Optimal adaptive stochastic control of large scale energy production under the influence of market risk

Version 160316

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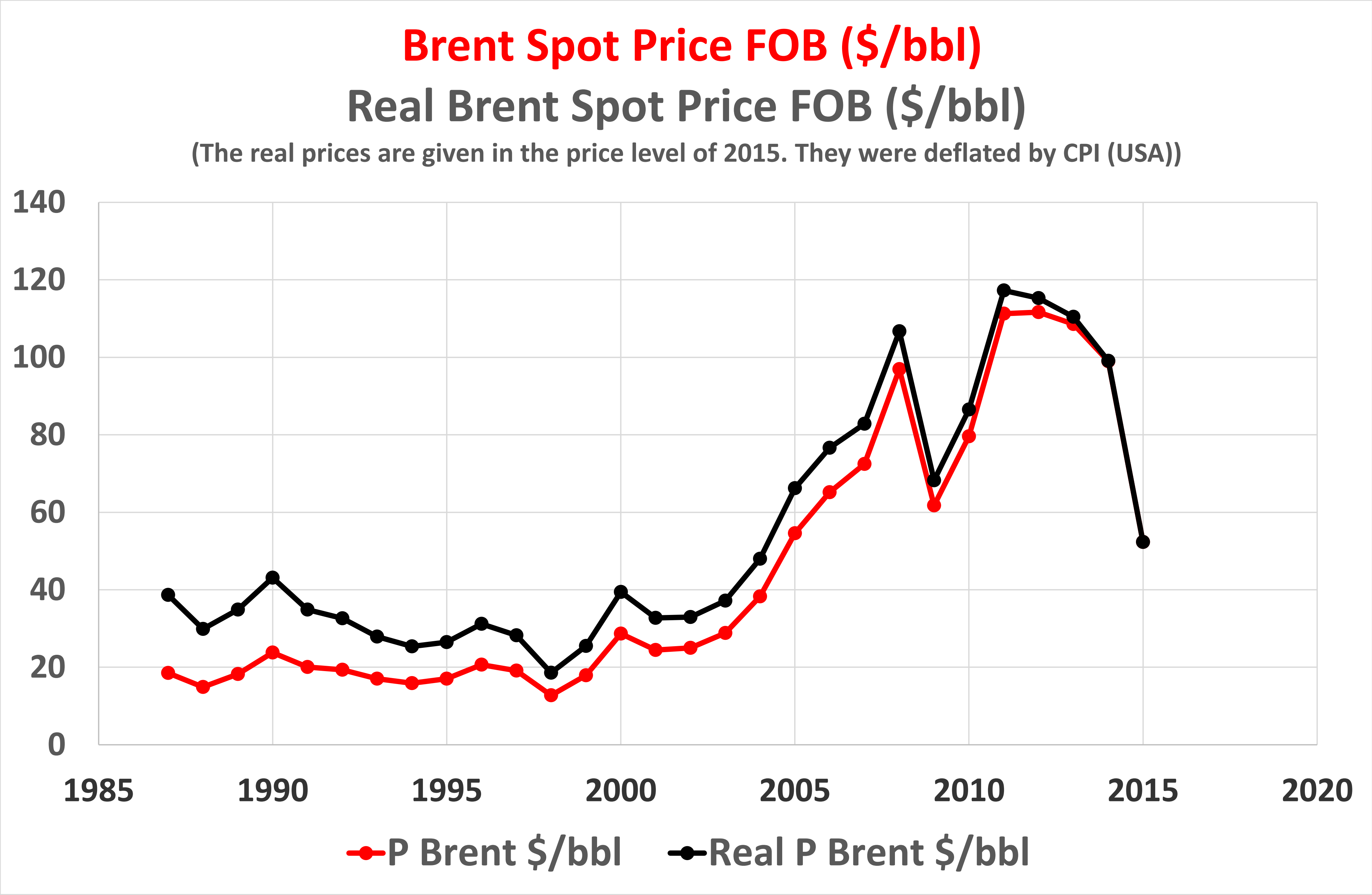
Abstract

The global energy market prices may be considered as stochastic processes. These prices may be strongly influenced (partly controlled) by large producers and cartels, since production levels influence the price level. Under such conditions, optimal dynamic production plans have to be optimized using adaptive controls, derived via stochastic control theory and/or stochastic dynamic programming. This paper presents alternative modelling options and a detailed analysis of a sample problem. It is possible to use stochastic dynamic programming as a master problem in combination with detailed multidimensional solutions to production and logistics problems for each state and stage. This can be done in finite time and also in infinite time, via stochastic dynamic programming in Markov chains with linear programming as a solution method. A sample problem is defined as a stochastic dynamic programming problem for optimal adaptive control of energy production under the influence of market risk and energy reserve constraints. General results concerning how the optimal adaptive production decisions and expected resource values are affected by increasing risk in the energy markets are reported. The main results are presented in connection to the present modelling results. General conclusions are presented and suggestions for future research are given.

Keywords: Optimal adaptive control; Stochastic dynamic programming; Market adapted oil production.


Brent Spot Price FOB ($/bbl)
Real Brent Spot Price FOB ($/bbl)
(The real prices are given in the price level of 2015. They were deflated by CPI (USA))
140
120
100
80
60
40
20
0
1985
1990
1995
2000
2005
2010
2015
2020
P Brent $/bbl
Real P Brent $/bbl

