
LPRINT "**********************************************************************"
LPRINT "PROGRAM PSTDP.BAS VERSION WITH 3 ORDER HARVEST COST"
LPRINT "STOCHASTIC DYNAMIC PROGRAMMING WITH OBJECTIVE FUNCTION APPROXIMATION"
LPRINT "VIA MULTIDIMENSIONAL (2 DIMENSIONS, ORDER 3) POLYNOMIALS"
LPRINT "LOHMANDER PETER 90-08-16"
LPRINT "**********************************************************************"
LPRINT " "
DEFDBL A-Z
DIM A(20, 21), C(20), F(9, 9)
DIM X1(50), X2(50), WOBS(50), WMAP(5, 5)
DIM WMAPTOT(5, 5, 5), WMAPMED(5, 5), WMAPDEV(5, 5)
DIM HMAT(5, 5, 5, 5), WMAT(5, 5, 5, 5)
DIM WMATMED(5, 5, 5), WMATDEV(5, 5, 5), HMATMED(5, 5, 5), HMATDEV(5, 5, 5)
IMAX = 10
JMAX = IMAX + 1
1 INPUT "NUMBER OF TIME PERIODS = "; TMAX
LPRINT "NUMBER OF TIME PERIODS = "; TMAX
INPUT "STANDARD DEV. OF STOCHASTIC COMPONENT IN PRICE PROCESS = "; PSTDEV
INPUT "REL. STANDARD DEV. OF GROWTH PROCESS (%) = "; QRSTDEV
LPRINT "PSTDEV = "; PSTDEV; "       QRSTDEV = "; QRSTDEV
INPUT "ARE POLYNOMIAL COEF. OUTPUT NEEDED ? (0 = NO, 1 = YES) "; COEFOUT
INPUT "NUMBER OF SAMPLE SETS ? "; NSAMP
INPUT "DISTANCE BETWEEN SYSTEMTIC SAMPLES ? "; SAMPDIS
LPRINT "NUMBER OF SAMPLES = "; NSAMP; " SYSTEMATIC DISTANCE = "; SAMPDIS
INPUT "DO YOU WANT TO RESTART THE PROGRAM  ? (0 = NO, 1 = YES) "; NSTA
IF NSTA = 1 THEN GOTO 1

REM *******************************************************************
REM HERE THE FIRST SAMPLE OF OBJECTIVE FUNCTION OBSERVATIONS IS INTRODUCED.
REM IT IS IMPORTANT THAT THE COEFFICIENT MATRIX OF THE EQUATION SYSTEM
REM WHICH IS PARTLY A RESULT OF THE SELECTED SAMPLE BELOW BECOMES
REM NONSINGULAR. THIS IS GENERALLY THE CASE WHEN THE COORDINATES BELOW
REM ARE USED.
REM *******************************************************************
```

```
X1(1) = 1
X2(1) = 1
X1(2) = 90
X2(2) = 5
X1(3) = 7
X2(3) = 90
X1(4) = 99
X2(4) = 95
X1(5) = 32
X2(5) = 25
X1(6) = 88
X2(6) = 44
X1(7) = 55
X2(7) = 1
X1(8) = 6
X2(8) = 52
X1(9) = 50
X2(9) = 50
X1(10) = 35
X2(10) = 75

REM ***************************************************************************
REM HERE, A SAMPLE SET NUMBER IS SELECTED AND THE OPTIMIZATION IS STARTED.
REM ***************************************************************************

FOR SAMP = 1 TO NSAMP
LPRINT " "
LPRINT " ++++++++++ SAMPLE NUMBER "; SAMP; " ++++++++++"
LPRINT " "
PRINT "SAMPLE SET = "; SAMP
REM THE OBJECTIVE FUNCTION OF PERIOD T+1 IS SET TO ZERO FOR ALL Q AND P
FOR I = 1 TO 20
C(I) = 0
NEXT I
REM THE SYSTEMATIC CHANGE OF THE SAMPLE IS PERFORMED
FOR I = 1 TO IMAX
IF SAMP > 1 THEN X1(I) = X1(I) + SAMPDIS
IF SAMP > 1 THEN X2(I) = X2(I) + SAMPDIS
NEXT I
```

