Fundamental Economic Functions

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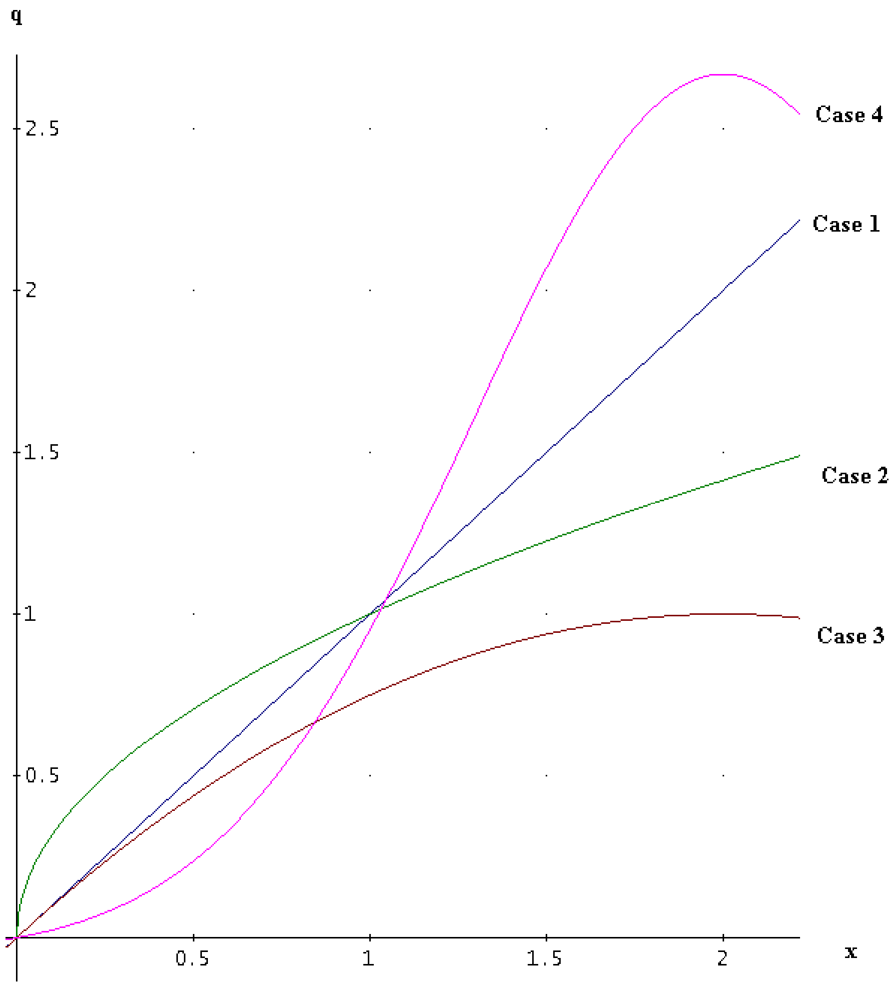


Figure 1. Alternative versions of the function q(x).

Case 1

$q = x$

$x = q$
$C(q) = F + rq$

Example:
$C = TC = 1 + q$

$$ATC = \frac{1+q}{q} = \frac{1}{q} + 1$$
$$AFC = \frac{1}{q}$$
$$AVC = \frac{q}{q} = 1$$
$$MC = \frac{dC}{dq} = 1$$