Optimal oil extraction

Optimal domestic oil logistics

Optimal oil refining

Optimal international trade and oil logistics

Optimal oil industry management

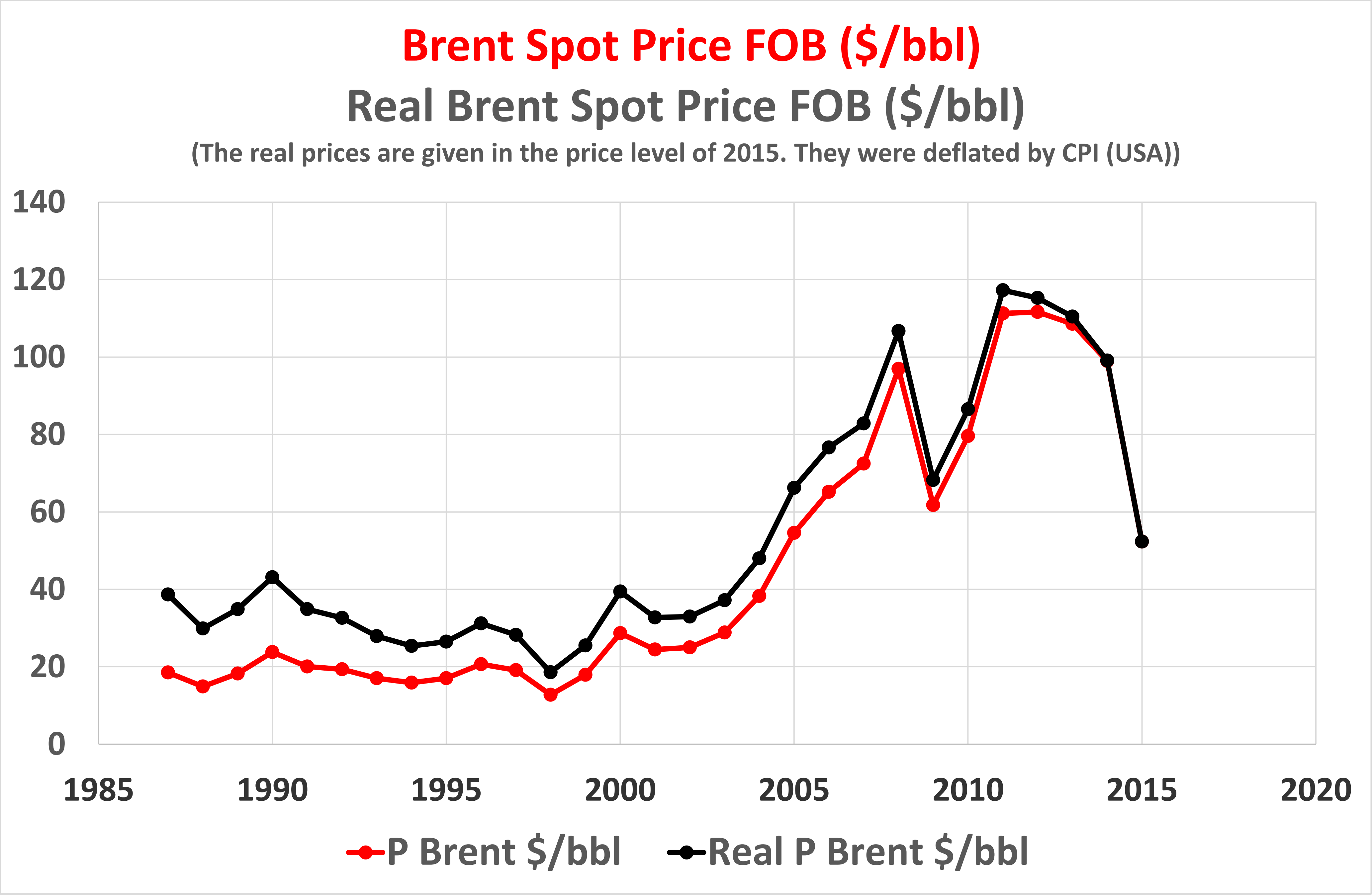








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120
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2020
P Brent $/bbl
Real P Brent $/bbl

$$f(t,s,m) = \max_{h \in H(t,s,m)} \left(\pi(h;t,s,m) + \sum_{n} \tau(n|m) f(t+1, s-h, n) \right)$$

$$\forall (t \le T, s, m)$$








Inclusion of:

Linear and quadratic programming sub problems

$$f(t,s,m) = \max_{h \in H(t,s,m)} \left(\max_{\substack{s.t.. \\ a_{11}x_1 + ... + \alpha_{1n}x_n \le C_1 \\ ... \\ a_{m1}x_1 + ... + \alpha_{mn}x_n \le C_m}} \pi(x_1,.,x_n;h,t,s,m) + \sum_n \tau(n|m) f(t+1, s-h, n) \right)$$

File = PL_OptOil_cases 160219, **OptOil Cases**, ***Peter Lohmander*** 2016-02-19

The global energy market prices may be considered as stochastic processes.

Since the stochastic properties of these prices are of fundamental importance to optimal management of
energy companies and to the economy in general, we start by investigating them.

Source of oil prices:

Independent Statistics & Analysis
U.S. Energy Information Administration

(The following graphs will show annual averages.)

https://www.eia.gov/dnav/pet/pet_pri_spt_s1_m.htm

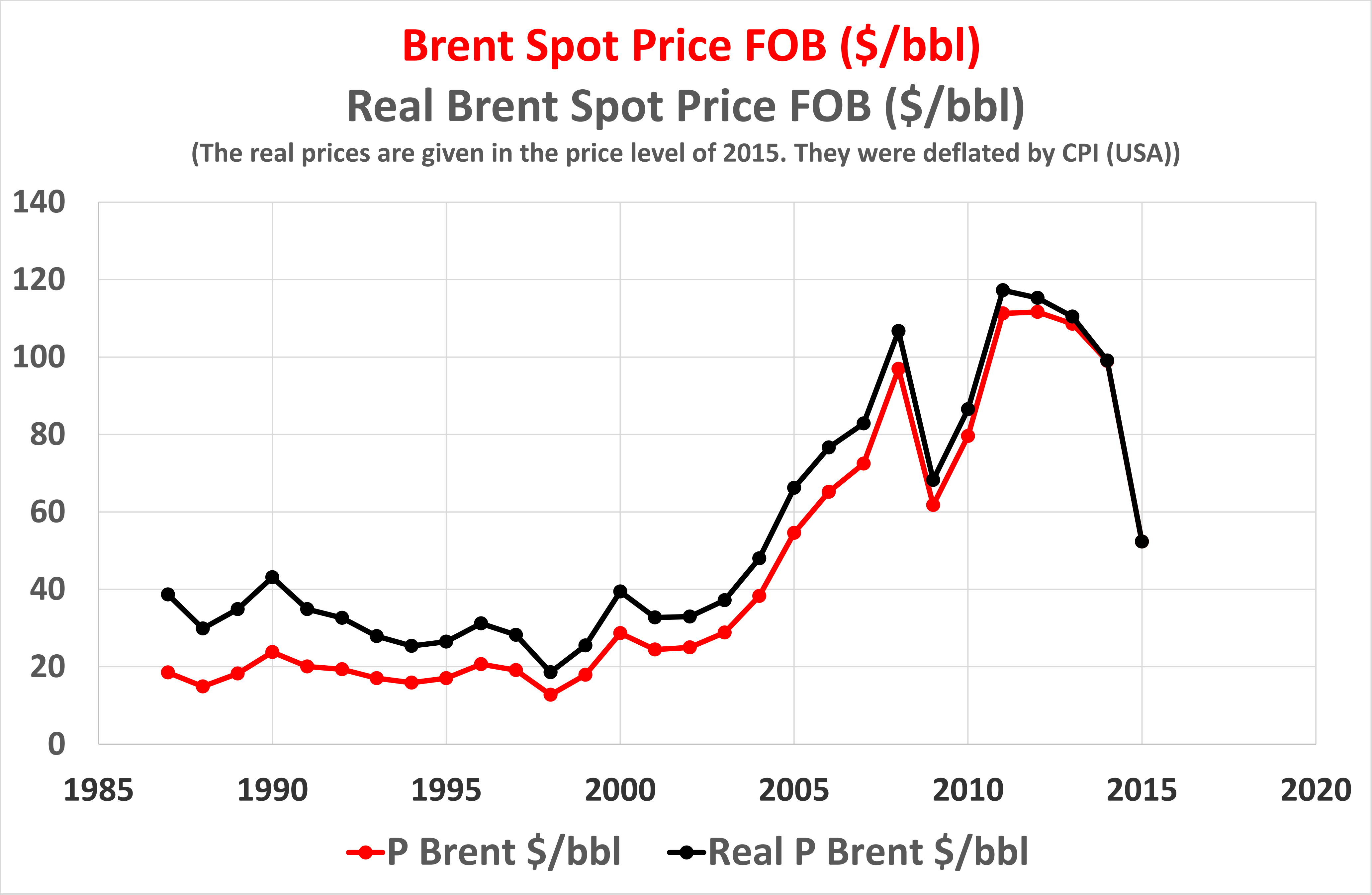
Inflation adjustments via CPI:

