INTERNATIONAL SYMPOSIUM ON SYSTEMS ANALYSIS AND MANAGEMENT DECISIONS IN FORESTRY

THE ECONOMICALLY OPTIMAL NUMBER OF PLANTS, THE DAMAGE PROBABILITY AND THE STOCHASTIC ROUNWOOD MARKET.

Peter Lohmander
Swedish University of Agricultural Sciences, Faculty of Forestry,
Dept. Forest Economics, S-901 83 Umeå, Sweden

← (290-314)

Editor
Gonzalo L. Paredes V.

Organizing Institutions

Forest Management Institute
Universidad Austral of Chile

Department of Industrial Engineering
Universidad of Chile

Proceedings of the Symposium
Villa del Rio Conference Center,
March 9–12, 1993, Valdivia, Chile

Valdivia, Chile, 1994

PREFACE

Since 1975, the Society of American Foresters' Systems Analysis Working Group has sponsored symposia with increasing frequency as the number of participants and the field itself has grown. Previous symposia were held in Athens, Georgia 1975 and 1985; Asilomar, Pacific Grove, California 1988: and Charleston, South Carolina 1991.

The conference held at the Austral University in Valdivia, Chile in 1993 is the first time it has been held outside the United States. Seventeen countries were represented by 125 attendees including: Canada, Austria, Argentina, Australia, New Zealand, Guatemala, Mexico, Finland, Norway, Sweden, Denmark, Peru, Slovenia, Brazil, Korea, United States, and Chile. Seventy-one presentations were given.

We would like to thank those who presented special topic tutorials and conferences:

Rene Alfaro, Canada
    Modeling forest pest and diseases
Peter Lohmander, Sweden
    Adaptative decision-making in forestry
Marku Kallio, Finland
    Equilibrium models in forestry and forest industries
Douglas Brodie, USA
    Silvicultural optimization at the stand level
David Martell, Canada
    Economics and operations research in fire management
Bruce Bare, USA
    Multicriteria decision-making in forest management

Prominent among program and operational leadership were Andres Weintraub, Antonio Grass, Beatriz Rojas and Clark Row.

A new feature of this symposium was simultaneous English/Spanish translation for half the presentations. A 4-day tour of industries, natural and exotic forest, nursery and field research areas, commencing in Valdivia and ending in Concepción, provided a technical and social capstone to the trip.

In assessing technical evolution and trends, this meeting's presentations show increasing emphasis on integer and spatial approaches with links to geographic information systems, stochastic modelling and adaptive control, and evolution of optimization and heuristics to handle problems of increasing complexity and dimensionality. The range of applications -industry, fire, wildlife, inventory, social forestry, along with silviculture and harvest scheduling- are important links to all fields of resource management modelling and economics.

We would like to thank our industrial sponsors and the professional forest resource and academic institutions in Chile that contributed to this symposium.

We look forward to the next symposium now scheduled for September 1994.

Gonzalo L. Paredes V. and J. Douglas Brodie
Program Committee

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Peter Lohmander
Swedish University of Agricultural Sciences, Faculty of Forestry,
Dept. Forest Economics, S-901 83 Umeå, Sweden

Abstract

The purpose of this paper is to determine the economically optimal number of plants in a plantation and to investigate how this is affected by the plant survival probability, the roundwood price risk and other parameters. The objective function is the expected present value. The future roundwood prices are described as a stochastic process. Future harvesting will be optimal and adaptive, conditional on future price observations.

The optimization problem is analytically and numerically investigated. Several new qualitative results are reported, such as: If the optimal number of plants initially is low (high), then the optimal number of plants decreases (increases) if the death probability increases marginally. In order to solve the optimization problem, it is necessary to know several qualitative and quantitative properties of forest growth. In particular, it is very important to know the functional form of the growth function and the signs of several derivatives. For this reason, empirically supported optimization is made at two levels.

First, the optimal investment intensity in Norway spruce plantations in northern Sweden is investigated as an empirically supported test case. Plant survival is assumed to be random.

Then, via adaptive optimization, it is shown that the optimal number of plants, and the expected optimal present value, are increasing functions of the future price risk. In order to support the multi stage adaptive optimization, a specific growth function is estimated. This is a nonlinear Markovian growth function with the number of stems and the volume per hectare as independent variables. It turns out that all estimated coefficients have biological interpretations and are strongly significant.

Acknowledgements

I gratefully acknowledge research grants from Brattåsstiftelsen för Skogsvetenskaplig Forskning, Cellulosaindustriens Stiftelse för Teknisk och Skoglig Forskning samt Utbildning, Domänverket, Lars-Erik Thunholms Stiftelse för Främjande av Vetenskaplig Forskning and Skogs- och Jordbrukets Forskningsråd. The very kind invitations by Professor Brodie, Corvallis, Oregon and by Professor Weintraub, Santiago, Chile, made it possible for me to participate in the conference. Lars-Erik Thunholms Stiftelse covered a part of my travel costs. Master of Science Peichen Gong, checked an earlier version of my manuscript.

1. General Introduction to Reforestration Economics

Reforestration is an economic issue. An economic issue, on the other hand, can contain several ingredients. We may in the reforestration issue consider costs, revenues, the capital market, physical damages and economic risks. We may also, in case this is of particular interest in the area, consider environmental questions. Environmental questions, on the other hand, is a complicated set of issues in itself. Here, we may consider the value of the forest and the environment to the human observer, the value of the forest as a wildlife habitat and hunting ground, the value of the preservation of different species and ecosystems etc..

In a more restrictive sense, excluding the environmental aspects, we may consider the reforestration issue as a classical investment problem. The reader should be well aware that present value maximization of the investment is appropriate in that case since this leads to maximization of the intertemporal consumption budget. Compare Johansson and Löfgren (1985, Ch. 1) or any book on investment theory.

In the work of Johansson and Löfgren (1985, Ch. 4), the forest investment problem is analyzed in a particular sense: The cost of the investment is constant. The physical development of the forest stand is not described as a function of the investment intensity, the number of plants per hectare, or the investment cost. Hence it is not possible to optimize the investment intensity. The problem under investigation is on the other hand to optimize the harvest age, the rotation age. That is the classical problem of forest economics.

In case we do not think that we already know everything about the future, which we of course do not, then we should consider risks. The reader is recommended to study Rothschild and Stiglitz (1970, 1971) and Hey (1981). Many general effects of risk are presented by that authors. As long as the investment represents a marginal fraction of the wealth of the investor, which is mostly the case when we discuss a particular reforestration area, then one can show that maximization of the expected present value is a appropriate. This issue and several related questions are analyzed in Lohmander (1987).

For this reason, the analysis in this paper will be entirely based on maximization of the expected present value. Of course, one may widen the scope of the analysis and include restrictions of different kinds that reflect particular industrial needs, availability of personel and transportation capacity. It has been very popular to analyze forestry and forest sector issues with highly detailed numerical models. On the other hand, the time it takes for the typical forest stand to grow up and to be harvested is usually very long. In Sweden, a typical rotation age is 100 years. That age is usually higher than the economic optimum.

We should be aware that we can not make reliable detailed multi decade predictions of technology inventions, of the taste of future consumers, of the size of the labour force or of political changes in our country and countries that influence our international trade.

Thus, there is no obvious reason why we should consider such predictions and associated restrictions when we study the reforestration problem. We may regard the relevant fraction of the future state of the world as the timber prices and harvesting costs. These, we regard as stochastic processes.

2. The present solution and the scientific literature

Let us make a short investigation of the presently applied principles in swedish reforestration. Then, we will look closer into the latest scientific results.

2.1. The administrative solution to the reforestration problem

What number of plants should we select in a forest plantation? What species or species combination should we choose? These are two of the more basic questions of forest management. Of course, depending on the main purpose of the plantation, several suggestions and rules can be made. In swedish forestry, as one example, the target according to law, has been to obtain at least a predetermined number of stems of a particular species in the young forest. If, for some reason, some damage has occured, then it has often been necessary to replant the area. There is no available economic background to the different and very detailed restrictions in the present swedish forest act. Most likely we will very soon have a completely new forest act in Sweden.

Officials from the Swedish Board of Forestry investigate the plantations when these reach a particular age. If the investigators find the number of stems to be too low according to their standards, they order the land owner to replant the area. If the land owner does not obey this order, the Board of Forestry undertakes the operation and sends the bill to the land owner.

In some cases, it has been considered necessary to take away also the plants that were not damaged, just in order to obtain a new stand in which all the plants have the same age after the replantation. In some cases, some years after the plantation of Scots pine, it has been found that the young forest contains only Norway spruce and birch. The reason is often that there is a dense moose population in the area and they eat young pine. Partial natural regeneration of Norway spruce and birch is very common in Sweden thanks to seeds from neighbour stands.
In such cases, one can often show that it would be better (with respect to the expected present value of the investment) to accept the observed young spruce/birch forest, even if this is not very dense, than to start all over again. Furthermore, if we start all over again, the result is once again uncertain. The Board of Forestry officials sometimes order the land owner to take away the spruce and birch and create a new pine plantation.

2.2. Relevant reforestration literature

The fundamental questions in forest economics, such as how many plants should we place in the ground and when should we harvest, have in the latest decade been given high attention in the literature. During the earlier years, the assumption of a deterministic world was usually made. Later on, the insight that the future development of prices and damages is not yet known, was taken into account.

In some articles it was assumed that the future development was indeed uncertain but that all decisions were taken at once. In some other studies it was assumed that several possible scenarios could occur. For each scenario, on the other hand, it was assumed that the decisions over time were optimized conditional on perfect information about all details of that scenario. We may call these studies "non adaptive", since it is not assumed that future decisions are optimized conditional on future information, which is not yet available.