```
PRINT "THE USED SAMPLE IS"
FOR I = 1 TO IMAX
PRINT USING "####.###"; I; X1(I); X2(I)
NEXT I

REM ***********************************************************************
REM HERE, THE SHAPE OF THE TWO DIMENSIONAL TRANSITION PROBABILITY DENSITY
REM FUNCTION RANDOM COMPONENT WITH DENSITY FUNCTION F(IQDEV,IPDEV) IS CALCULATED.
REM       IQDEV AND IPDEV DENOTE INDICES OF DEVIATION AND (5,5) MEANS
REM THAT NO DEVIATION AT ALL TAKES PLACE FROM THE EXPECTED TRANSITION.
REM ***********************************************************************

FSUM = 0
FOR I = 1 TO 9
FOR J = 1 TO 9
II = EXP(-(I - 5) * (I - 5) / 2)
JJ = EXP(-(J - 5) * (J - 5) / 2)
F(I, J) = II * JJ
FSUM = FSUM + F(I, J)
NEXT J
NEXT I
FOR I = 1 TO 9
FOR J = 1 TO 9
F(I, J) = F(I, J) / FSUM
NEXT J
NEXT I

REM ***************************************************************************
REM HERE, THE TIME STARTS TO GO FROM TMAX AND BACKWARDS. IN EVERY TIME
REM PERIOD, THE OPTIMAL HARVEST LEVEL IS DETERMINED VIA MAXIMIZATION OF THE
REM EXPECTED PRESENT VALUE (THE INSTANT PROFIT + THE EXPECTED FUTURE PROFITS
REM CONDITIONAL ON OPTIMAL DECISIONS THEN). THE EXPECTED FUTURE PROFITS
REM ARE EXPRESSED AS A TWO DIMENSIONAL THIRD ORDER POLYNOMIAL OF THE TWO
REM STATE DIMENSIONS OF THE STATE SPACE, NAMELY Q (=X1) AND P (=X2).
REM WHEN THE CALCULATIONS START (AT TMAX), THEN ALL COEFFICIENTS OF THE
REM "FUTURE PROFIT POLYNOMIAL" C(I) TAKE THE VALUE O. THE POLYNOMIAL IS
REM SEQUENTIALLY UPDATED.
REM THE MAXIMIZATIONS ARE PERFORMED FOR 10 DIFFERENT COMBINATIONS OF Q AND P.
REM THIS IS EXACTLY THE AMOUNT OF OBSERVATIONS NEEDED OF THE EXPECTED
REM PRESENT VALUE FUNCTION WHEN THIS WILL BE APPROXIMATED BY THE POLYNOMIAL!
REM AFTER MAXIMIZATION, THE OPTIMAL VALUE IS STORED IN WOBS(I).
REM ***************************************************************************
```

```
FOR T = TMAX TO 1 STEP (-1)
LPRINT " "
LPRINT "***************************"
LPRINT " TIME = "; T
LPRINT "***************************"
LPRINT " "
PRINT "SAMPLE SET = "; SAMP; " TIME PERIOD = "; T
DISCOUNT = EXP(-.05 * T)
FOR I = 1 TO IMAX
Q = X1(I)
P = X2(I)
PROFMAX = 0
HOPT = 0
FOR H = 1 TO Q
PROF = (H * P - .02 * H * H * H) * DISCOUNT
     FOR IQDEV = 1 TO 9
     QDEV = (IQDEV - 5) / 100 * (Q - H) * QRSTDEV
     Q2 = (Q - H) * 1.1 + QDEV
     FOR IPDEV = 1 TO 9
     PDEV = (IPDEV - 5) * PSTDEV
     P2 = 25 + .5 * P + PDEV
          W2 = C(1) + C(2) * Q2 + C(3) * P2 + C(4) * Q2 * Q2
          W2 = W2 + C(5) * Q2 * P2 + C(6) * P2 * P2 + C(7) * Q2 * Q2 * Q2
          W2 = W2 + C(8) * Q2 * Q2 * P2 + C(9) * Q2 * P2 * P2
          W2 = W2 + C(10) * P2 * P2 * P2
          IF W2 < 0 THEN W2 = 0
               PROB = F(IQDEV, IPDEV)
PROF = PROF + PROB * W2
     NEXT IPDEV
     NEXT IQDEV
IF PROF > PROFMAX THEN HOPT = H
IF PROF > PROFMAX THEN PROFMAX = PROF
NEXT H
WOBS(I) = PROFMAX
NEXT I
```