Table 24. Historical Consumer Price Index for All Urban Consumers (CPI-U): U. S. city average, all items-Continued

CPI Detailed Report
Data for January 2016

Editors
Malik Crawford
Jonathan Church
Bradley Akin

http://www.bls.gov/cpi/cpid1601.pdf



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(The real prices are given in the price level of 2015. They were deflated by CPI (USA))
140
120
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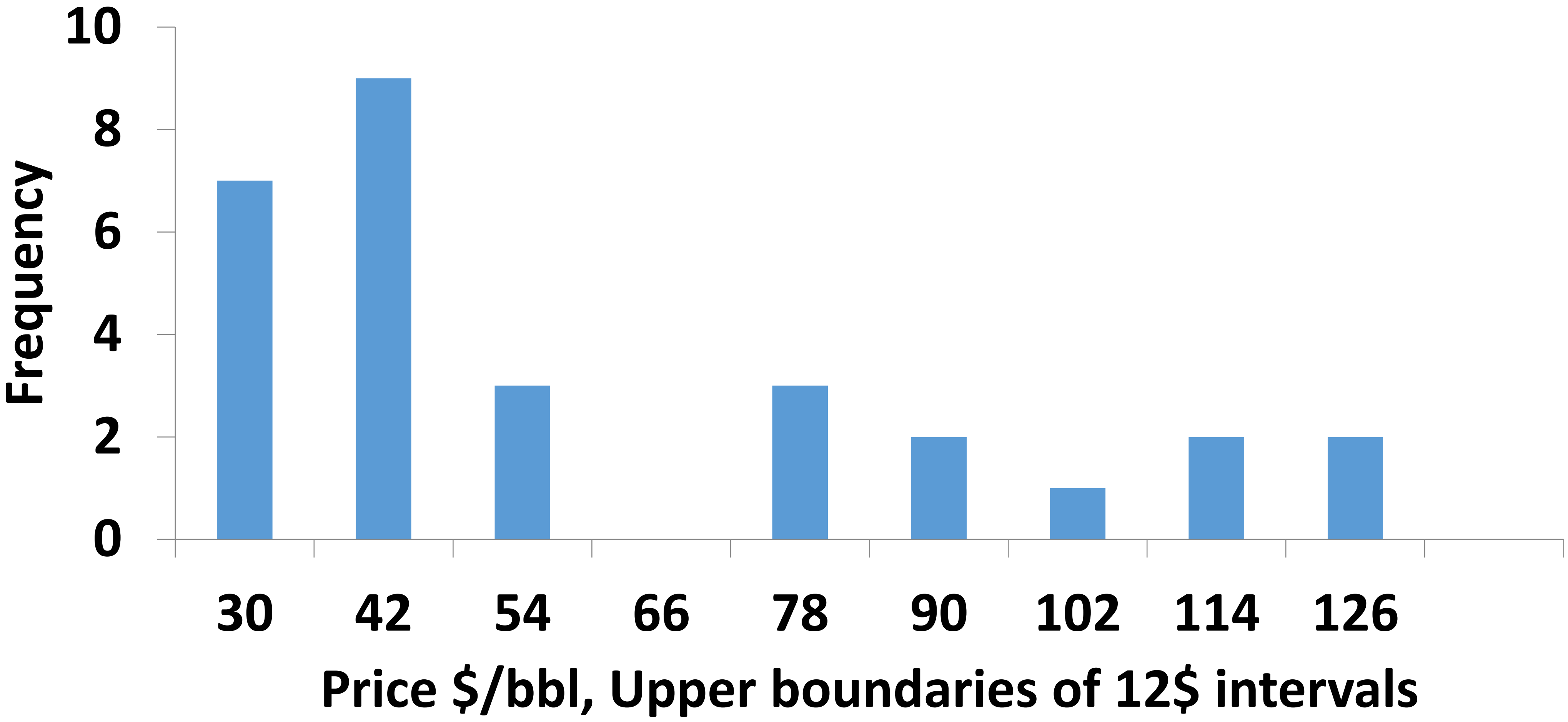
P Brent $/bbl
Real P Brent $/bbl

Frequency distribution of the inflation adjusted (real) spot price of crude oil (Brent) $/bbl. (Years 1987-2015, Price level of 2015)