Some of the classical nonadaptive studies in the areas of: optimal investment, thinning and rotation age, are: Chang (1983), Brodie and Haight (1985), Johansson and Löfgren (1985), Nautiyal and Williams (1990), Reed and Apaloo (1991), Solberg and Haight (1991), Taylor and Fortson (1991), Teeter and Caulfield (1991), Filius and Dul (1992), Gove (1992), Jonsson and Jonsson (1992) and Valsta (1992).

Several of these papers include management guidelines and solutions that may be very useful when we want to compare our adaptive solutions to the deterministic equivalents. Several of the results hold also in adaptive situations where the level of uncertainty is low. In many cases, however, we should be aware that the solutions are very different from the earlier solutions, when we accept that the future is not yet perfectly known and that we will take advantage of the future information when we take our future decisions. The adaptive approach has become more and more common in forest management during the latest years. Risvand (1976) wrote an early paper in which the future is described as stochastic and the future decisions are optimized conditional on future information.

Risvand used a discrete stochastic Markov chain to describe the price movements. In an analysis by Lohmander (1985), the future prices are distributed according to continuous probability density functions. The pulse harvesting problem, optimal rotation in forestry, was investigated. It was found that new properties of the growth function are important when we use adaptive optimization of the harvest year. In a stochastic timber market, it is important to create "harvest year flexibility". It is important that the forest stand can survive unexpectedly high stand density and age when we sometimes have to wait for the timber price to increase. This, in turn, affects the optimal selection of species in reforestration.

Then, the author extended the analysis to continuous harvesting problems and situations where stochastic windthrows take place. Spatial considerations became important. In later years, a number of studies in the field of adaptive forest management were conducted by the author, most of which may be found in the reference list.

Several other authors have also started to use the adaptive approach and found it very interesting and useful in forest management. Among such papers, we find: Kaya and Buongiorno (1987, 1989), Brazee and Mendelsohn (1988), two impressive articles by Haight (1990) and (1991) and by Carlsson (1992). Gong (1992) has taken the first steps to extend the adaptive analysis to cover multiobjective optimization.

3. Structure of the paper

The optimal number of plants in a forest plantation is under investigation in this paper. The objective is to maximize the expected present value of the investment. Economic parameters such as the rate of interest in the capital market, the cost per plant and the roundwood price are explicitly treated. The production function is discussed in detail. Two phenomena of great economic importance will be considered. These are physical damages and stochastic markets.

In section 4., the optimal number of plants will be analyzed within a single decision problem: First you decide the number of plants and then you follow your plan, performing the harvest at a predermined point in time.

In section 5., we look deeper into the consequences of a changing plant survival probability. In section 6., the optimal intensity in Norway spruce plantations in northern Sweden is determined.

In section 7., stochastic timber prices and rough market adapted harvesting are introduced in the following way: When the "harvest point in time" occurs, the timber price and the variable harvesting cost are observed. If it is found that it is more profitable to harvest than to leave the stand in the forest for ever, a harvest takes place. Otherwise, we just forget the area.

In sections 8. and 9., a multi stage market adapted decision model is introduced. As always the main question concerns the optimal investment intensity. The harvest section of the problem is however dynamic: The decision at a particular point in time is to accept the observed price and to harvest the stand or to let the stand grow at least one more period.

4. Optimal intensity in the single decision problem

Equation (1) is the present value optimization problem of this section. π denotes the present value and n is the (initial) number of plants per hectare. The costs contain one fix part, F, and one variable cost, cn. c is the marginal plant cost. At time t, perhaps 100 years after the investment, it is time to harvest. The discount factor is written in exponential form where r is the rate of interest in the capital market. P is the "net price", a short notation for "price minus variable (relevant harvesting and transportation) costs per cubic metre". L is the value of the land released for sales or other purposes after harvest. V denotes the volume per hectare if the number of survived stems, N, is 2000. U(xn) denotes the relative production, where x is the survival probability of the plants in the first dangerous years of the life of the plantation. Hence, U*V is the volume per hectare in the stand. N = xn.

$$\max_{n} \pi(n) = -F - cn + e^{-rt}[PU(xn)V_t + L] \quad (1)$$

We assume, with strong empirical support, that U'(N) > 0 and U''(N) < 0. The assumption of decreasing marginal return to scale is very common also in most other economic theory.

Note in particular that (1) is consistent with a forest investment with no thinnings. One reason is that forestry in many areas is more profitable without thinnings. This has been shown by Wieslander (1986) and by Bjurulf and Freij (1986).

Furthermore, the very dominating fraction of the profits from forestry comes from the final fellings also in traditional Swedish forestry with thinnings. The suggested model will also make the qualitative results easy to obtain and to interpret.

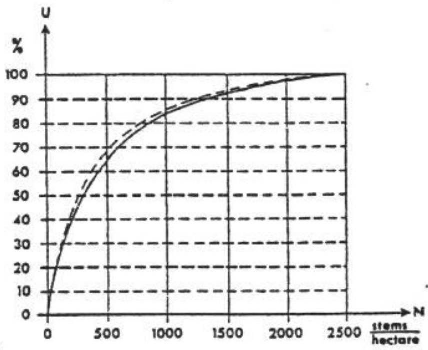

Figure 1.
The relative production function. Sweden.
The graph shows the relative yield level, U, for different numbers of stems per hectare, N. (The rotation age is 80 years.)
Solid line = Scots pine (Pinus silvestris), site index = T24.
Dotted line = Norway spruce (Picea abies), site index = G32.
Source: Elfving (1985).

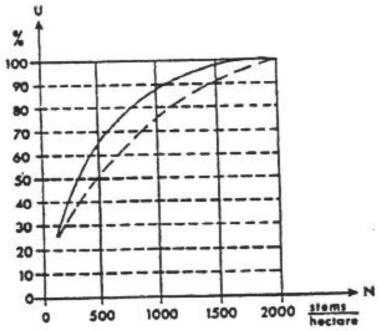

Figure 2.
The relative production function. Norway. The graph shows the relative yield level, U, for different numbers of individuals per hectare, N. The species is Norway spruce (Picea abies). Solid line = regular spacing. Dotted line = irregular spacing. Source: Braastad (1983).

Let us optimize the initial number of plants. We take the first order derivative and set this equal to zero in (2). From (2) we get (3). (3) is very useful since we can instantly draw a graph and determine the optimal number of plants. In Figure 3. we see how this is done:

$$\frac{\delta \pi}{\delta n} = -c+e^{-rt}P\frac{\delta U}{\delta N}xV_t = 0 \qquad (2)$$

$$\frac{\delta U}{\delta n} = \frac{\delta U}{\delta N}x = \frac{ce^{rt}}{PV_t} \qquad (3)$$

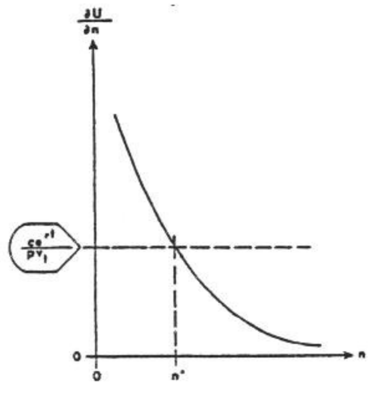

Figure 3.
Graph for determination of the optimal number of plants. The graph is based on [illegible] result reported in (3). We find that the optimal number of plants is an increasing function of the net price, P, and the site index (V is an increasing function of the site index). The optimal number of plants is a decreasing function of the marginal plant cost, c, and the rate of interest, r.

(4) shows that the solution is a unique maximum.

$$\frac{\delta^2 \pi}{\delta n^2} = e^{-rt}P\frac{\delta^2 U}{\delta N^2}x^2V_t < 0 \qquad (4)$$

Suppose that we make a change in an arbitrary parameter z and adjust the optimal number of plants accordingly. (5), (6) and (7) show how we can determine the qualitative effects of parameter changes on the optimal intensity decision. We simply have to determine the signs of a series of mixed second order derivatives.

$$\frac{\delta^2 \pi^*}{\delta n^2}dn^* + \frac{\delta^2 \pi^*}{\delta n \delta z}dz = 0 \qquad (5)$$

$$\frac{\delta n^*}{\delta z} = -\frac{\left(\frac{\delta^2 \pi^*}{\delta n \delta z}\right)}{\left(\frac{\delta^2 \pi^*}{\delta n^2}\right)} \qquad (6)$$

$$sgn\left(\frac{\delta n^*}{\delta z}\right) = sgn\left(\frac{\delta^2 \pi^*}{\delta n \delta z}\right) \qquad (7)$$

Let us determine the signs of these mixed second order derivatives:

$$\frac{\delta^2 \pi}{\delta n \delta c} = -1 < 0 \qquad (8)$$

$$\frac{\delta^2 \pi}{\delta n \delta r} = -te^{-rt}P\frac{\delta U}{\delta N}xV_t < 0 \qquad (9)$$

$$\frac{\delta^2\pi}{\delta n\delta P} = e^{-rt}\frac{\delta U}{\delta N}xV_t > 0 \qquad (10)$$

$$\frac{\delta^2\pi}{\delta n\delta x} = e^{-rt}PV_t\left[\frac{\delta U}{\delta N}+\frac{\delta^2 U}{\delta N^2}xn\right] \gtreqless 0 \qquad (11)$$

$$\frac{\delta^2\pi}{\delta n\delta V_t} = e^{-rt}P\frac{\delta U}{\delta N}x > 0 \qquad (12)$$

As a consequence of (7) and (8) - (12), a number of qualitative results can be stated. Compare also Figure 3. The optimal number of plants is a decreasing function of the marginal plant cost,

$$\frac{\delta n^*}{\delta c} < 0 \qquad (13)$$

a decreasing function of the rate of interest

$$\frac{\delta n^*}{\delta r} < 0 \qquad (14)$$

and an increasing function of the net price, P.