```
REM ***************************************************************************
REM HERE, THE OPTIMAL CONTROL DECISIONS ARE CALCULATED FOR A COLLECTION
REM OF STATE COORDINATES. THESE RESULTS ARE NOT USED IN THE ESTIMATION OF
REM THE OBJECTIVE FUNCTION. THEY ARE USED IN AN ANALYSIS OF THE SENSITIVITY
REM OF THE OPTIMAL CONTROL DECISION TO THE STATE COORDINATES AND TO THE
REM OBJECTIVE FUNCTION SAMPLE SET SELECTION. THE OBJECTIVE FUNCTION
REM (THE SUM OF THE INSTANT PROFIT IN PERIOD T + THE EXPECTED PRESENT
REM  VALUE OF FUTURE PROFITS) IS CALCULATED AND PLACED IN MATRIX
REM WMAT(SAMPLE,T,QINDEX,PINDEX). THE OPTIMAL CONTROL DECISIONS ARE
REM STORED IN HMAT(SAMPLE,T,QINDEX,PINDEX).
REM ***************************************************************************

FOR QI = 1 TO 5
Q = 20 + QI * 10
FOR PI = 1 TO 5
P = 20 + PI * 10
PROFMAX = 0
HOPT = 0
FOR H = 1 TO Q
PROF = (H * P - .02 * H * H * H) * DISCOUNT
     FOR IQDEV = 1 TO 9
     QDEV = (IQDEV - 5) / 100 * (Q - H) * QRSTDEV
     Q2 = (Q - H) * 1.1 + QDEV
     FOR IPDEV = 1 TO 9
     PDEV = (IPDEV - 5) * PSTDEV
     P2 = 25 + .5 * P + PDEV
          W2 = C(1) + C(2) * Q2 + C(3) * P2 + C(4) * Q2 * Q2
          W2 = W2 + C(5) * Q2 * P2 + C(6) * P2 * P2 + C(7) * Q2 * Q2 * Q2
          W2 = W2 + C(8) * Q2 * Q2 * P2 + C(9) * Q2 * P2 * P2
          W2 = W2 + C(10) * P2 * P2 * P2
          IF W2 < 0 THEN W2 = 0
               PROB = F(IQDEV, IPDEV)
PROF = PROF + PROB * W2
     NEXT IPDEV
     NEXT IQDEV
IF PROF > PROFMAX THEN HOPT = H
IF PROF > PROFMAX THEN PROFMAX = PROF
NEXT H
HMAT(SAMP, T, QI, PI) = HOPT
WMAT(SAMP, T, QI, PI) = PROFMAX
NEXT PI
NEXT QI
```

```
REM ***********************************************************************
REM HERE, THE OPTIMAL DECISIONS AND THE OPTIMAL EXPECTED PRESENT VALUE OF
REM INSTANT AND FUTURE PROFITS ARE PRINTED FOR 5*5 DIFFERENT COORDINATES
REM IN QUANTITY - PRICE STATE SPACE.
REM ***********************************************************************