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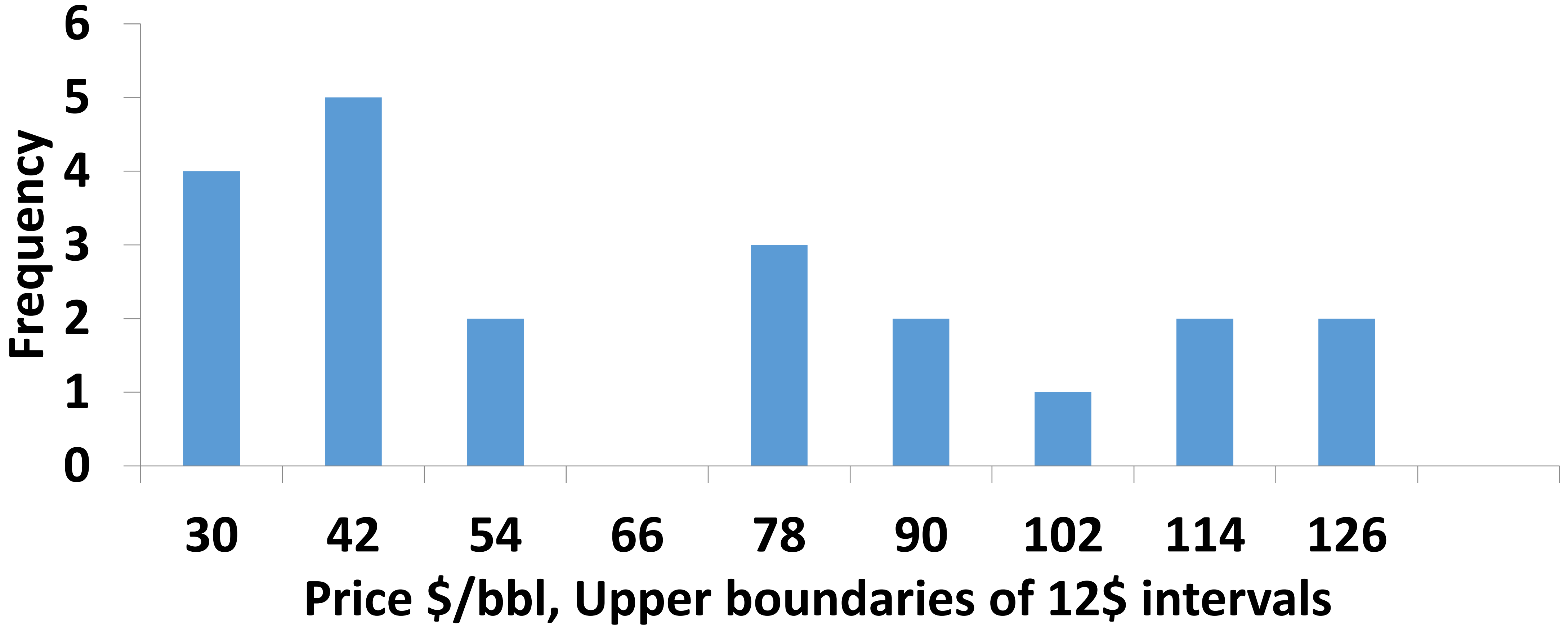
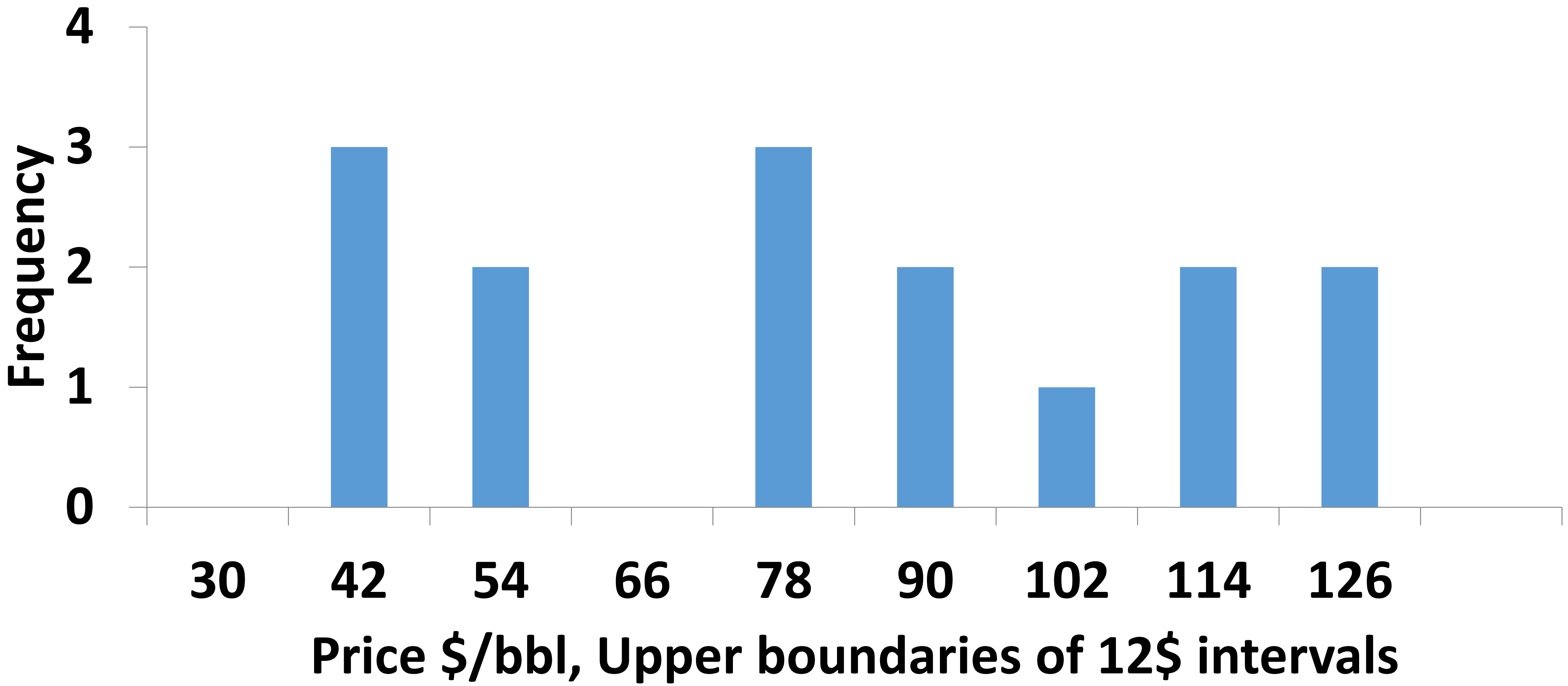


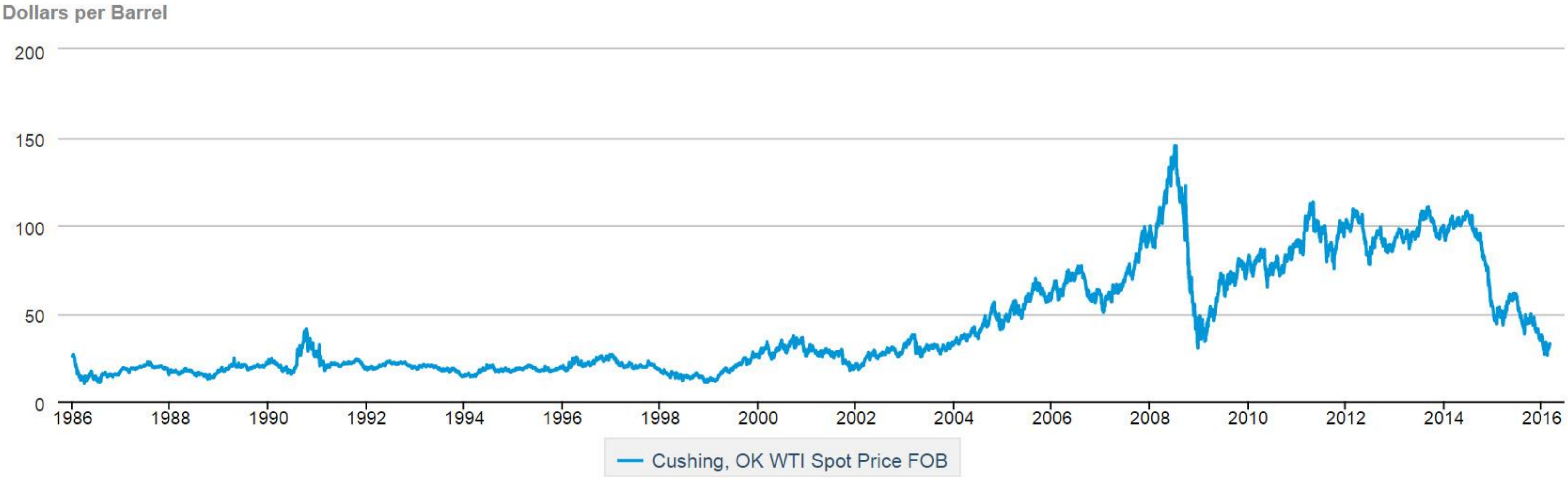
Frequency distribution of the inflation adjusted (real) spot price of crude oil (Brent) $/bbl. (Years 2001-2015, Price level of 2015)