$$\frac{\delta n^*}{\delta P} > 0 \qquad (15)$$

In the general case it is not clear if the optimal number of plants increases, is unchanged or decreases if the plant survival probability increases! This result may seem strange to the reader who is used to definite answers to this question. This phenomenon will be analyzed and explained in greater detail in the following section.

$$\frac{\delta n^*}{\delta x} \gtreqless 0 \qquad (16)$$

If the site index increases, we should have more plants per hectare.

$$\frac{\delta n^*}{\delta V_t} > 0 \qquad (17)$$

5. Intensity and the survival probability - A deeper investigation

Now, we will make a deeper investigation of the effects of a changing survival probability, x. In section 4. we found that the optimal investment intensity increases, is unchanged or decreases if the survival probability increases. Of course, it is important to understand more about this perhaps unexpected result. In order to obtain such understanding, we may start by investigating a particular case. In (18), we assume that the relative production function is quadratic. For obvious reasons, a > 0, b < 0.

$$U(N) = U(xn) = a(xn) + b(xn)^2 \qquad (18)$$

(18) may be rewritten as (19).

$$U(xn) = axn + bx^2n^2 \qquad (19)$$

The contribution of the marginal plant, n, is given in (20).

$$\frac{\delta U}{\delta n} = ax + 2bx^2n \qquad (20)$$

If we set the derivative (20) equal to zero, we may determine the number of plants which maximizes the relative production function. This is determined in (21). No doubt, the solution is unique. This expression will soon be used.

$$n_0 = -\frac{a}{2bx} \qquad (21)$$

The mixed second order derivative of the relative production function with respect to the number of plants, n, and the survival probability, x, is given in (22). We find obvious similarities between (20) and (22).

$$\frac{\delta^2 U}{\delta n \delta x} = a + 4bxn \tag{22}$$

Assume, for a while, that every plant survives. Then, we get:

$$(x=1) \rightarrow \frac{\delta U}{\delta n} = a + 2bn \tag{23}$$

$$(x=1) \rightarrow \frac{\delta^2 U}{\delta n \delta x} = a + 4bn \tag{24}$$

In this situation where x = 1, the marginal relative production function (23) is strictly positive for values of n < -a/2b, equal to zero for n = -a/2b and strictly negative for n > -a/2b. The functions (23) and (24) have the same strictly positive value for n = 0. Furthermore, the negative slope of (24) in the n dimension is 2 times the negative slope of (23) in the same dimension. We may conclude that the sign of (24) can be described via (25), (26) and (27).

$$\left(x=1,\ n<\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta^2 U}{\delta n \delta x}>0\right) \tag{25}$$

$$\left(x=1,\ n=\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta^2 U}{\delta n \delta x}=0\right) \tag{26}$$

$$\left(x=1,\ n>\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta^2 U}{\delta n \delta x}<0\right) \tag{27}$$

We recall equation (11). Clearly, when x increases, $\delta U/\delta n$ increases, is unchanged or decreases. Now, we reconsider (7), (25), (26) and (27). Obviously we can draw the conclusions presented in (28), (29) and (30). We may conclude our latest findings in the following way: If the optimal number of plants is low, then the optimal number of plants decreases if the death probability increases. If the optimal number of plants is high, then the optimal number of plants increases if the death probability increases. Compare Figure 4..

The "common sense" solution that we should always compensate for increasing plant mortality by increasing the number of plants is wrong.

The marginal plant cost, the timber net price, the rate of interest in the capital market, the site index and the initial survival probability determine in which direction we should change the optimal number of plants in the presence of increasing plant mortality.

$$\left(x=1,\ n^*<\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta n^*}{\delta x} > 0\right) \tag{28}$$

$$\left(x=1,\ n^*=\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta n^*}{\delta x}=0\right) \tag{29}$$

$$\left(x=1,\ n^*>\frac{n_0}{2}\right) \rightarrow \left(\frac{\delta n^*}{\delta x}<0\right) \tag{30}$$

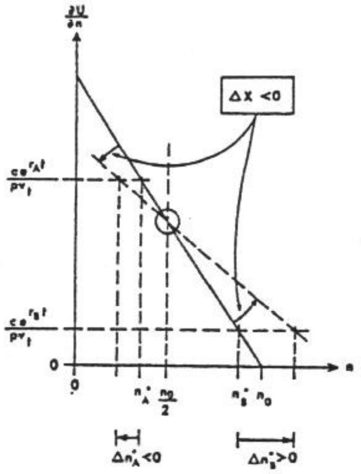

Figure 4.
Changes in the optimal number of plants when the death probability increases from 0 (the survival probability x decreases from 1). We observe that the optimal number of plants increases (decreases) if the initially optimal number of plants is high (low).

6. The optimal intensity in Norway spruce in northern Sweden

This section will concentrate the attention to Norway spruce plantations in northern Sweden. We will assume that thinnings are not profitable (which often can be shown to be the case in the area) and are not undertaken. In any case, the optimal number of plants per hectare is low according to the preliminary optimizations. For this reason, it is obviously physically possible to avoid thinnings if this is desirable from economic points of view.

Clearly, one could utilize complicated and detailed growth functions. However, the intention of this section is to make a complete analysis of the problem in a way which is easy to control and modify by the reader. All used assumptions are based on information which is found in the reference list. The choice of all parameter values in this section will be discussed in detail.

The sensitivity of the optimal investment intensity to the different parameter values will be analyzed. It will turn out that the applied transparent approach is not obviously worse than optimization based on very complicated growth functions. It is not very critical to the derived results if we make some growth estimation errors (The growth model is simple and rough compared to many modern growth models.). It is very important, on the other hand, to make precise estimations of the economic parameters, in particular the net price and the rate of interest.

6.1. The relative production function

Now the time has come to look closer into the real world situation. The just presented methods will be applied to a plantation in northern Sweden. First of all we have to determine the functional form and the parameters of the relative production function U(xn). We assume that the function estimated in Norway by Braastad (1983), shown in Figure 2., is relevant also in Sweden. We will assume that the distribution of plants after the initial death of some young plants is random and that the stand can be described as "regularly spaced" according to the definition by Braastad.

Braastad also reports a similar function for irregularly spaced stands which is slightly different. Of course, that function may be used instead where appropriate. One way to search for approximating functions is to try poynomials of different degrees, starting from the lowest possible degree. It is essential to make the polynomial approximation satisfy some important properties of the original function.

First, it is reasonable to let U = 0 when the number of stems, N, is 0. This result is obtained if there is no constant in the polynomial.

Second, U has its maximum when the number of stems is 2000. We let the first order derivative of U with respect to N be zero for N = 2000. Third, since the maximum of U is 1, we know that U should take the value 1 when N = 2000. These conditions are sufficient to determine a second order polynomial. In the following equations in this section, n is expressed in the unit 1000 plants per hectare. The following function is found:

$$U_2(xn) = 1(xn) - \frac{1}{4}(xn)^2 \qquad \textbf{(31)}$$

If we want to use a more flexible tool, we have the third order polynomial. Then, we need one more observation in order to determine the function. We select the value of U when there are 1000 stems per hectare. According to Figure 2., U(1000) = 0.874. The third order polynomial becomes:

$$U_3(xn) = 1.496(xn) - 0.746(xn)^2 + 0.124(xn)^3 \qquad \textbf{(32)}$$

However, since the behaviour of the function is important to describe in detail for low values of the stand density, we want to use the fact that U takes the value 0.650 when there are 500 stems per hectare. U(500) = 0.650. The fourth order polynomial becomes (33).

$$U_4(xn) = A(xn) + B(xn)^2 + C(xn)^3 + D(xn)^4 \qquad \textbf{(33)}$$

The parameters are given in (34).

$$A = 2.0151095,\quad B = -1.7842,\quad C = 0.772882,\quad D = -0.1297725 \qquad \textbf{(34)}$$

The latest function (33) is found to approximate Figure 2. very well. It is selected for the coming analysis and is found in Figure 5..

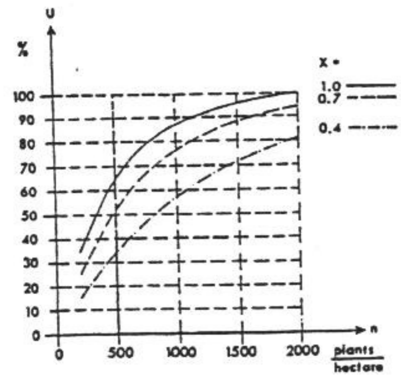

Figure 5.
The relative production function. Approximation via a fourth order polynomial.

The marginal contribution is (35).

$$\frac{\delta U}{\delta n} = Ax + 2Bx^2n + 3Cx^3n^2 + 4Dx^4n^3 \qquad (35)$$

The marginal contribution is plotted in Figure 6.

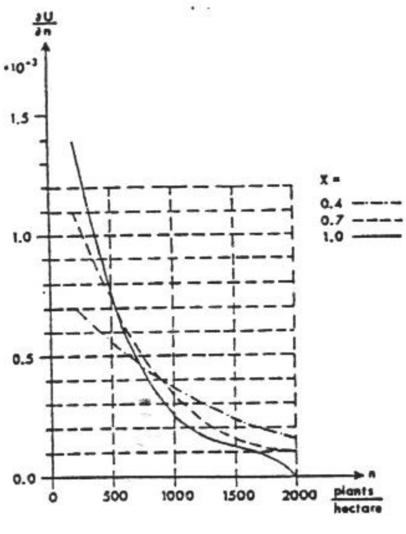

Figure 6..
The marginal contribution to the relative production.
The first derivative of the forth order polynomial approximation.

We may compare the qualitative properties of Figure 6. to those of Figure 4.. In both cases, the different functions representing different levels of plant survival, intersect. Hence, also in this more complicated case with an empirically justified fourth order approximation of the relative production function, this is true:

- If the optimal number of plants is low, and the survival probability decreases marginally, then we should have less plants.