LPRINT ""
LPRINT " "
LPRINT "THE OPTIMAL CONTROL DECISIONS ARE THE FOLLOWING FOR DIFFERENT  "
LPRINT "COORDINATES IN THE STATE SPACE."
LPRINT "SAMPLE SET NUMBER = "; SAMP; ",   T = "; T
LPRINT "THE VOLUME AND PRICE DIMENSIONS CORRESPOND TO ROWS AND COLUMNS. "
LPRINT "30 L.E. VOLUME L.E. 70, 30 L.E. PRICE L.E. 70 "
LPRINT "-----------------------------------------------------------------"
FOR QI = 1 TO 5
FOR PI = 1 TO 5
LPRINT USING "########"; HMAT(SAMP, T, QI, PI);
NEXT PI
LPRINT " "
LPRINT " "
NEXT QI

LPRINT " "
LPRINT " "
LPRINT "SAMPLE SET NUMBER = "; SAMP; ",   T = "; T
LPRINT "THE PRESENT VALUE OF THE PROFIT IN PERIOD "; T; " + THE EXPECTED "
LPRINT "FUTURE PROFITS WHEN THE OPTIMAL DECISIONS ARE MADE "
LPRINT "------------------------------------------------------------------"
LPRINT " "
FOR QI = 1 TO 5
FOR PI = 1 TO 5
LPRINT USING "########"; WMAT(SAMP, T, QI, PI);
NEXT PI
LPRINT " "
LPRINT " "
NEXT QI
LPRINT " "
```

```
REM ************************************************************************
REM HERE, THE OBJECTIVE FUNCTION OBSERVATIONS ARE USED TO PRODUCE THE
REM COEFFICIENTS OF THE LINEAR EQUATION SYSTEM WHICH IS SOLVED IN ORDER
REM TO DETERMINE THE MULTIDIMENSIONAL POLYNOMIAL WHICH APPROXIMATES
REM THE OBJECTIVE FUNCTION. THE EQUATION SYSTEM IS STORED IN MATRIX A.
REM ************************************************************************

FOR I = 1 TO IMAX
A(I, 1) = 1
A(I, 2) = X1(I)
A(I, 3) = X2(I)
A(I, 4) = X1(I) * X1(I)
A(I, 5) = X1(I) * X2(I)
A(I, 6) = X2(I) * X2(I)
A(I, 7) = X1(I) * X1(I) * X1(I)
A(I, 8) = X1(I) * X1(I) * X2(I)
A(I, 9) = X1(I) * X2(I) * X2(I)
A(I, 10) = X2(I) * X2(I) * X2(I)
A(I, 11) = WOBS(I)
NEXT I
```

```
REM *********************************************************************
REM HERE, THE LINEAR EQUATION SYSTEM IS SOLVED VIA THE GAUSS METHOD,
REM REF. WITTMEYER-KOCK, I., ELDEN, L., LÄROBOK I NUMERISKA METODER,
REM UNIVERSITETET I LINKÖPING, MAT.INST., 581 83 LINKÖPING, 1986
REM *********************************************************************

FOR I = 1 TO IMAX
ROWCOEFMAX = I
FOR K = (1 + I) TO IMAX
IF ABS(A(K, I)) > ABS(A(ROWCOEFMAX, I)) THEN ROWCOEFMAX = K
NEXT K
FOR L = I TO JMAX
C1 = A(I, L)
C2 = A(ROWCOEFMAX, L)
A(I, L) = C2
A(ROWCOEFMAX, L) = C1
NEXT L
FOR II = (I + 1) TO IMAX
COEF = A(II, I) / A(I, I)
FOR J = I TO JMAX
A(II, J) = A(II, J) - COEF * A(I, J)
NEXT J
NEXT II
NEXT I
FOR IBACK = 1 TO IMAX
I = IMAX - IBACK + 1
FOR II = 1 TO (I - 1)
COEF = A(II, I) / A(I, I)
FOR J = 1 TO JMAX
A(II, J) = A(II, J) - COEF * A(I, J)
NEXT J
NEXT II
NEXT IBACK
FOR I = 1 TO IMAX
A(I, JMAX) = A(I, JMAX) / A(I, I)
C(I) = A(I, JMAX)
NEXT I
```

```
REM ************************************************************************
REM PRINTOUT OF THE ESTIMATED POLYNOMIAL COEFFICIENTS
REM ************************************************************************