View History: Daily Weekly Monthly Annual   Download Data (XLS File)

Cushing, OK WTI Spot Price FOB

DOWNLOAD



THOMSON REUTERS

Chart Tools

no analysis applied ▼

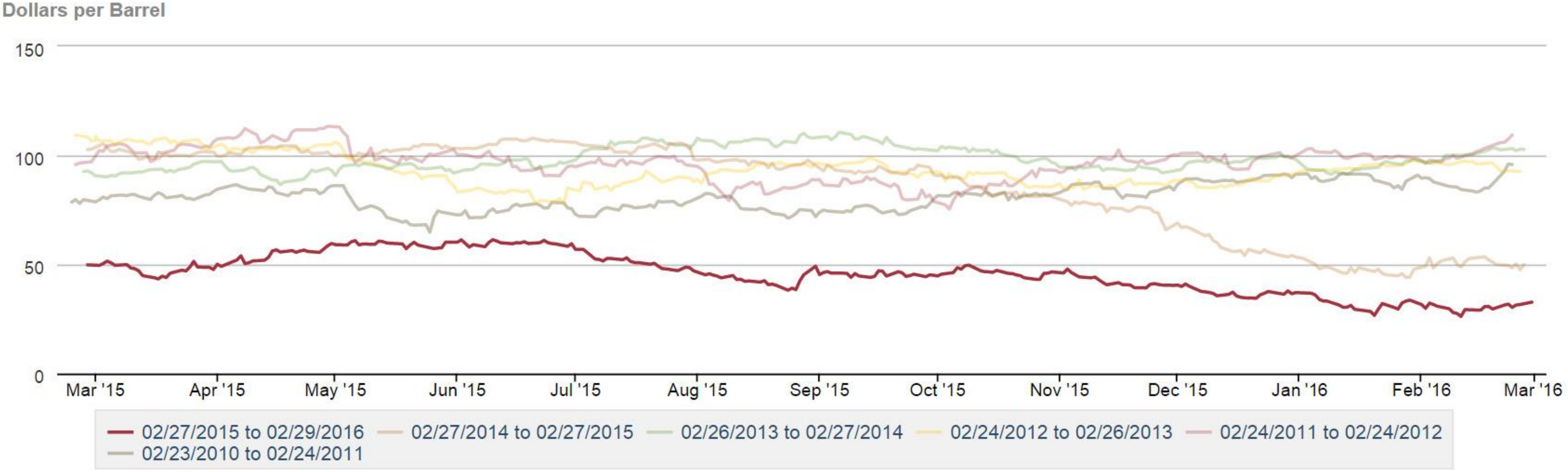
This series is available through the EIA open data API and can be downloaded to Excel or embedded as an interactive chart or map on your website.

http://www.eia.gov/dnav/pet/hist/LeafHandler.ashx?n=PET&s=RWTC&f=D

View History: ◉ Daily ○ Weekly ○ Monthly ○ Annual   Download Data (XLS File)