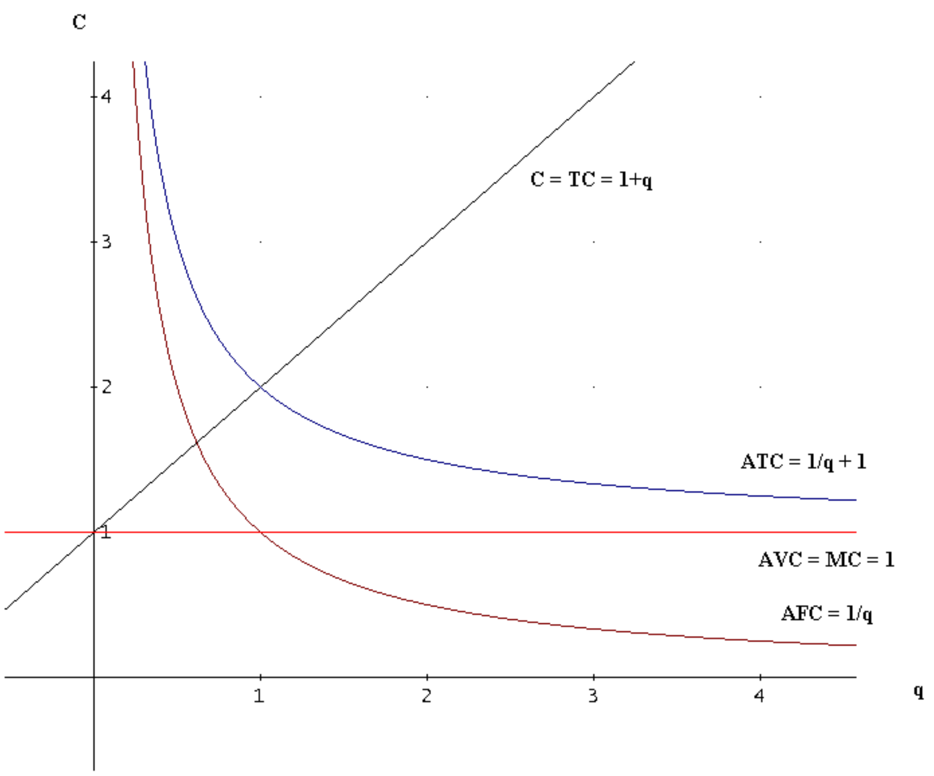


Figure 2. CASE 1.

Case 2

$q = \sqrt{x}$

$x = q^2$

$C(q) = F + rq^2$

Example:

$C = TC = 1 + q^2$

$$ATC = \frac{1+q^2}{q} = \frac{1}{q} + q = q^{-1} + q$$

$$AFC = \frac{1}{q}$$

$$AVC = \frac{q^2}{q} = q$$

$$MC = \frac{dC}{dq} = 2q$$

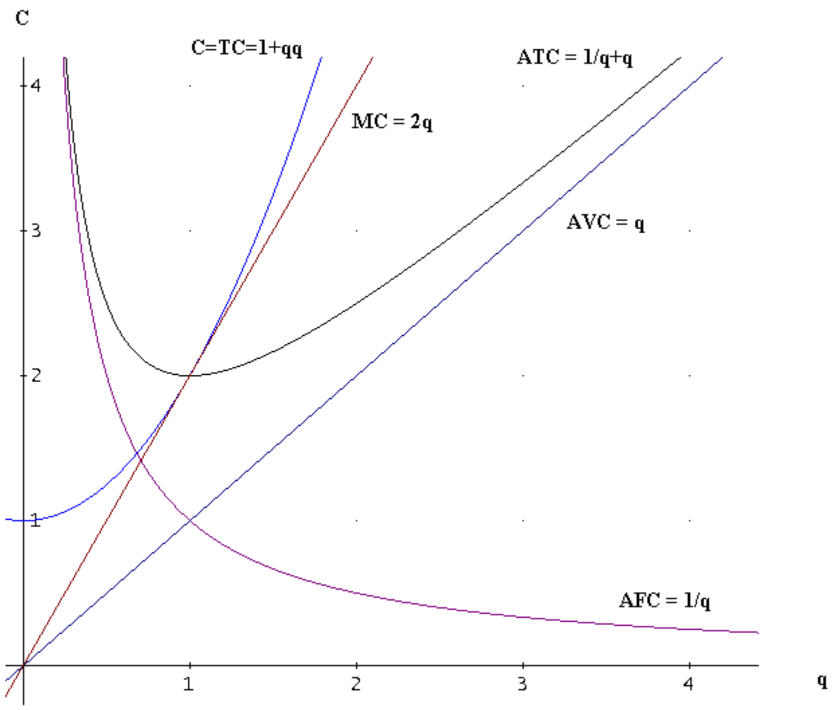


Figure 3. CASE 2.

Case 3

$$q = x - \frac{1}{4}x^2 \quad 0 \le x \le 2$$

$$-\frac{1}{4}x^2 + x - q = 0$$

$$x^2 - 4x + 4q = 0$$

$$x = -\left(\frac{-4}{2}\right) \overset{(+)}{-} \sqrt{\left(\frac{-4}{2}\right)^2 - 4q}$$

$$x = 2 - \sqrt{4 - 4q}$$

$$x = 2 - 2\sqrt{1-q}$$

$$x = 2 - 2(1-q)^{\frac{1}{2}}$$

$$C(q) = F + rx(q)$$

$$C(q) = F + r\left(2 - 2(1-q)^{\frac{1}{2}}\right)$$

Example:

$$C = TC = 1 + \left(2 - 2(1-q)^{\frac{1}{2}}\right)$$

$$ATC = \frac{1 + \left(2 - 2(1-q)^{\frac{1}{2}}\right)}{q}$$

$$AFC = \frac{1}{q}$$

$$AVC = \frac{2 - 2(1-q)^{\frac{1}{2}}}{q}$$

$$MC = \frac{dC}{dq} = -(1-q)^{-\frac{1}{2}}(-1) = \frac{1}{\sqrt{1-q}}$$

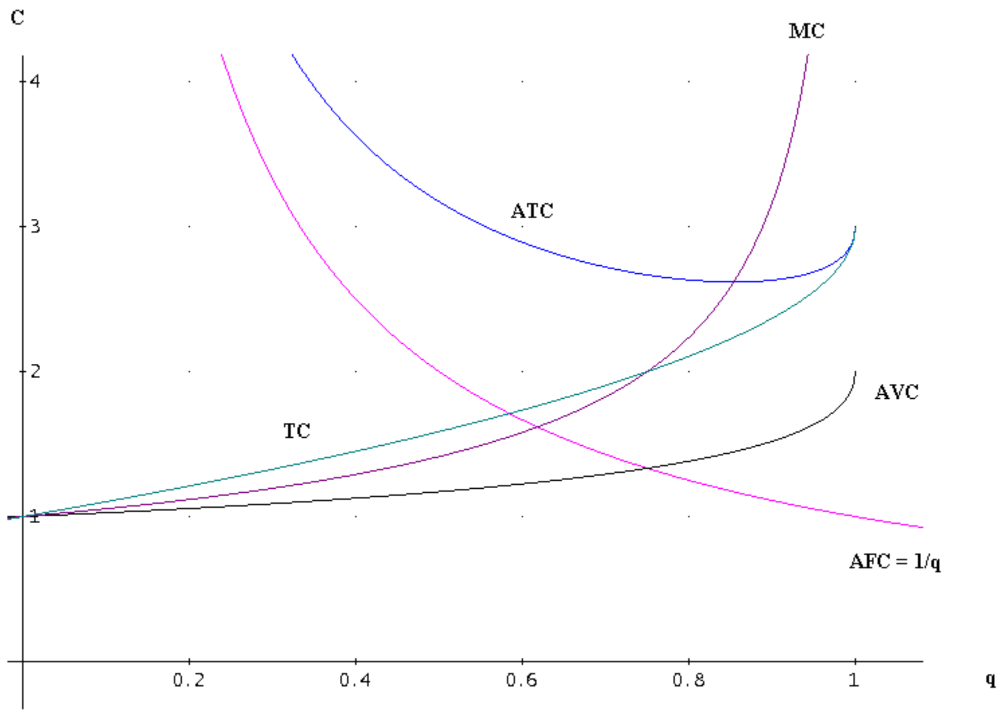


Figure 4. CASE 3.

Case 4

$$q = e^{-3+4x-x^2} - k \quad , \quad k = e^{-3}$$

$$q + k = e^{-3+4x-x^2}$$

$$\ln(q+k) = -3 + 4x - x^2$$

$$x^2 - 4x + 3 + \ln(q+k) = 0$$

$$x = 2 - \sqrt{1 - \ln(q + e^{-3})}$$

$$x = 2 - (1 - \ln(q+k))^{\frac{1}{2}}$$

$$MC = \frac{dC}{dq} = -\frac{1}{2}(1 - \ln(q+k))^{-\frac{1}{2}}\left(\frac{-1}{q+k}\right)(1)$$

$$MC = \frac{dC}{dq} = \frac{1}{2(q+k)}(1 - \ln(q+k))^{-\frac{1}{2}}$$

$$MC = \frac{dC}{dq} = \frac{1}{2(q+k)\sqrt{1 - \ln(q+k)}}$$

$$x = 2 - \sqrt{4 - 3 - \ln(q+k)}$$

$$x = 2 - \sqrt{1 - \ln(q + e^{-3})}$$

$$x = 2 - (1 - \ln(q+k))^{\frac{1}{2}}$$

$$C(q) = F + rx(q)$$

$$C(q) = F + r\left(2 - \sqrt{1 - \ln(q + e^{-3})}\right)$$

Example:

$$C = TC = 1 + \left(2 - \sqrt{1 - \ln(q + e^{-3})}\right)$$

$$ATC = \frac{1 + \left(2 - \sqrt{1 - \ln(q + e^{-3})}\right)}{q}$$

$$AFC = \frac{1}{q}$$

$$AVC = \frac{2 - \sqrt{1 - \ln(q + e^{-3})}}{q}$$

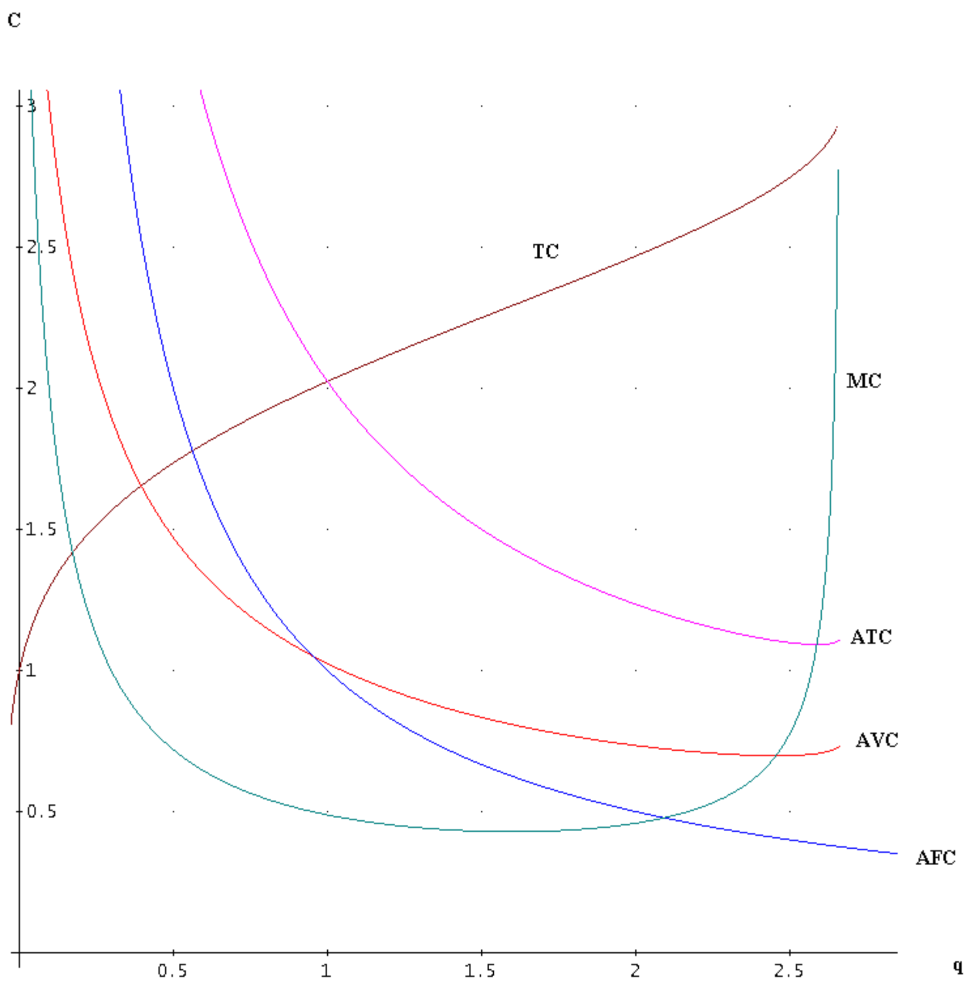
$$x = 2 - \sqrt{1 - \ln(q + e^{-3})}$$

$$x = 2 - (1 - \ln(q + k))^{\frac{1}{2}}$$

$$MC = \frac{dC}{dq} = -\frac{1}{2}(1 - \ln(q + k))^{-\frac{1}{2}}\left(\frac{-1}{q + k}\right)(1)$$

$$MC = \frac{dC}{dq} = \frac{1}{2(q + k)}(1 - \ln(q + k))^{-\frac{1}{2}}$$

$$MC = \frac{dC}{dq} = \frac{1}{2(q + k)\sqrt{1 - \ln(q + k)}}$$



<u>Figure 5.</u> CASE 4.

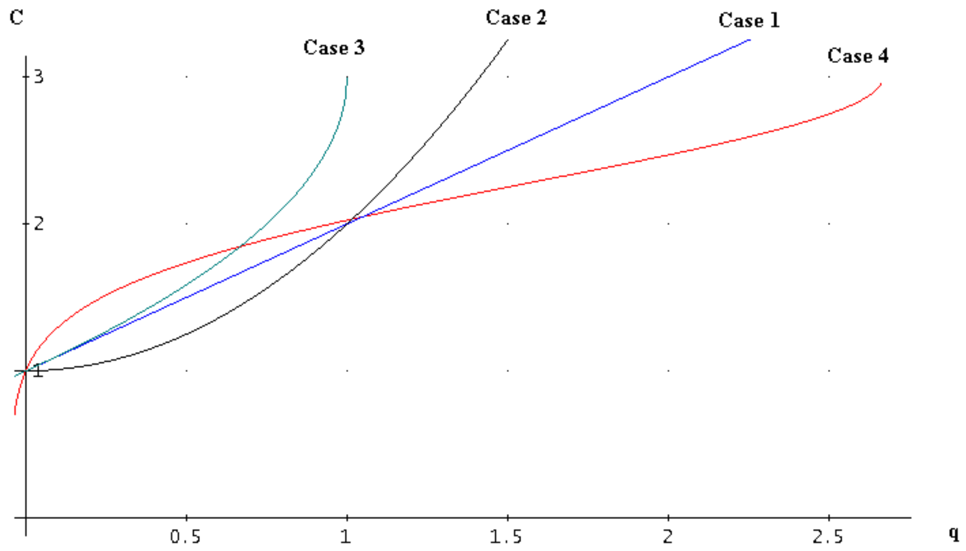


Figure 6. C(q) = TC(q) = total cost functions (for the different cases).