L5 (By Peter Lohmander 2009-09-28)

Quadratic programming and other multi dimensional nonlinear programming. Focus on problems with concave objective functions and convex feasible sets. General theory, application examples, analytical solutions and numerical solutions via computer programming.
Portfolio theory according to Markowitz with applications in separate document. Regression and curve fitting with flexible nonlinear functions, different constraints and objective functions.

Useful links:

http://www.lohmander.com/AppPro/L4c.doc

http://www.lohmander.com/AppPro/L4c.pdf

http://en.wikipedia.org/wiki/True_variance

http://en.wikipedia.org/wiki/Bessel%27s_correction

```
! Curve fitting via direct parameter
optimization;
! Peter Lohmander 090928;

MODEL:

 SETS:
     OBS/1..5/: X, Y, EstY, Dev;
 ENDSETS

 DATA:
    X = 0  10  20  30  40;
    Y = 30 50  66  80  92;
 ENDDATA

! Number of observations;
 NK = @SIZE( OBS);

@for(OBS(i): EstY = a + b*X(i));

@for(OBS(i): Dev(i) = EstY(i) - Y(i));

obj = @SUM(OBS(i): Dev(i)*Dev(i));

min = obj;

@free(a);
@free(b);
@for(OBS(i): @free(Dev(i)));
@for(OBS(i): @free(EstY(i)));

EST_STDEV = @SQRT( obj/(NK-1));

END
```

Local optimal solution found.
Objective value: 23.60000
Infeasibilities: 0.2486900E-13
Total solver iterations: 39

Variable	Value	Reduced Cost
NK	5.000000	0.000000
A	32.80000	0.000000
B	1.540000	0.000000
OBJ	23.60000	0.000000
EST_STDEV	2.428992	0.000000
X(1)	0.000000	0.000000
X(2)	10.00000	0.000000
X(3)	20.00000	0.000000
X(4)	30.00000	0.000000
X(5)	40.00000	0.000000
Y(1)	30.00000	0.000000
Y(2)	50.00000	0.000000
Y(3)	66.00000	0.000000
Y(4)	80.00000	0.000000
Y(5)	92.00000	0.000000
ESTY(1)	32.80000	0.000000
ESTY(2)	48.20000	0.000000
ESTY(3)	63.60000	0.000000
ESTY(4)	79.00000	0.000000
ESTY(5)	94.40000	0.000000
DEV(1)	2.800000	0.000000
DEV(2)	-1.800000	0.000000
DEV(3)	-2.400000	0.000000
DEV(4)	-1.000000	0.000000
DEV(5)	2.400000	0.000000

```
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MODEL:

 SETS:
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 ENDSETS

 DATA:
    X = 0  10  20  30  40;
    Y = 30 50  66  80  92;
 ENDDATA

! Number of observations;
 NK = @SIZE( OBS);

@for(OBS(i): EstY = a + b*X(i) + c*X(i)*X(i));

@for(OBS(i): Dev(i) = EstY(i) - Y(i));

obj = @SUM(OBS(i): Dev(i)*Dev(i));

min = obj;

@free(a);
@free(b);
@free(c);
@for(OBS(i): @free(Dev(i)));
@for(OBS(i): @free(EstY(i)));

EST_STDEV = @SQRT( obj/(NK-1));

END
```

Local optimal solution found.

Objective value:	0.4571429
Infeasibilities:	0.1421085E-13
Total solver iterations:	40

Variable	Value	Reduced Cost
NK	5.000000	0.000000
A	30.22857	0.000000
B	2.054286	0.000000
C	-0.1285714E-01	0.000000
OBJ	0.4571429	0.000000
EST_STDEV	0.3380617	0.000000
X(1)	0.000000	0.000000
X(2)	10.00000	0.000000
X(3)	20.00000	0.000000
X(4)	30.00000	0.000000
X(5)	40.00000	0.000000
Y(1)	30.00000	0.000000
Y(2)	50.00000	0.000000
Y(3)	66.00000	0.000000
Y(4)	80.00000	0.000000
Y(5)	92.00000	0.000000
ESTY(1)	30.22857	0.000000
ESTY(2)	49.48571	0.000000
ESTY(3)	66.17143	0.000000
ESTY(4)	80.28571	0.000000
ESTY(5)	91.82857	0.000000
DEV(1)	0.2285714	0.3245112E-08
DEV(2)	-0.5142857	-0.1094799E-07
DEV(3)	0.1714286	0.000000
DEV(4)	0.2857143	0.1299537E-07
DEV(5)	-0.1714286	0.000000

```
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MODEL:

 SETS:
     OBS/1..5/: X, Y, EstY, Dev;
 ENDSETS

 DATA:
    X = 0  10  20  30  40;
    Y = 30 50  66  80  92;
 ENDDATA

! Number of observations;
 NK = @SIZE( OBS);

@for(OBS(i): EstY = a + b*X(i) +
c*@SQRT(X(i)));

@for(OBS(i): Dev(i) = EstY(i) - Y(i));

obj = @SUM(OBS(i): Dev(i)*Dev(i));

min = obj;

@free(a);
@free(b);
@free(c);
@for(OBS(i): @free(Dev(i)));
@for(OBS(i): @free(EstY(i)));

EST_STDEV = @SQRT( obj/(NK-1));

END
```