Cushing, OK WTI Spot Price FOB

DOWNLOAD

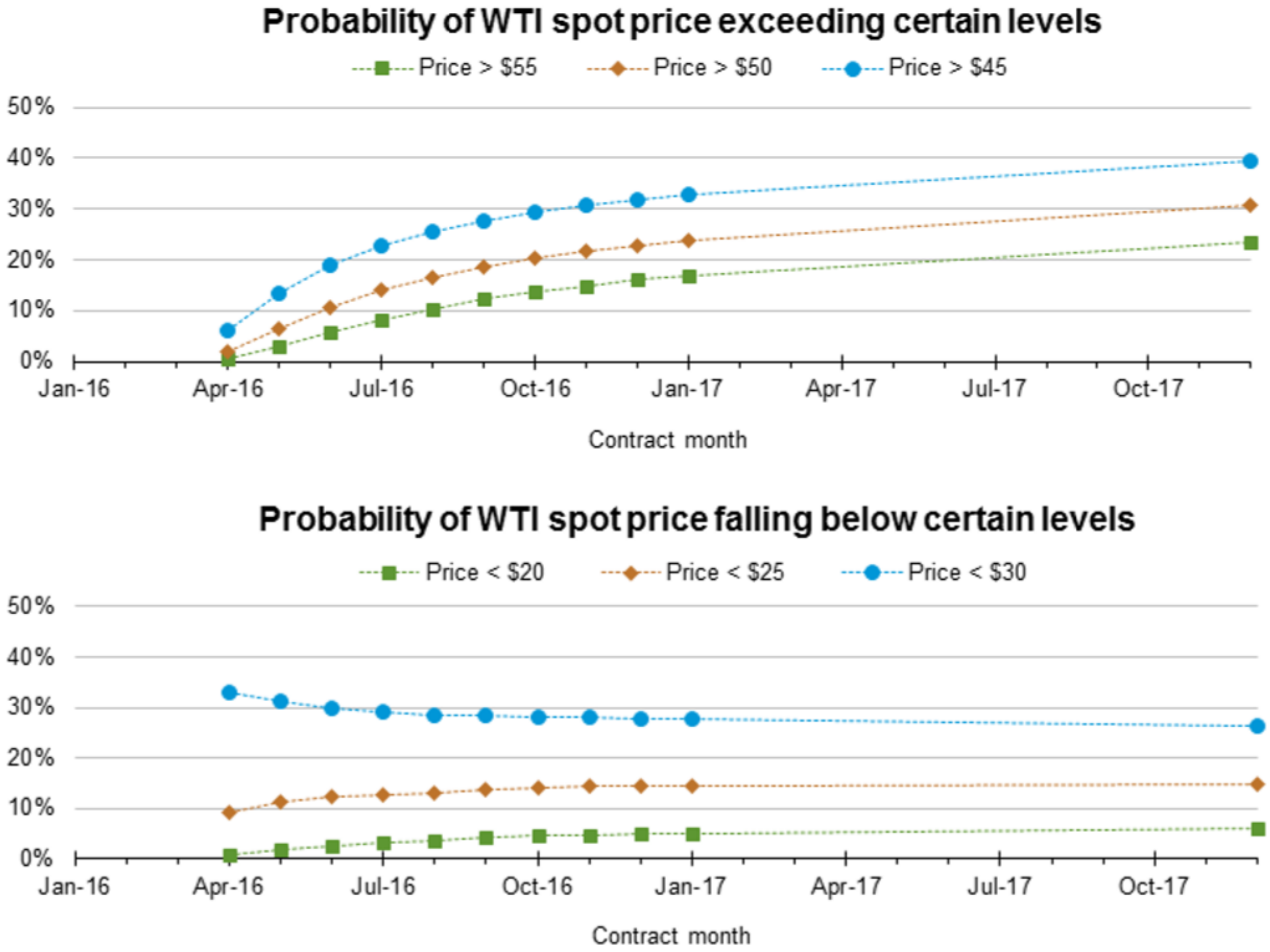


THOMSON REUTERS

Chart Tools

5 Year Seasonal Analysis ▼

This series is available through the EIA open data API and can be downloaded to Excel or embedded as an interactive chart or map on your website.



Notes: Probability values calculated using NYMEX market data for the five trading days ending February 4, 2016.
Values not calculated for months with little trading in "close-to-the-money" options contracts.

Source: EIA Short-Term Energy Outlook, February 2016, and CME Group (http://www.cmegroup.com)

What method is used by EIA to calculate the probabilities?

- Option prices are used to determine the parameters of a probability density function (and indirectly the probabilities of the oil price to be higher or lower than different limits.)
- Of course, one could also use other methods. For instance, one could look at the distribution of errors of price predictions in earlier periods.

On the next pages, some sections are copied from:

www.eia.doe.gov
Energy Information Administration

October 2009

Short-Term Energy Outlook Supplement:
Energy Price Volatility and Forecast Uncertainty[1]

- **Confidence intervals** for expected prices can be calculated using a variety of **alternative techniques**, including estimates based on past price volatility, statistical analysis of past forecast errors, or estimates of parameter uncertainty in an econometric energy price forecasting equation.

- Such backward-looking approaches, notwithstanding their merits, cannot reflect changes in current market conditions and expectations that may lead to greater or lesser uncertainty about the future at any given time.

- The STEO (Short-Term Energy Outlook) will instead focus on a measure of **uncertainty derived from** the New York Mercantile Exchange (NYMEX) light sweet **crude oil options** and natural gas options markets.

- EIA will **derive confidence intervals** around expected futures prices **using** the "implied volatilities" of these options. Implied volatility is nothing more than a **standard deviation** for expected returns **embedded in the option's price**.

- If an **option's price** is observed in the market, then a pricing model can be **"run backwards" to calculate the volatility embedded** in that price. This represents a market-cleared estimate of implied volatility, i.e., a buyer and seller have agreed on the value of an option.

- A particular type of random walk is **assumed** in the B-S-M and Black models, known as a **geometric Wiener process**.

- In such a process, the likelihood of a 1-percent upward move in an asset's price is equal to the likelihood of a 1-percent downward move over a very small time increment.

- The most an asset can lose is 100% of its value (i.e., the price distribution is bounded at zero).

- This means returns would be normally distributed, with constant volatility, while absolute **prices would be log-normally distributed** at the option's expiry.

$$\ln\left(f_{(t+dt),k} / f_{t,k}\right) = \mu_k dt + \sigma_k z \sqrt{dt}$$

$$\ln\left(f_{(t+dt),k} / f_{t,k}\right) = \mu_k dt + \sigma_k z\sqrt{dt}$$, where

$\ln[\bullet]$ = Napierian logarithm, or natural logarithm

$f_{t,k}$ = observed futures price at time = t for the kth-nearby contract

$f_{(t+dt),k}$ = futures price at $t + dt$ for the kth-nearby contract ($dt > 0$)

μ_k = mean logarithmic return

dt = infinitesimal change in time (Δt, as $\Delta t \to 0$)

$\mu_k dt$ = the "drift" term

σ_k = standard deviation of the kth-nearby contract's returns

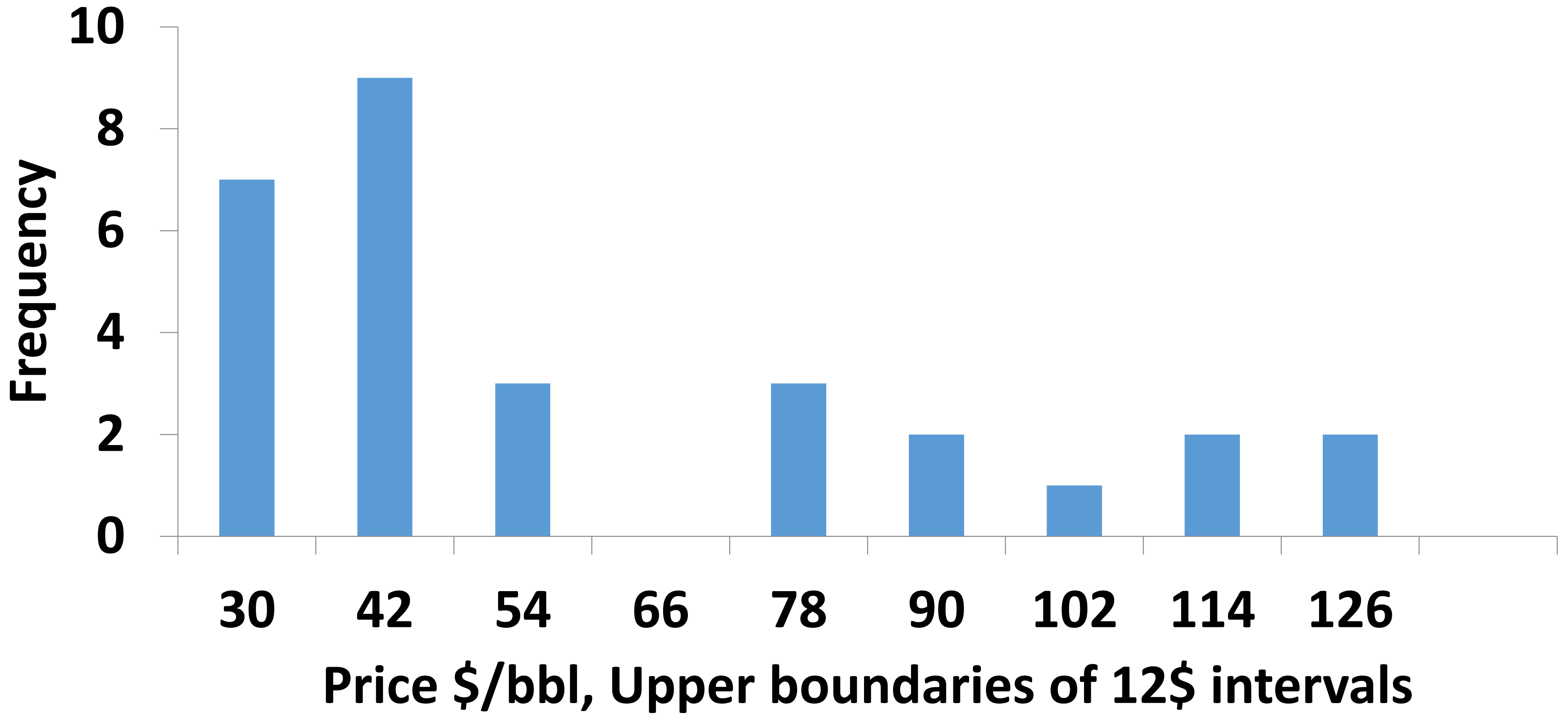
z = standard normal random variable with mean = 0, var = 1