- If the optimal number of plants is high, and the survival probability decreases marginally, then we should have more plants.

What do we mean by "low" or "high" numbers of plants in this context? Well, the answer is more complicated than before. The boundaries can be exactly calculated and given in analytical form. It is much easier, however, to look at the graphs in Figure 6..

6.2. Other relevant empirical information

As we recall from (3), when we optimize the number of plants, we need to know the values of the following parameters: The marginal plant cost, c, the rate of interest, r, the harvest age, t, the net price per cubic metre, P, and the volume per hectare, V, at the age of harvest (if the number of plants that survive is 2000).

The cost per plant

Andersson (1993) estimates that the marginal cost (per plant) is in the interval 1.60 SEK - 2.00 SEK in northern Sweden. The value 2.00 SEK holds for small reforestration areas and 1.60 holds for large areas. The cost of the plant only, included in the marginal cost, is 1.00 SEK for the species Norway spruce and Scots pine.

The reason for the economy of scale is that efficient machines, which help the labour to carry the plants, are profitable to use and transport to the site when the reforestration area is large. Andersson (1993) states that the marginal cost generally is higher in the rest of Sweden.

The National Board of Forestry (1992, Table 13.1) reports that the average cost of artificial and natural regeneration is 5 511 SEK per hectare in Sweden. If we include the cost of site preparation, the average regeneration cost is 7 347 SEK. This information is not inconsistent with the information gained from Andersson (1993) since 2000 plants per hectare represents a typical Swedish plantation.

The time of the final felling

The National Board of Forestry (the forest act) (1987) includes detailed restrictions concerning the lowest allowed age of the trees when the final felling takes place. In the rest of this section, we will assume that the species is Norway spruce and that the site index is typical in Northern Sweden, G16. In this case, the trees must not be harvested before the age of 120 years. When the rate of interest is higher than 1%, one can mostly show that the economically optimal rotation age is lower than 120 years. Hence, we will assume that the harvest age, t, is exactly 120 years in this example.

The volume per hectare at the time of the final felling

Sveriges Skogsvårdsförbund (1978), page 425, informs us that the total production of a Norway spruce G16 stand until the age of 120 years is 241 cubic metre per hectare. Then, the total thinning of 57 cubic metre is included. The initial number of stems per hectare is 1698. It is well known that the total growth is only marginally reduced via weak thinnings. Compare Figures 1. and 2.. A rough estimate is that the volume per hectare at the age of 120 years is 250 cubic metre (if the initial number of stems per hectare is 2000 and no thinnings are undertaken).

According to The National Board of Forestry (1992), (Table 6.8) the mean annual fellings in the period 85/86 - 89/90 were 9.9 million cubic metres in northern Sweden and 61.2 million cubic metres in all of Sweden.

The forest land area subject to final harvest (Table 6.9) was
48 000 hectares in northern Sweden and 184 000 hectares in all of Sweden during the same period.

We may divide the harvest volumes by the harvest areas and get the following ratios:
- Northern Sweden, 206 cubic metre per hectare
- All of Sweden, 333 cubic metre per hectare.

The net price per cubic metre in the final felling

The National Board of Forestry (1992, Figure 13.1) tells us that the following costs and prices per cubic metre were true in Sweden during the harvest season 1990/91: Gross value = 290 SEK, Harvest cost = 127 SEK, Stumpage value = 163 SEK.

We will assume that the net price in our example is 150 SEK per cubic metre. In northern Sweden, the distance from the forest stand to the industry is usually large. The volume per hectare is usually low. These are two reasons why the costs per cubic metre on the average are higher than in the rest of Sweden.

Does the spacing matter very much to the quality? Does the quality influence the price very much?

The National Board of Forestry (1992, Table 11.1) shows that the price of pine timber is reduced by 35% if the quality is reduced from high (OS) to low (V).

The price of spruce timber is only reduced by 9% if the quality is reduced from high to low. The quality is mainly determined by the size and number of branches per log. It is not exactly known how sensitive the quality is to the number of plants per hectare. In a dense stand, we get smaller branches. On the other hand, the logs will become smaller. Furthermore, there are indications that with low numbers of plants per hectare (which are the results of the preliminary optimizations in this paper), marginal changes in the spacing will not affect the quality distribution very much. The quality will be of the lower level in any case.

In our example, where we study spruce, the quality premium is, as mentioned, only 9%. Hence, we will in the rest of this section assume that the quality effects of the spacing may be forgotten.

We should be aware that the net price varies very much with the location. In places very far from the mills, the net price is close to zero or negative. There, we should be aware that also small changes in the net price, according to Figure 3., strongly affect the optimal number of plants. If the net price changes with a particular percentage, this influences the optimal number of plants exactly as much as if the volume per hectare in the final felling (with 2000 plants per hectare) changes with the same percentage.

In distant places, where the harvest and transportation costs are of the same order of magnitude as the revenues, also small absolute changes in the price level cause dramatic relative changes in the local net price. The local net price may often change sign. We should, for these reasons, accept the regional differences and the importance of the economic environment and adjust the investment intensity accordingly. On the other hand, which will be discussed in the following sections, we can not be sure that our predictions of the future net price levels in different regions are correct. The investment intensities should be based on this insight.

The rate of interest

The rate of interest is the most difficult parameter. The fundamental importance of this parameter to most values and decisions in forestry, such as the length of the rotation age, has been discussed earlier by for instance Lohmander (1990b).

In the present example, we study prices and costs in real terms. We should also utilize the real rate of interest. In different historical periods and in different countries, the rate of interest has taken very different values. In our optimization, we should as always in present value calculations, use the rate of interest of the best alternative investment.

The real rate of interest will in the example (without any empirical support!) be given the values 1%, 2% and some marginal changes from these values. If the best alternative gives you more or less than 1% or 2% over a 120 year period, the relevant figure should of course be selected.

6.3. The optimal solution and the sensitivity to the assumptions

Via (3) and Figure 6., we may derive the results found in Table 1.

						Opt # plants		
							x=	
c	r	t	P	V	$\delta U/\delta n$ *10^-3	0.4	0.7	1.0
2	1	120	150	250	.18	1830	1380	1180
2.2	1	120	150	250	.19	1770	1360	1150
2	1.1	120	150	250	.20	1700	1320	1130
2	1	132	150	250	.20	1700	1320	1130
2	1	120	165	250	.16	1980	1500	1300
2	1	120	150	275	.16	1980	1500	1300
2	2	120	150	250	.59	430	640	600
2.2	2	120	150	250	.65	290	570	560
2	2.2	120	150	250	.75	(100)	480	500
2	2	132	150	250	.75	(100)	480	500
2	2	120	165	250	.53	560	710	660
2	2	120	150	275	.53	560	710	660

Table 1.
The optimal number of plants per hectare. Row 1 and row 7 are the "standard cases" where the parameters suggested by the investigation described in the text are used. In the other rows, one parameter at a time has been changed from the "standard case value" by 10%.

Some of the results which we partly find in Table 1. are the following: The optimal number of plants is extremely sensitive to the rate of interest. If the rate of interest 3% is used, the optimal solution is to plant less than 100 plants per hectare. In such cases, however, plantations in the area are questioned. Seeds from neighbour stands will most likely give us a much more dense new forest. A deeper investigation of the alternative natural regeneration is strongly suggested.

If the rate of interest changes from 2% to 1%, the optimal number of plants increases very much. A highly interesting and perhaps surprising conclusion, however consistent with the qualitative analysis, is that:
If the rate of interest is 2%, then we should have less plants if the plant survival probability decreases from 100% to 40%. If the rate of interest is only 1%, then we should have more plants if the plant survival probability decreases.

All of the results from the qualitative analysis in the earlier sections hold also in the numerically specified example of Norway spruce in northern Sweden.

7. The optimal intensity and the price risk

In this section, we will study a first version of an adaptive optimization model. The first stage decision is the number of plants. When that decision is taken, we are aware that the future timber price may be higher, equal to or lower than the present price. We assume that the future price is distributed according to a probability distribution. The second stage decision is: Harvest or leave the stand for ever. The simple rule of the second stage decision is that if the harvest profit (including the released land value) is positive, then we harvest. Otherwise, we just leave the stand. In mountain areas in particular, it sometimes happens that the price is lower than the harvest and transportation cost per cubic metre. In fact, there are always some geographical regions where the costs are at least as high as the revenues from harvesting. The boundaries of these regions however shift over time as the world market prices, the infra structure and the technology in industry and transportation change.

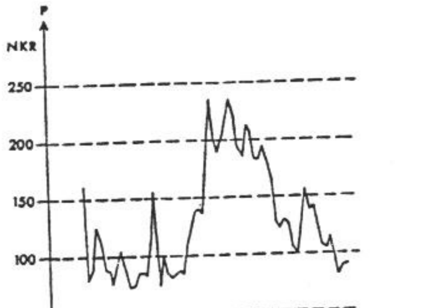

Figure 7.
Real net prices in Norway.
(Mean price - variable costs) deflated by consumer price index (The year 1979 has the index 100.) , Norwegian crowns per cubic metre. Assumptions: Norway spruce, 35% high quality timber, 30% low quality timber, 35% pulpwood.
Source: Lohmander (1987).

In Figure 7., we observe that the net price may change considerably over a rotation period. Afterwards, we may say that the high profitability in forestry during the period 1950 to 1965 was caused by the economic expansion and building activities in Europe after the second world war. However, in 1920, we could hardly predict the second world war and its economic consequences.
In 1973, we experienced the "oil crisis". We could not predict the behaviour of the oil cartel, the oil crisis and the consequences for the price of wood. Wood is an important energy substitute.