IF COEFOUT = 0 THEN GOTO 2
LPRINT "ESTIMATED COEFFICIENTS OF EXPECTED PRESENT VALUE POLYNOMIAL "
LPRINT "------------------------------------------------------------ "
FOR I = 1 TO IMAX
LPRINT "COEF("; I; ") = "; C(I)
NEXT I
LPRINT " "
2 REM

REM ************************************************************************
REM THIS IS A TEST AND A 2-D MAP OF THE ESTIMATED POLYNOMIAL.
REM ************************************************************************

FOR QINDEX = 1 TO 5 STEP 1
Q = 20 + QINDEX * 10
FOR PINDEX = 1 TO 5 STEP 1
P = 20 + PINDEX * 10
W = C(1) + C(2) * Q + C(3) * P + C(4) * Q * Q + C(5) * Q * P
W = W + C(6) * P * P + C(7) * Q * Q * Q + C(8) * Q * Q * P
W = W + C(9) * Q * P * P + C(10) * P * P * P
WMAP(QINDEX, PINDEX) = W
WMAPTOT(SAMP, QINDEX, PINDEX) = W
NEXT PINDEX
NEXT QINDEX
LPRINT " "
LPRINT " SAMPLE SET NUMBER = "; SAMP; " T = "; T
LPRINT " WMAP(QINDEX,PINDEX) IS A 2-D MAP OF THE OBJECTIVE FUNCTION LEVEL."
LPRINT " THE VOLUME AND PRICE DIMENSIONS CORRESPOND TO ROWS AND COLUMNS."
LPRINT " 30 L.E. VOLUME L.E. 70, 30 L.E. PRICE L.E. 70 "
LPRINT " THE POLYNOMIAL WHICH IS SHOWN HERE IS USED IN THE PERIOD BEFORE "
LPRINT " THIS PERIOD AS AN APPROXIMATION OF THE TRUE EXPECTED VALUE FUNCTION."
LPRINT " ------------------------------------------------------------------- "
LPRINT " "
FOR QINDEX = 1 TO 5
FOR PINDEX = 1 TO 5
LPRINT USING "########"; WMAP(QINDEX, PINDEX);
NEXT PINDEX
LPRINT " "
LPRINT " "
NEXT QINDEX
NEXT T
NEXT SAMP
```

```
REM ************************************************************************
REM STATISTICAL INVESTIGATION OF THE MEAN AND STDV OF THE ESTIMATED
REM OBJECTIVE FUNCTION IN PERIOD 1
REM ************************************************************************

IF NSAMP = 1 THEN STOP
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMAPMED(Q, P) = 0
WMAPDEV(Q, P) = 0
NEXT P
NEXT Q

FOR S = 1 TO NSAMP
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMAPMED(Q, P) = WMAPMED(Q, P) + WMAPTOT(S, Q, P) / NSAMP
NEXT P
NEXT Q
NEXT S

FOR S = 1 TO NSAMP
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMAPDEV(Q, P) = WMAPDEV(Q, P) + (WMAPTOT(S, Q, P) - WMAPMED(Q, P)) ^ 2
NEXT P
NEXT Q
NEXT S

FOR Q = 1 TO 5
FOR P = 1 TO 5
WMAPDEV(Q, P) = (WMAPDEV(Q, P) / (NSAMP - 1)) ^ .5
NEXT P
NEXT Q
```

```
LPRINT " "
LPRINT " MEAN VALUES OF THE OBJECTIVE FUNCTION IN DIFFERENT COORDINATES"
LPRINT " IN PERIOD 1 (THE UPPER TABLE) AND THE STANDARD DEVIATIONS (THE"
LPRINT " LOWER TABLE)."
LPRINT " THE POLYNOMIAL WHICH IS SHOWN HERE SHOULD HAVE BEEN USED AS AN"
LPRINT " APPROXIMATION OF THE TRUE EXPECTED VALUE FUNCTION IN THE PERIOD"
LPRINT " BEFORE THE FIRST PERIOD IF SUCH A PERIOD WOULD HAVE EXISTED."
LPRINT " (THE VOLUME AND PRICE DIMENSIONS CORRESPOND TO ROWS AND COLUMNS,"
LPRINT " 30 L.E. VOLUME L.E. 70, 30 L.E. PRICE L.E. 70)"
LPRINT " -------------------------------------------------------------------"
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; WMAPMED(Q, P);
NEXT P
LPRINT " "
LPRINT " "
NEXT Q