$\sigma_k z\sqrt{dt}$ = random-shock[17]

- The **lognormal assumption** for asset prices, which translates into a normal assumption for log returns, has been a **source of debate** since at least the early 1960s.

- Numerous researchers have noted the distribution of daily returns of many assets is leptokurtic, i.e., “**fat-tailed**” and peaked.

Frequency distribution of the inflation adjusted (real) spot price of crude oil (Brent) $/bbl. (Years 1987-2015, Price level of 2015)



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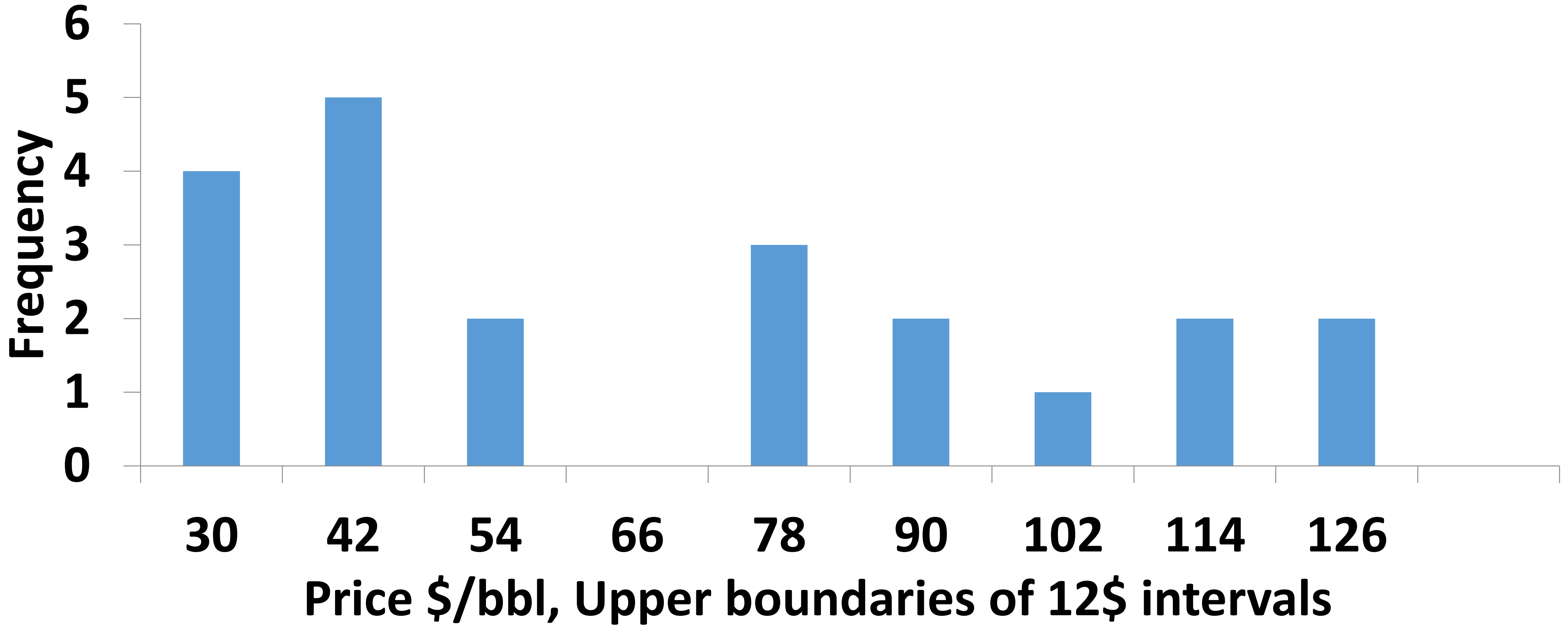

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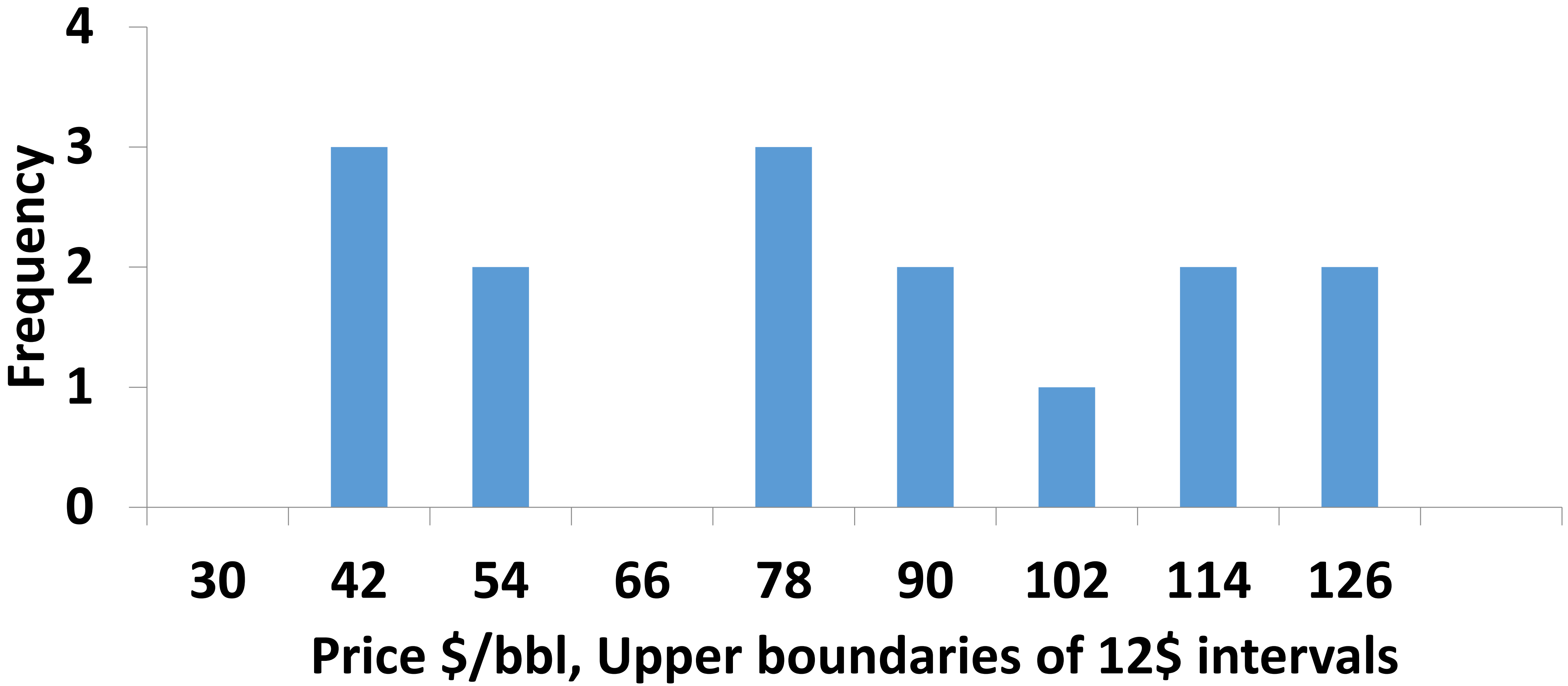