Local optimal solution found.
Objective value: 1.597419
Infeasibilities: 0.1421085E-13
Total solver iterations: 44

Variable	Value	Reduced Cost
NK	5.000000	0.000000
A	29.87300	0.000000
B	1.050304	0.000000
C	3.272480	0.000000
OBJ	1.597419	0.000000
EST_STDEV	0.6319451	0.000000
X(1)	0.000000	0.000000
X(2)	10.00000	0.000000
X(3)	20.00000	0.000000
X(4)	30.00000	0.000000
X(5)	40.00000	0.000000
Y(1)	30.00000	0.000000
Y(2)	50.00000	0.000000
Y(3)	66.00000	0.000000
Y(4)	80.00000	0.000000
Y(5)	92.00000	0.000000
ESTY(1)	29.87300	0.000000
ESTY(2)	50.72454	0.000000
ESTY(3)	65.51406	0.000000
ESTY(4)	79.30624	0.000000
ESTY(5)	92.58215	0.000000
DEV(1)	-0.1269951	0.000000
DEV(2)	0.7245371	0.4566189E-08
DEV(3)	-0.4859361	0.000000
DEV(4)	-0.6937586	0.1167033E-07
DEV(5)	0.5821527	0.000000

```
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MODEL:
 SETS:
     OBS/1..5/: X, Y, EstY, Dev;
 ENDSETS

 DATA:
    X = 0  10  20  30  40;
    Y = 30 50  66  80  92;
 ENDDATA

! Number of observations;
 NK = @SIZE( OBS);

@for(OBS(i): EstY = a + b*X(i) + c*(X(i))^D);
@for(OBS(i): Dev(i) = EstY(i) - Y(i));

obj = @SUM(OBS(i): Dev(i)*Dev(i));
min = obj;

@free(a);
@free(b);
@free(c);
@free(D);

[con1] 1.2 <= D;
[con2] D <= 3;
[con3] c < 0.01;

@for(OBS(i): @free(Dev(i)));
@for(OBS(i): @free(EstY(i)));
EST_STDEV = @SQRT( obj/(NK-1));

END
```

Local optimal solution found.
Objective value: 0.1331149E-01
Infeasibilities: 0.1421085E-13
Total solver iterations: 32

Variable	Value	Reduced Cost
NK	5.000000	0.000000
A	30.00462	0.000000
B	2.602434	0.000000
C	-0.2428732	0.000000
D	1.397378	0.000000
OBJ	0.1331149E-01	0.000000
EST_STDEV	0.5768773E-01	0.000000
X(1)	0.000000	0.000000
X(2)	10.00000	0.000000
X(3)	20.00000	0.000000
X(4)	30.00000	0.000000
X(5)	40.00000	0.000000
Y(1)	30.00000	0.000000
Y(2)	50.00000	0.000000
Y(3)	66.00000	0.000000
Y(4)	80.00000	0.000000
Y(5)	92.00000	0.000000
ESTY(1)	30.00462	0.000000
ESTY(2)	49.96498	0.000000
ESTY(3)	66.07942	0.000000
ESTY(4)	79.92777	0.000000
ESTY(5)	92.02322	0.000000
DEV(1)	0.4615481E-02	0.000000
DEV(2)	-0.3502402E-01	0.5883413E-08
DEV(3)	0.7941945E-01	-0.1577499E-07
DEV(4)	-0.7222876E-01	0.000000
DEV(5)	0.2321784E-01	-0.7658420E-08

Row	Slack or Surplus	Dual Price
CON1	0.1973778	0.000000
CON2	1.602622	0.000000
CON3	0.2528732	0.000000