Let us turn to the description of prices as stochastic processes!
More explicitly, we assume that there are s possible net price levels which can be given the order 1,...,m, m+1,...,s. The price levels 1,...,m are below μ and the rest of the price levels are above μ. (μ is the value of P which makes the term in the right hand side bracket of (36) take the value zero.) The probability of a particular price level, P_k, is denoted by g_k. The expected optimal present value is given in (36).

$$\max_{n} \pi_1(n) = -F - cn + e^{-rt}\sum_{k=m+1}^{s}\left[P_k U(xn)V_t + L\right]g_k \tag{36}$$

A (slightly restricted form of a) mean preserving spread, compare Rothschild and Stiglitz (1970), may be defined by (37), (38), (39) and (40).

$$dE(P) = g_A\, dP_A + g_B\, dP_B = 0 \tag{37}$$

$$1 \le A \le m, \quad m+1 \le B \le s \tag{38}$$

$$g_A > 0, \quad g_B > 0 \tag{39}$$

$$dP_A < 0, \quad dP_B > 0 \tag{40}$$

The above equations imply that we take some probability density from the lower (left) part of the distribution, g_A, and move that further down (to the left), $dP_A<0$. At the same time we take some probability density from the upper (right) part of the distribution, g_B, and move that to the right, $dP_B>0$. The expected price is not changed in the process. By this method, it is possible to define and describe increasing price risk in a practical way. We observe in particular that the expected present value, our objective function (36), is not affected by the change in the low price value. The high price shift, on the other hand, affects the value of the objective function,

The first stage (investment intensity) first order optimum condition is given in (41).

$$\frac{\delta \pi_1}{\delta n} = -c + e^{-rt}\frac{\delta U}{\delta N} x V_t \sum_{k=m+1}^{s} P_k g_k = 0 \quad (41)$$

We observe in (42) that we have a unique maximum.

$$\frac{\delta^2 \pi_1}{\delta n^2} = e^{-rt}\frac{\delta^2 U}{\delta N^2} x^2 V_t \sum_{k=m+1}^{s} P_k g_k < 0 \quad (42)$$

From (7) we remember that we have to derive mixed second order derivatives in order to determine in what direction the optimal number of plants is affected by changing parameter values. When we investigate the effects of increasing price risk, (43) is relevant.

$$\frac{\delta^2 \pi_1}{\delta n \delta P_B} = e^{-rt}\frac{\delta U}{\delta N} x V_t \, g_B > 0 \quad (43)$$

Clearly, the optimal number of plants increases if the future price risk increases, which is also stated in (44).

$$\frac{\delta n^*}{\delta P_B} > 0 \quad (44)$$

How is the objective function affected by increasing price risk? We have to investigate this via equation (45). Let θ be the sign of total derivatives!

$$\frac{\theta \pi_1^*}{\theta P_B} = \frac{\delta \pi_1^*}{\delta P_B} + \frac{\delta \pi_1^*}{\delta n}\frac{\delta n^*}{\delta P_B} \quad (45)$$

However, since n is optimally chosen, the second part of the expression (45) is equal to zero. Hence, we have (46):

$$\frac{\theta \pi_1^*}{\theta P_B} = e^{-rt} U(xn^*) V_t g_B > 0 \quad (46)$$

The interesting conclusions from this section are that the optimal number of plants and the expected present value of the investment are strictly increasing functions of the future price risk.

If the future price happens to become very high, then the marginal value of the plant and the total profit become high. If, on the other hand, the future price becomes very low, then we may just leave the stand. The harvest profit (including the released land value) is hence bounded from below by the limit zero. Hence, the expected present value increases from from increasing price variability. In mountain areas, far from the industry, these issues and results deserve strong attention.

8. The adaptive multi stage decision problem

Now the time has come to approach a more detailed description of the available options. The intertemporal structure of information and decisions becomes important. We will assume that the first stage decision, the investment intensity, is of the same kind as before. The harvesting decision, on the other hand, is a sequential, multi period problem. The timber price (net price) is a stochastic process which is sequentially observed. This process is stationary and time is discrete.

The time distance between the individual periods in the problem is sufficiently long to make sure that the prices in different periods can be described as independent random variables. Figure 8. gives a modern example of such a "random price series" from Sweden.

We assume that we can approximate these random prices by a discrete probability distribution. The probabilities are denoted by g. As in the latest section, there are s different possible price levels. The index of the price is k. These possible price outcomes can be ordered from the lowest (k = 1) to the highest possible price (k = s).
In period t, we can select to harvest or to wait at least one more period. We can derive the optimal reservation price, q, the price which makes the present value of instant harvesting equal to the expected present value of waiting at least one more period. The explicit derivation of that reservation price will not be made in this section. In earlier publications, such as Lohmander (1987), (1988b) etc. such derivations are made. In the numerical section 9. of this paper, the reservation prices will also be derived.

Equation (47) contains a partly simplified version of the relevant optimization problem. (There, t is the first period when harvesting is possible.) The optimal reservation price q is such that $\alpha < q < (\alpha+1)$. If we select to harvest, then we get the present value of the harvest and the released land value. If we wait at least one more period, then we get the expected present value in the next period. That expected present value is denoted by W_{t+1}. The exact value of W_{t+1} may be calculated via explicit stochastic dynamic programming. That is not necessary in this qualitative discussion. Calculations of a similar type can be found in Lohmander (1987), (1988b) and elsewhere. In the numerical section 9. of this paper, W_{t+1} will be numerically calculated via stochastic dynamic programming.

$$\max_{n} \ \pi_2(n) = -F-cn+\sum_{k=1}^{\alpha} W_{t+1}(n)\, g_k + e^{-rt}\sum_{k=\alpha+1}^{s} [P_k U_t(xn)\, V_t+L] g_k \quad (47)$$

Considering the structure of the problem, we may optimize the number of plants. This is done via (48).
We should be aware that the derivative of the expected present value in the next period with respect to the number of plants can be shown to be positive.

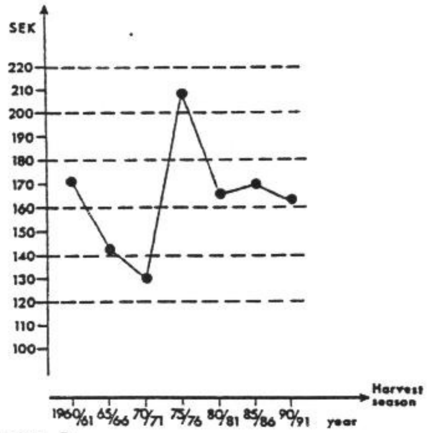

Figure 8.
Real net round wood prices in Sweden (price minus harvest cost per cubic metre). The prices have been adjusted via the consumer price index series to the price level of 1990. The mean price is 164.60 SEK and the standard deviation (with 6 degrees of freedom) is 24.30 SEK. Sources: Statistiska Centralbyrån (1991), The National Board of Forestry (1992), Table 13.1.

Such derivations are however very page consuming and will not be made here. The intuitive explanation is simply that if the number of plants increases, the harvest profitability increases in every future period. Hence, the derivative should be positive.

$$\frac{\delta \pi_2}{\delta n} = -c+\sum_{k=1}^{\alpha} \frac{\delta W_{t+1}(n)}{\delta n}\, g_k + e^{-rt}\sum_{k=\alpha+1}^{s} P_k \frac{\delta U_t}{\delta N} x V_t g_k = 0 \quad (48)$$

In a similar way we may conclude that the second order derivative of the expected future present value with respect to the number of plants is strictly negative. This is the case since the relative production function is a strictly concave function of the number of plants in every time period.

$$\frac{\delta^2 \pi_2}{\delta n^2} = \sum_{k=1}^{\alpha} \frac{\delta^2 W_{t+1}(n)}{\delta n^2} g_k + e^{-rt}\sum_{k=\alpha+1}^{s} P_k \frac{\delta^2 U_t}{\delta N^2} x^2 V_t g_k < 0 \quad (49)$$

As a consequence, the optimum is a unique maximum.

Now, we come to the most interesting question: In what direction is the optimal number of plants affected by increasing risk in the price probability distribution in period t? Again, we use a "mean preserving spread" to describe the increasing risk.

One possible price, the price level with index D, is moved to a higher level. This price is above the reservation price in period t. Another price, which is lower than the reservation price in period t, is moved to a lower level.

In that way the expected price in period t remains unchanged. Only the higher possible price level affects the objective function, in a way which is indicated by (50).

$$\frac{\delta^2 \pi_2}{\delta n \delta P_D} = e^{-rt} \frac{\delta U_t}{\delta N} x V_t g_D > 0 \qquad (50)$$

Differentiation of the first order optimum condition gives (51):

$$\frac{\delta^2 \pi_2^*}{\delta n^2} dn^* + \frac{\delta^2 \pi_2^*}{\delta n \delta P_D} dP_D = 0 \qquad (51)$$

In (52) we find that the optimal number of plants is a strictly increasing function of increasing price risk in period t.

$$\frac{\delta n^*}{\delta P_D} = - \frac{\left[\frac{\delta^2 \pi_2^*}{\delta n \delta P_D} \right]}{\left[\frac{\delta^2 \pi_2^*}{\delta n^2} \right]} > 0 \qquad (52)$$

Since the number of plants is optimally chosen, we instantly find that the optimal expected present value also is a strictly increasing function of the price risk in period t.

$$\frac{\theta \pi_2^*}{\theta P_D} = e^{-rt} U_t (x n^*) V_t g_D > 0 \qquad (53)$$

9. The numerical version of the adaptive multi stage model

Here, a numerical version of the adaptive multi stage optimization model will be presented. The program calculates the optimal expected present value per hectare for different numbers of stems, N. When the number of stems and the site index (H40, height in meters at the age of 40 years) is known, the volume per hectare is calculated for every possible future harvest period. The periods are 5 years long and in the last period under consideration the trees are 200 years old. The optimization method is stochastic dynamic programming.