LPRINT " "
LPRINT " +++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; WMAPDEV(Q, P);
NEXT P
LPRINT " "
LPRINT " "
NEXT Q

REM ************************************************************************
REM STATISTICAL INVESTIGATION OF THE EFFECTS OF DIFFERENT SAMPLE SETS ON
REM THE OPTIMAL SOLUTION (DECISION AND OBJECTIVE FUNCTIONS). THE MEANS
REM AND THE STANDARD DEVIATIONS OF THE OPTIMAL CONTROLS AND THE OBJECTIVE
REM FUNCTION ARE CALCULATED VIA THE RESULTS BASED ON THE DIFFERENT
REM SAMPLE SETS.
REM ************************************************************************
```

```
IF NSAMP = 1 THEN STOP
FOR T = 1 TO TMAX
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMATMED(T, Q, P) = 0
WMATDEV(T, Q, P) = 0
HMATMED(T, Q, P) = 0
HMATDEV(T, Q, P) = 0
NEXT P
NEXT Q
NEXT T

FOR S = 1 TO NSAMP
FOR T = 1 TO TMAX
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMATMED(T, Q, P) = WMATMED(T, Q, P) + WMAT(S, T, Q, P) / NSAMP
HMATMED(T, Q, P) = HMATMED(T, Q, P) + HMAT(S, T, Q, P) / NSAMP
NEXT P
NEXT Q
NEXT T
NEXT S

FOR S = 1 TO NSAMP
FOR T = 1 TO TMAX
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMATDEV(T, Q, P) = WMATDEV(T, Q, P) + (WMAT(S, T, Q, P) - WMATMED(T, Q, P)) ^ 2
HMATDEV(T, Q, P) = HMATDEV(T, Q, P) + (HMAT(S, T, Q, P) - HMATMED(T, Q, P)) ^ 2
NEXT P
NEXT Q
NEXT T
NEXT S

FOR T = 1 TO TMAX
FOR Q = 1 TO 5
FOR P = 1 TO 5
WMATDEV(T, Q, P) = (WMATDEV(T, Q, P) / (NSAMP - 1)) ^ .5
HMATDEV(T, Q, P) = (HMATDEV(T, Q, P) / (NSAMP - 1)) ^ .5
NEXT P
NEXT Q
NEXT T
```

```
LPRINT " "
LPRINT " "
LPRINT "++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"
LPRINT "HERE THE OBJECTIVE FUNCTION AND DECISION LIST STARTS"
LPRINT "++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++"
LPRINT " "
LPRINT "THE VOLUME AND PRICE DIMENSIONS CORRESPOND TO ROWS AND COLUMNS"
LPRINT "30 L.E. VOLUME L.E. 70, 30 L.E. PRICE L.E. 70"
LPRINT " "

FOR T = 1 TO TMAX
LPRINT "TIME PERIOD = "; T; ", OBJECTIVE FUNCTION VALUES ="
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; WMATMED(T, Q, P);
NEXT P
LPRINT " "
NEXT Q
LPRINT " "

LPRINT "TIME PERIOD = "; T; ", OBJECTIVE FUNCTION STANDARD DEVIATIONS ="
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; WMATDEV(T, Q, P);
NEXT P
LPRINT " "
NEXT Q
LPRINT " "
```

```
LPRINT "TIME PERIOD = "; T; ", OPTIMAL CONTROL = "
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; HMATMED(T, Q, P);
NEXT P
LPRINT " "
NEXT Q
LPRINT " "