The net price per cubic metre is assumed to follow the function (54):

$$P = y_0 + y_t t + y_N N + e_t \qquad (54)$$

The price is a stochastic process. The expected value of the error component, epsilon, is zero. The probability density function of the error is a uniform or a triangular distribution. The net price probability density function $f_t(P)$ can be determined from (54).

For each possible choice of N, we may describe the objective function (the investment cost is excluded) in an arbitrary period by (55):

$$W_t = \int_{-\infty}^{q_t^*} W_{t+1} f_t(P)\, dP + \int_{q_t^*}^{\infty} e^{-rt} [P V_t + L]\, f_t(P)\, dP \qquad (55)$$

Compare the discrete price version of (55) shown in (47). In (47), on the other hand, the investment costs are included in the expression. In (55), we assume that the investment level, the number of plants, is fixed for each optimization.

Of course the objective function is reduced by the investment cost in the optimizations for every possible investment level. This is however not explicitly seen in equation (55).

In this version of the problem, the reservation price in period t is denoted q_t. If the price is lower, then we wait. If the price is higher, then we harvest. If the price is exactly q_t, then we are indifferent between the two possible decisions: to harvest and to wait.

Note that from now on, we do not explicitly treat the number of stems in the recursive equations. We assume, on the other hand, that the growth of the volume is a function of the volume, the number of stems per hectare and the site index.

Since the number of stems is constant within the dynamic optimizations, the volume per hectare can be described as a function of time only. Now, we should optimize the reservation price q_t:

$$\frac{\delta W_t}{\delta q_t} = \left[W_{t+1} - e^{-rt}(q_t V_t + L)\right] f_t(q_t) = 0 \quad (56)$$

We assume, in this derivation, that all prices have strictly positive probabilities, $f_t(q_t) > 0$. Then, we find that the term in the square bracket of the expression in (56) must be equal to zero. The second order condition of a unique maximum can be found to be fullfilled in (57):

$$\frac{\delta^2 W_t}{\delta q_t^2} = -e^{-rt} V_t f_t(q_t) < 0 \quad (57)$$

From (56) we may derive the optimal reservation price:

$$e^{-rt}\left[q_t^* V_t + L\right] = W_{t+1} \quad (58)$$

(58) leads to (59):

$$q_t^* = \frac{e^{rt} W_{t+1} - L}{V_t} \quad (59)$$

Now, we have too practical formulae, (55) and (59), which give us the optimal expected present values and the optimal reservation prices in the different periods via backward recursion.

9.1 The growth function of the numerical model

In order to support the more detailed multi stage optimization model, a more detailed growth model is needed. The report of Braastad (1983) contains several forest production tables from which interesting growth data may be collected. The ambition in this paper is to extract general, numerically useful and efficient growth models that can be understood from biological theory.

It is found that a rather simple functional form of the growth function fits the data very well:

$$\frac{\delta V}{\delta t} = a + bV + cV^2 + d\sqrt{NV} \quad (60)$$

Note in particular that time (age) is not an independent variable. If we know the present state, the volume and the number of stems per hectare, then the growth can be calculated. We have a Markov model.

We should expect that growth is strictly positive also for very low levels of the volume per hectare. In very young stands, the timber volume, is very low in relation to the mass of the (growth producing) needles. We should expect that $a > 0$.

As the volume increases from a very low initial value, the mass of the needles increases. Hence growth is an increasing function of the volume per hectare. We should expect that $b > 0$.

We expect a decreasing marginal return. The marginal (growth) value of the first volume (and needle) unit (tree) is higher than the marginal value of the later volume (and needle) units (trees). If the stand becomes very dense, the growth factors such as light and water become limiting. We expect that $c < 0$.

There is a positive synergy effect: The marginal (growth) value of the "volume", V, increases if the volume is distributed between many stems. If there are many trees distributed over the area, then the crowns more effectively utilize the sunlight and the roots use the available water with higher efficiency than if there are only a few big trees. We may also conclude that the marginal (growth) value of the number of stems, N, increases if these stems represent more volume (and needles). Again, we should expect decreasing returns to scale also in this situation. As a consequence, the square root shape of the function seems adequate. We should expect that $\hat{d} > 0$.

The growth function parameters a, b, c and d have been determined for the different site indices (according to Braastad) H40 = 11, 14, 17, 20 and 23. (H40 = τ means that the trees reach the height τ when the age is 40 years.) These are found in Table 2.. The value of R^2 varies in the interval 90% - 99%. In all the cases, the estimated parameters have the expected signs and are strongly significant.

Of course, we should be aware that the very high values of the correlation coefficients partly may be explained by the fact that the "data" is derived from production tables. A large fraction of the random variation in the original empirical data is most likely excluded when we replace the original data by that tables. However, if we accept the production tables, we should also accept (60).

Site (H40)	a	b	c	d	R^2	SEE
11	1.356 (13.7)	.9179 (6.46)	-.3213 (-7.95)	0.6467 (15.3)	.957	.168
14	1.380 (15.5)	.7662 (7.49)	-.2908 (-13.4)	1.118 (31.3)	.989	.142
17	1.812 (13.4)	.3668 (2.94)	-.2233 (-11.2)	1.577 (27.1)	.979	.279
20	1.611 (12.3)	.4309 (3.70)	-.2800 (-16.8)	2.153 (39.8)	.990	.256
23	1.485 (10.1)	.5948 (4.75)	-.3296 (-20.4)	2.704 (42.5)	.993	.295

Table 2.
Estimated parameters of the growth function (60). The t-values are written in brackets below the parameter values. The adjusted R^2 values and the standard deviation of the error (SEE) are given for each function. The estimations are based on the following numbers of observations: 42 (H40 = 11m), 36 (H40 = 14m), 42 (H40 = 17m), 39 (H40 = 20m) and 36 (H40 = 23m). The following units and scales are used: Growth (1 cubic metre per hectare and year), volume, V, (100 cubic metre per hectare), number of stems, N, (100 stems per hectare). The variable V, volume per hectare, is assumed to take the value (volume before growth + volume after growth)/2 in the estimations. All data are found in the appendix. In Figure 9. we find a plot of the estimated growth function and in Figure 10. we can follow the time path of the volume per hectare.

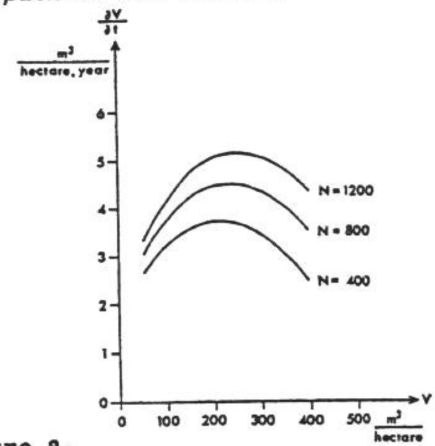

Figure 9.
The estimated growth function. Site index H40 = 11m. Norway spruce.

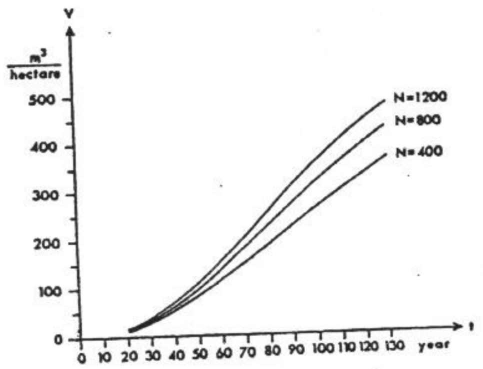

Figure 10.
The time path of the volume per hectare according to the estimated growth function. Site index H40 = 11m. Norway spruce.

9.2. A case study with some numerical results

In order to derive results with the numerical model, it is necessary to make specific numerical assumptions. We let the expected net price take the value 164.60 SEK. This is the average net price in Sweden during the period

1960 - 1990 according to Figure 8. In the deterministic case, we will assume that this net price is true in every future period and optimize the decisions accordingly. We will however also utilize the variability information reported in Figure 8. The standard deviation of the net prices was found to be 24.30 SEK. In the stochastic case, we will assume that this degree of variation is true also in the future and optimize the decisions accordingly. (We should be aware that the estimated autocorrelation of the net prices in Figure 8. is very low and not significantly different from zero. Hence, we assume zero autocorrelation.)

More specifically, the assumptions are: The site index (H40) is 11m, the species is Norway spruce, and the expected net price per cubic metre is 164.6 SEK. The standard deviation of the net price per cubic metre is 0 SEK in the deterministic case and 24.30 SEK in the stochastic case. A uniform probability density function is used. (A triangular probability density function is also tested.) As a result, the maximum deviation from the expected value is 42.0876 (SEK). The fix plantation cost is 2000 SEK per hectare and the marginal plant cost is 2 SEK. The real rate of interest is 2.5 %.

The physical development of a stand with this site index is found in Figure 9. and Figure 10.. In the deterministic case, the optimal present value is 1 927 SEK per hectare and the optimal investment intensity is 600 stems per hectare. The harvest should take place at the age of 65 years. Note that this harvest age is very much lower than the lowest harvest age which is accepted by the present forest act in Sweden. In the stochastic case, the optimal expected present value is 2 653 SEK per hectare and the optimal investment intensity is 900 stems per hectare. Since we accept that future prices can not yet be perfectly predicted, we can not yet determine the optimal harvest age. The optimal reservation prices and the harvest age probability distribution are found in Figure 11. and Figure 12.. We find that the most likely harvest age is 60 or 65 years. However, it is possible that we should harvest ten years earlier or more than 30 years later! The future timber market will determine the optimal decision. It turns out that the qualitative results are the same for the two possible functional forms of the probability density function (even (uniform) distribution and triangular distribution).

The important conclusion, that we should increase the number of plants under the influence of future price risk, has been verified. We may conclude that the results from the numerical optimizations seem reasonable in the light of the qualitative analysis. Furthermore, the numerical dynamic multi stage model makes it possible to analyze problems in high detail.

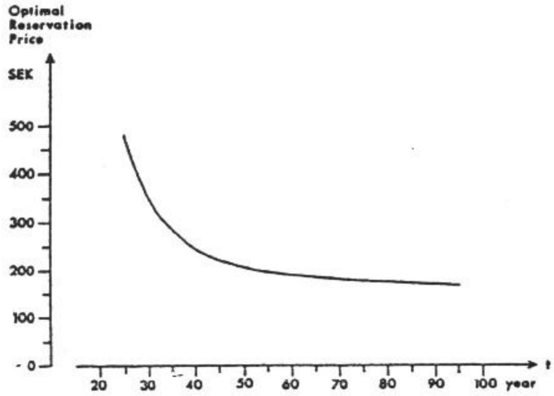

Figure 11.
The optimal reservation price in the stochastic case as a function of the age of the stand. Site index: H40 = 11m. Expected net price per cubic metre: 164.60 SEK.. Standard deviation of the net price: 24.30 SEK. Real rate of interest: 2.5 %

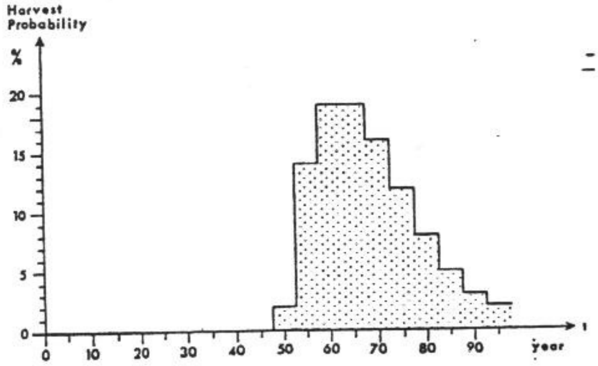

Figure 12.
The harvest age probability distribution in the stochastic case. The assumptions are the same as those reported in connection to Figure 11..

10. Conclusions

We can not make perfect predictions of the future state of the world and the round wood prices. We must accept this fact when we undertake our investments, the forest plantations.

We have found that the optimal initial decision, the number of plants per hectare, is an increasing function of the degree of future price risk. This general conclusion is true in the investigated adaptive cases. Several other general results have also been derived and reported.

The particular problems of spruce plantations in northern Sweden have been given special attention. Several explicit suggestions are given that may considerably increase the profitability. The optimal number of plants is very sensitive to the future price risk in the numerically investigated case. Furthermore, the optimal number of plants is sensitive to most other "traditional and deterministic" parameters of forest economics such as the rate of interest, the expected price level and the marginal plant cost. Very similar numerical results have been reported by Solberg and Haight (1991).
Are there strong reasons to assume that the reported results should have been very different if more detailed assumptions would have been made? Was it a mistake not to include thinnings in the analysis?

We may make a rough comparision with the results reported by Solberg and Haight (1991). They calculate the economically optimal forest management program for Norway spruce using deterministic assumptions. For every investigated site index, they find that the net present value of the forest investment is less than 3% higher if optimal thinnings are undertaken, than if there are no thinnings at all. They do not consider the fixed cost per thinning occation (the set up costs and the costs of machine and labour transportation) in their analysis. Still, they find that thinnings are not very profitable.

We may draw the conclusion that forestry without thinnings often gives a higher present value than forestry with thinnings, in particular if there is no "adaptive reason" why we should make thinning decisions. This conclusion is consistent with the results reported by Bjurulf and Freij (1986) and by Wieslander (1986).

Very important reasons why we should make species selective thinnings in multi species forestry, have been reported by Lohmander (1992a), (1992c) and (1993) and by Carlsson (1992).
We may conclude, in the single species case, that forestry without thinnings may be all right in several cases.

In the future we may want to consider the effects of spacing on the timber quality in detail. In the numerical analysis of this paper, it was possible to show that such details are presently not very interesting.

Solberg and Haight (1991) came to a similar conclusion. One method to obtain very dense stands rather cheaply, stands that may give very high timber quality, is of course natural regeneration. Lähde (1991), Ackzell (1992), Johansson (1992) and Malimbwi et al. (1992) are examples of this interest. As long as we are interested in plantation forestry, however, we have to accept that the economically optimal investment intensity often is very low. Norway spruce will give rather good timber quality anyway and the methods suggested in this paper are appropriate.

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Numerical Appendix

In this appendix, the multi stage adaptive optimization program is included. It is written in Quick Basic and may be used on a personal computer. Most parameters may be chosen by the user. You just have to answer questions when the program starts. Other parameters may be altered by modifications of the source code. This should not be too difficult since the code contains explaining remarks. Furthermore, the parameter names correspond to those used in the main text of this paper.

Typical results from this program are presented in the main text.

```
REM RISINT9.bas
REM Lohmander Peter 93-02-05, 10.23

DIM V(41), W(41), QOPT(41), PM(41), PMO(41), PLEFT(41)
DIM K(5, 4), VOL(200), STEMS(200)

CLS
REM L = Land value after harvest.
L = 2000
REM ST = Time after plantation, years, until the estimated
REM growth function starts to generate the volume development.
ST = 15
W(41) = 0
PRINT " RESULTS from RISINT9, by Lohmander Peter 93-02-05"
PRINT " **************************************************"
PRINT "          P(T,N) = PCON + PT*T + PN*N + ERROR(T)"
INPUT "  PCON, PT, PN = ", PCON, PT, PN
INPUT "  ERROR DISTR. = (1 = even, 2 = triangular)", DISTR
INPUT "     ERROR MAX. = ", ACON

INPUT "FIX and VARIABLE costs in plantation = ", FCPLANT, VCPLANT
INPUT "                        RATE OF INTEREST = ", R
INPUT "Do you want final economic output only ? (YES = 1, NO = 0)", EOO
IF EOO = 1 THEN GOTO 56
INPUT "Do you want to see the volume path ? (YES = 1, NO = 0)", VOLPATH
56 REM

REM Here, we will use a stem number dependent growth function.
INPUT "Site Index H40 = (11, 14, 17, 20 or 23) = ", SITE
S = 1 + (SITE - 11) / 3

REM Here, the growth function coefficients are feeded into
REM the matrix K(Siteind, Coef).
FOR SI = 11 TO 23 STEP 3: SITEIND = 1 + (SI - 11) / 3
FOR COEF = 1 TO 4
READ K(SITEIND, COEF)
NEXT COEF: NEXT SI

DATA 1.356, .9179, -.3213, .4467
DATA 1.380, .7762, -.2908, 1.118
DATA 1.812, .3668, -.2233, 1.577
DATA 1.611, .4309, -.2800, 2.153
DATA 1.485, .5948, -.3296, 2.704

REM Here, the stem number loop starts.
FOR NSTEMS = 400 TO 1200 STEP 100
N = NSTEMS
NIND = 1 + (N - 400) / 100
STEMS(1) = N
VOL(1) = 0

FOR T = 2 TO ST
REM Here, we give a very small volume to the area during the first
REM years of the plants. The figure will not affect the calculations.
VOL(T) = 1
STEMS(T) = STEMS(T - 1)
NEXT T

VOL(ST + 1) = .02 * STEMS(1)

FOR T = (ST + 1) TO 200
T0 = T - 1
VV = VOL(T0)

REM Thinning strategy: If V > 1000, then make a 20% thinning.
REM This, however, seldom happens.
STEMS(T) = STEMS(T0)
IF VV > 1000 THEN STEMS(T) = STEMS(T0) * .8
IF VV > 1000 THEN VOL(T) = VOL(T0) * .8

N0 = STEMS(T) / 100: V0 = VV / 100
growth = K(S, 1) + K(S, 2) * V0 + K(S, 3) * V0 ^ 2
growth = growth + K(S, 4) * (V0 * N0) ^ .5

VOL(T) = VOL(T0) + growth
NEXT T

IF EOO = 1 THEN GOTO 44
IF VOLPATH = 0 THEN GOTO 44
PRINT ""
PRINT "      YEAR   VOL/HA     STEMS        H40 = "; SITE; "m."
PRINT ""
PRINT USING "######.##"; 1; VOL(1); STEMS(1)
FOR T = 10 TO 200 STEP 10
PRINT USING "######.##"; T; VOL(T); STEMS(T)
NEXT T

INPUT "NEXT TABLE?", XX
44 REM

FOR T = 1 TO 40 STEP 1
TT = T * 5
V(T) = VOL(TT)
NEXT T

REM ******************************************************************

51 REM
IF EOO = 1 THEN GOTO 52
PRINT ""
PRINT "DISTR = "; DISTR; " ACON = "; ACON; " R = "; R;
PRINT ""
52 REM

REM ******************************************************************

FOR TT = 200 TO 5 STEP -5

T = TT / 5
QOPT(T) = (EXP(R * TT) * W(T + 1) - L) / V(T)
Q = QOPT(T)

REM Calculation of the price distribution parameters A and B.
EPRICE = PCON + PT * TT + PN * NSTEMS
A = EPRICE - ACON
B = EPRICE + ACON

REM Are the prices evenly distributed with support in the
REM interval (A,B)? Then the distribution parameter
REM DISTR = 1. If the prices have a symetric triangular
REM distribution with support in the interval (A,B), DISTR = 2.

IF DISTR = 2 THEN GOTO 100

REM -----------------------------------------------------------------

IF Q < A THEN GOTO 2


GOTO 3
2 REM
PM(T) = 1
W(T) = EXP(-R * TT) * ((A + B) / 2 * V(T) + L)
GOTO 10

REM -----------------------------------------------------------------

3 IF (A < Q AND Q < B) THEN GOTO 4
GOTO 5
4 REM
PM(T) = (B - Q) / (B - A)
W(T) = (1 - PM(T)) * W(T + 1)
W(T) = W(T) + PM(T) * EXP(-R * TT) * ((Q + B) / 2 * V(T) + L)
GOTO 10

REM -----------------------------------------------------------------

5 IF B < Q THEN GOTO 6
GOTO 10
6 REM
PM(T) = 0
W(T) = W(T + 1)

10 REM
REM ------------------ END OF EVEN DISTRIBUTION ----------------------

NEXT TT
GOTO 200

REM ******************************************************************
100 REM The case of a triangular price distribution
REM ******************************************************************

C = (A + B) / 2

IF Q < A THEN GOTO 102
GOTO 103
102 REM
PM(T) = 1
W(T) = EXP(-R * TT) * (C * V(T) + L)
GOTO 10

REM -----------------------------------------------------------------

103 IF (A < Q AND Q < C) THEN GOTO 104
GOTO 105
104 REM
PM(T) = 1 - (Q - A) ^ 2 / (2 * (C - A) ^ 2)
K11 = ((Q ^ 2 / 2 - A * Q) - (A ^ 2 / 2 - A ^ 2))
K12 = V(T) * C ^ 3 / 3 + (L - A * V(T)) * C ^ 2 / 2 - A * L * C
K13 = V(T) * Q ^ 3 / 3 + (L - A * V(T)) * Q ^ 2 / 2 - A * L * Q
K14 = -V(T) * B ^ 3 / 3 + (V(T) * B - L) * B ^ 2 / 2 - L * B ^ 2
K15 = -V(T) * C ^ 3 / 3 + (V(T) * B - L) * C ^ 2 / 2 - L * B * C
W(T) = W(T + 1) * K11 + EXP(-R * TT) * (K12 - K13 + K14 - K15)
W(T) = 1 / (C - A) ^ 2 * W(T)
GOTO 10

REM -----------------------------------------------------------------

105 IF (C < Q AND Q < B) THEN GOTO 106
GOTO 107
106 REM
```

```
PH(T) = (B - Q) ^ 2 / (2 * (C - A) ^ 2)
K11 = (C ^ 2 / 2 - A * C) - (A ^ 2 / 2 - A ^ 2)
K12 = (B * Q - Q ^ 2 / 2) - (B * C - C ^ 2 / 2)
K13 = -V(T) * B ^ 3 / 3 + (V(T) * B - L) * B ^ 2 / 2 + L * B ^ 2
K14 = -V(T) * Q ^ 3 / 3 + (V(T) * B - L) * Q ^ 2 / 2 + L * B * Q
W(T) = W(T + 1) * (K11 + K12) + EXP(-R * TT) * (K13 - K14)
W(T) = 1 / (C - A) ^ 2 * W(T)
GOTO 10

REM ------------------------------------------------------------------

107 IF B < Q THEN GOTO 108
GOTO 10
108 REM
PH(T) = 0
W(T) = W(T + 1)
GOTO 10

REM ------------------ END OF TRIANGULAR DISTRIBUTION ------------------

200 REM **** FORWARD CALCULATION OF HARVEST PROBABILITIES ************

PLEFT(1) = 1
FOR T = 1 TO 40
PHO(T) = PLEFT(T) * PH(T)
PLEFT(T + 1) = PLEFT(T) - PHO(T)
NEXT T

REM ********** RESERVATION PRICE RESULTS ARE PRINTED. ****************

IF EOO = 1 THEN GOTO 53
PRINT "      AGE      W(T)   RES.P. PROB(HARV)"
FOR T = 1 TO 19
TT = T * 5
UTP = QOPT(T)
IF UTP > (10 * PCON) THEN UTP = 99999
PRINT USING "#######.##"; TT; W(T); UTP; PHO(T)
NEXT T
INPUT "Next table ?", XX

53 REM

REM ********* IMPORTANT INVESTMENT INTENSITY RESULTS *******************
IF NSTEMS > 400 THEN GOTO 62
PRINT ""
PRINT "   NSTEMS      W(1) E(Present value)"
PRINT ""
62 REM
PROFIT = -FCPLANT - VCPLANT * NSTEMS + W(1)
PRINT USING "#########."; NSTEMS; W(1); PROFIT

NEXT NSTEMS
```