LPRINT "TIME PERIOD = "; T; ", STANDARD DEVIATION IN OPTIMAL CONTROL = "
LPRINT " "

FOR Q = 1 TO 5
FOR P = 1 TO 5
LPRINT USING "#######.#"; HMATDEV(T, Q, P);
NEXT P
LPRINT " "
NEXT Q
LPRINT " "
LPRINT "------------------------------------------------------------------"

NEXT T

END
```

Here the computer output is shown:

```
*****************************************************************************
PROGRAM PSTDP.BAS VERSION WITH 3 ORDER HARVEST COST
STOCHASTIC DYNAMIC PROGRAMMING WITH OBJECTIVE FUNCTION APPROXIMATION
VIA MULTIDIMENSIONAL (2 DIMENSIONS, ORDER 3) POLYNOMIALS
LOHMANDER PETER 90-08-16
*****************************************************************************

NUMBER OF TIME PERIODS =  5
PSTDEV =  10        QRSTDEV =  30
NUMBER OF SAMPLES =  3  SYSTEMATIC DISTANCE =  3.1




+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
HERE THE OBJECTIVE FUNCTION AND DECISION LIST STARTS
+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

THE VOLUME AND PRICE DIMENSIONS CORRESPOND TO ROWS AND COLUMNS
30 L.E. VOLUME L.E. 70, 30 L.E. PRICE L.E. 70

TIME PERIOD =  1 , OBJECTIVE FUNCTION VALUES =

   1384.4   1470.8   1601.9   1778.0   1985.0
   1714.3   1835.3   2009.4   2220.5   2459.3
   2004.9   2173.3   2386.0   2631.0   2901.4
   2269.1   2480.7   2730.0   3008.6   3310.8
   2504.3   2755.3   3039.9   3351.8   3685.9

TIME PERIOD =  1 , OBJECTIVE FUNCTION STANDARD DEVIATIONS =

     11.3     25.4     54.0     80.4    106.5
      9.4     12.6     34.1     56.1     78.3
     20.4      5.3     16.6     34.6     53.5
     28.0     15.1      5.3     16.7     32.2
     35.1     24.9     13.7      5.9     15.2

TIME PERIOD =  1 , OPTIMAL CONTROL =

      1.0      3.3      9.7     14.0     17.3
      1.0      7.0     12.0     16.0     19.0
      3.7     10.0     14.0     18.0     20.3
      8.0     12.3     16.0     19.0     22.0
     10.7     14.7     17.7     20.7     23.3