File = appenel.vp

1(4)

93-02-05

Empirical appendix

The data used in the estimations of the growth function parameters are found in this appendix. There are five different site indices: H40 = 11, 14, 17, 20 and 23.

The data have been constructed from the production tables published by Braastad (1983).

The variables are:

N	Number of stems per hectare.
V	Volume (cubic metre) per hectare.
dVdt	Volume growth (per hectare and year) until the next revision.
YN	Number of years until the next revision.

Source: Braastad, H., Yield level in Picea abies stands with low initial density and irregular spacing, Norwegian Forest Research Institute, 7/83, 1983

Site index: H40 = 11m

N	V	dVdt	YN
1200	38	3.7	5
1177	56	4.0	5
1154	75	4.0	5
1131	94	4.2	5
1109	114	4.4	5
1087	135	4.6	5
1066	157	4.7	5
1045	178	4.7	5
1025	200	4.7	5
1005	222	4.7	5
985	242	4.6	5
966	263	4.6	5
947	283	4.6	5
929	303	4.8	5
800	25	2.8	5
785	39	3.1	5
770	54	3.1	5
755	69	3.4	5
740	85	3.7	5
726	102	3.9	5
712	121	4.0	5
698	139	4.1	5
685	159	4.2	5
672	178	4.2	5
659	197	4.2	5
646	216	4.2	5
634	234	4.3	5
622	253	4.5	5
400	12	1.8	5
393	21	2.0	5
386	31	2.2	5
379	41	2.4	5
372	53	2.7	5
365	66	2.9	5
358	79	3.0	5
351	94	3.2	5
344	108	3.3	5
338	124	3.3	5
332	139	3.4	5
326	155	3.5	5
320	170	3.6	5
314	187	3.8	5

Site index: H40 = 14m

N	V	dVdt	YN
1200	53	5.5	5
1177	80	5.6	5
1154	107	6.1	5
1131	136	6.5	5
1109	167	6.8	5
1087	199	7.0	5
1066	232	7.1	5
1045	265	7.2	5
1025	298	7.1	5
1005	330	7.1	5
985	362	6.9	5
966	393	6.8	5
800	35	4.0	5
785	55	4.2	5
770	75	4.7	5
755	98	5.1	5
740	122	5.5	5
726	148	5.7	5
712	175	5.9	5
698	202	6.0	5
685	230	6.1	5
672	258	6.1	5
659	286	6.1	5
646	313	6.0	5
400	17	2.4	5
393	29	2.7	5
386	42	3.1	5
379	57	3.5	5
372	74	3.8	5
365	92	4.0	5
358	111	4.2	5
351	131	4.4	5
344	151	4.5	5
338	172	4.6	5
332	194	4.7	5
326	215	4.8	5

Site index: H40 = 17m

N	V	dVdt	YN
1200	41	6.7	3
1186	61	6.9	2
1177	75	7.4	5
1154	110	8.2	5
1131	150	8.9	5
1109	193	9.3	5
1087	237	9.5	5
1066	282	9.6	5
1045	327	9.5	5
1025	371	9.4	5
1005	414	9.3	5
985	456	9.1	5
966	496	8.9	5
947	535	8.7	5
800	27	4.8	3
791	41	5.0	2
785	51	5.4	5
770	77	6.2	5
755	107	6.9	5
740	140	7.3	5
726	175	7.6	5
712	211	7.8	5
698	248	7.9	5
685	285	8.0	5
672	321	7.9	5
659	358	7.9	5
646	393	7.8	5
634	428	7.7	5
400	13	2.7	3
396	22	2.9	2
393	27	3.3	5
386	43	3.9	5
379	62	4.5	5
372	84	4.9	5
365	107	5.2	5
358	132	5.5	5
351	158	5.7	5
344	184	5.8	5
338	212	5.9	5
332	239	6.0	5
326	266	6.0	5
320	294	6.1	5

Site index H40 = 20m

N	V	dVdt	YN
1200	59	8.7	3
1186	84	9.1	3
1172	111	9.9	2
1163	130	10.5	2
1154	150	11.2	3
1141	183	12.1	5
1119	241	12.6	5
1097	301	12.7	5
1076	361	12.5	5
1055	419	12.2	5
1034	476	11.9	5
1014	530	11.5	5
994	581	11.1	5
800	39	6.1	3
791	56	6.5	3
782	76	7.2	2
776	90	7.7	2
770	105	8.4	3
761	129	9.2	5
746	173	9.8	5
732	220	10.1	5
718	268	10.2	5
704	316	10.2	5
690	364	10.1	5
677	410	9.9	5
664	455	9.7	5
400	19	3.4	3
396	29	3.8	3
392	40	4.3	2
389	49	4.7	2
386	58	5.2	3
382	73	5.8	5
375	101	6.4	5
368	132	6.8	5
361	164	7.1	5
354	198	7.2	5
347	231	7.3	5
341	265	7.3	5
335	299	7.4	5

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N	V	dVdt	YN
1200	56	9.9	2
1191	76	10.2	2
1182	96	11.6	3
1168	130	12.7	2
1159	155	13.9	3
1146	195	15.3	5
1124	270	16.1	5
1102	346	16.1	5
1080	423	15.8	5
1059	497	15.2	5
1038	568	14.4	5
1018	634	13.6	5
800	37	6.9	2
794	51	7.2	2
788	65	8.3	3
779	89	9.3	2
773	107	10.3	3
764	137	11.6	5
749	193	12.5	5
735	253	12.9	5
721	315	12.9	5
707	376	12.7	5
693	435	12.4	5
680	492	12.0	5
400	18	3.8	2
397	26	4.0	2
394	34	4.8	3
390	48	5.5	2
387	59	6.2	3
383	77	7.2	5
376	112	8.0	5
369	150	8.6	5
362	191	8.9	5
355	233	9.0	5
348	275	9.0	5
342	318	8.9	5