TIME PERIOD =  1 , STANDARD DEVIATION IN OPTIMAL CONTROL =

      0.0      2.5      1.5      2.0      1.5
      0.0      2.0      1.0      1.0      1.0
      1.5      1.0      1.0      1.0      0.6
      1.0      0.6      1.0      1.0      1.0
      0.6      0.6      0.6      0.6      0.6
```

TIME PERIOD = 2 , OBJECTIVE FUNCTION VALUES =

1156.7	1248.2	1381.2	1544.6	1728.5
1460.9	1583.5	1752.1	1951.5	2174.7
1734.0	1901.0	2107.9	2345.3	2608.3
1977.2	2186.2	2431.0	2706.3	3008.4
2178.7	2425.5	2706.6	3018.5	3358.6

TIME PERIOD = 2 , OBJECTIVE FUNCTION STANDARD DEVIATIONS =

7.8	22.3	46.0	64.3	79.0
28.1	10.9	16.0	28.1	37.4
38.9	23.8	13.9	11.1	13.9
45.3	34.0	25.8	21.3	21.4
47.7	38.0	31.3	28.4	30.0

TIME PERIOD = 2 , OPTIMAL CONTROL =

1.0	5.7	12.0	15.7	18.7
1.3	7.3	12.7	16.0	18.7
4.7	10.3	14.3	17.3	20.0
9.3	13.3	16.3	19.3	21.3
13.0	16.7	19.3	22.0	24.0

TIME PERIOD = 2 , STANDARD DEVIATION IN OPTIMAL CONTROL =

0.0	4.5	3.0	2.5	2.5
0.6	2.5	2.1	2.0	2.1
1.2	1.5	1.5	1.5	1.0
0.6	0.6	0.6	0.6	0.6
0.0	0.6	0.6	0.0	0.0

TIME PERIOD = 3 , OBJECTIVE FUNCTION VALUES =

957.3	1103.1	1256.7	1417.6	1587.4
1171.7	1364.4	1571.7	1791.5	2024.6
1367.6	1596.4	1847.2	2116.3	2403.8
1542.2	1796.3	2080.0	2388.9	2721.3
1693.6	1961.7	2268.0	2606.2	2973.9

TIME PERIOD = 3 , OBJECTIVE FUNCTION STANDARD DEVIATIONS =

10.4	5.8	6.4	7.5	12.5
24.0	19.1	18.6	22.3	29.5
30.9	27.6	28.4	32.5	40.2
34.1	31.1	31.9	36.3	43.6
37.4	32.6	32.5	36.0	42.6

TIME PERIOD = 3 , OPTIMAL CONTROL =

8.3	11.7	13.3	15.3	16.3
10.0	14.0	16.0	18.0	19.3
11.7	15.7	18.3	20.3	22.0
13.7	18.0	21.0	23.0	25.0
15.0	19.7	23.0	25.0	27.0

TIME PERIOD = 3 , STANDARD DEVIATION IN OPTIMAL CONTROL =

2.5	2.1	1.5	1.5	1.5
1.0	1.0	1.0	1.0	0.6
0.6	0.6	0.6	0.6	0.0
0.6	0.0	0.0	0.0	0.0
0.0	0.6	0.0	0.0	0.0

TIME PERIOD = 4 , OBJECTIVE FUNCTION VALUES =

688.4	880.7	1092.4	1317.3	1549.5
830.8	1075.4	1339.7	1617.1	1901.7
893.9	1181.8	1489.7	1811.7	2141.6
885.4	1206.1	1548.7	1906.7	2274.0
834.9	1173.0	1538.2	1921.8	2316.9

TIME PERIOD = 4 , OBJECTIVE FUNCTION STANDARD DEVIATIONS =

31.5	39.9	44.7	45.8	42.5
22.8	31.6	37.5	39.4	37.8
11.1	20.6	27.1	30.3	29.1
1.1	8.3	15.2	18.9	18.8
7.8	0.9	4.9	8.0	8.0

TIME PERIOD = 4 , OPTIMAL CONTROL =

11.7	13.3	15.3	17.0	18.3
17.0	19.0	20.7	22.3	23.7
21.3	23.3	25.0	26.7	28.3
24.0	26.3	28.3	30.0	31.7
25.0	27.7	30.0	32.0	34.0

TIME PERIOD = 4 , STANDARD DEVIATION IN OPTIMAL CONTROL =

0.6	0.6	0.6	0.0	0.6
1.0	1.0	0.6	0.6	0.6
0.6	0.6	0.0	0.6	0.6
0.0	0.6	0.6	0.0	0.6
0.0	0.6	0.0	0.0	0.0

TIME PERIOD = 5 , OBJECTIVE FUNCTION VALUES =

348.2	536.2	749.4	981.3	1214.9
348.2	536.2	749.4	984.9	1241.3
348.2	536.2	749.4	984.9	1241.3
348.2	536.2	749.4	984.9	1241.3
348.2	536.2	749.4	984.9	1241.3

TIME PERIOD = 5 , OBJECTIVE FUNCTION STANDARD DEVIATIONS =

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0

TIME PERIOD = 5 , OPTIMAL CONTROL =

22.0	26.0	29.0	30.0	30.0
22.0	26.0	29.0	32.0	34.0
22.0	26.0	29.0	32.0	34.0
22.0	26.0	29.0	32.0	34.0
22.0	26.0	29.0	32.0	34.0

TIME PERIOD = 5 , STANDARD DEVIATION IN OPTIMAL CONTROL =

0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
0.0	0.0	0.0	0.0	0.0