Option theory for uniform probability density functions by Peter Lohmander

(Motivated by international prices of crude oil)

The parameters of a uniform price probability density function may be derived via European call option prices. The rate of interest in the capital market is r and t is the time until the option expires.

exp(-rt)

$$y = k\int_{q}^{\infty}(x-q)f(x)dx$$

Option price
= Expected value
of the option

Strike Price

Oil price

Probability density function
of the oil price

$$y = k\int_{q}^{\infty}(x-q)\,f(x)\,dx$$

Option price = Expected value of the option

$$y = k\int_{q}^{m+h}(x-q)\frac{1}{2h}\,dx$$

We assume a uniform probability density function. Expected price = m. Maximum deviation, up or down, is h.

$$y = k\int_{q-q}^{m+h-q} x\frac{1}{2h}\,dx$$

$$y = \frac{k}{2h}\int_{0}^{m+h-q} x\,dx$$

$$y = \frac{k}{2h}\left(\frac{(m+h-q)^2}{2} - 0\right)$$

$$y = k\frac{(m+h-q)^2}{4h}$$

$$\frac{dy}{dq} = \frac{k}{2h}(m+h-q)(-1)$$

$$(q < m+h) \Rightarrow \left(\frac{dy}{dq} < 0\right)$$

The price of the option is a strictly decreasing function of the strike price.

$$y = k\frac{\left(m+h-q\right)^2}{4h}$$

$$\frac{dy}{dh} = k\frac{2\left(m+h-q\right)\cdot 4h-\left(m+h-q\right)^2\cdot 4}{16h^2}$$

$$\frac{dy}{dh} = k\frac{8h\left(m+h-q\right)-4\left(m+h-q\right)^2}{16h^2}$$

$$\frac{dy}{dh} = k\frac{\left(m+h-q\right)\left(8h-4m-4h+4q\right)}{16h^2}$$

$$\frac{dy}{dh} = \frac{k(m+h-q)(8h-4m-4h+4q)}{16h^2}$$

$$\frac{dy}{dh} = \frac{k(m+h-q)(h-m+q)}{4h^2}$$

$$\left.\begin{array}{l}(q<m+h) \Rightarrow (0<m+h-q) \\ (-h+m<q) \Rightarrow (0<h-m+q)\end{array}\right\} \Rightarrow \frac{dy}{dh} > 0$$

The option price is a strictly increasing function of the price standard deviation, which is a strictly increasing function of h.

Let us derive the value of h (which can give the standard deviation etc.) from the option price!

$$y = \frac{k(m+h-q)^2}{4h} \qquad z = \frac{y}{k} \qquad z = \frac{(m+h-q)^2}{4h}$$

$$4hz = (m+h-q)^2$$

$$4hz = m^2 + h^2 + q^2 + 2mh - 2mq - 2hq$$

$$m^2 + h^2 + q^2 + 2mh - 2mq - 2hq - 4hz = 0$$

$$h^2 + 2mh - 2hq - 4hz + m^2 + q^2 - 2mq = 0$$

$$h^2 + 2(m - q - 2z)h + \left(m^2 + q^2 - 2mq\right) = 0$$

$$h^2 + Ph + Q = 0$$

$$h_1 = -\frac{P}{2} + \sqrt{\left(\frac{P}{2}\right)^2 - Q}$$

$$h_2 = -\frac{P}{2} - \sqrt{\left(\frac{P}{2}\right)^2 - Q}$$

$$h_1 = 2z + q - m + \sqrt{\left(2z + q - m\right)^2 - \left(m^2 + q^2 - 2mq\right)}$$

$$h_2 = 2z + q - m - \sqrt{\left(2z + q - m\right)^2 - \left(m^2 + q^2 - 2mq\right)}$$

$$h_1 = 2z + q - m + \sqrt{4z^2 + q^2 + m^2 + 4zq - 4zm - 2qm - m^2 - q^2 + 2mq}$$

$$h_2 = 2z + q - m - \sqrt{4z^2 + q^2 + m^2 + 4zq - 4zm - 2qm - m^2 - q^2 + 2mq}$$

$$h_1 = 2z + q - m + \sqrt{4z^2 + 4zq - 4zm}$$

$$h_2 = 2z + q - m - \sqrt{4z^2 + 4zq - 4zm}$$

$$h_1 = 2z + q - m + 2\sqrt{z(z + q - m)}$$

$$h_2 = 2z + q - m - 2\sqrt{z(z + q - m)}$$

$h_1 \vee h_2$?

$$(m=q) \Rightarrow z = \int_0^h x \frac{1}{2h} dx = \frac{1}{2h}\left(\frac{h^2}{2} - 0\right) = \frac{h}{4}$$

h = 4z

$$(m=q) \Rightarrow \left(h_1 = 2z + 2\sqrt{z(z)} = 4y\right)$$

h = 4z

$$(m=q) \Rightarrow \left(h_2 = 2z - 2\sqrt{z(z)} = 0\right)$$

h = 0 (wrong)

This formula gives the "spread parameter" h of a uniform price probability density function.

$$h = 2z + q - m + 2\sqrt{z(z + q - m)}$$

$$z = \frac{y}{k} = \frac{y}{e^{-rt}} = ye^{rt}$$

m-h = lowest possible price

m+h = highest possible price

y = option price

q = strike price

m = expected price

r = rate of interest

t = time until expiration

https://www.eia.gov/forecasts/steo/uncertainty/pdf/uncertainty_past_wti.pdf

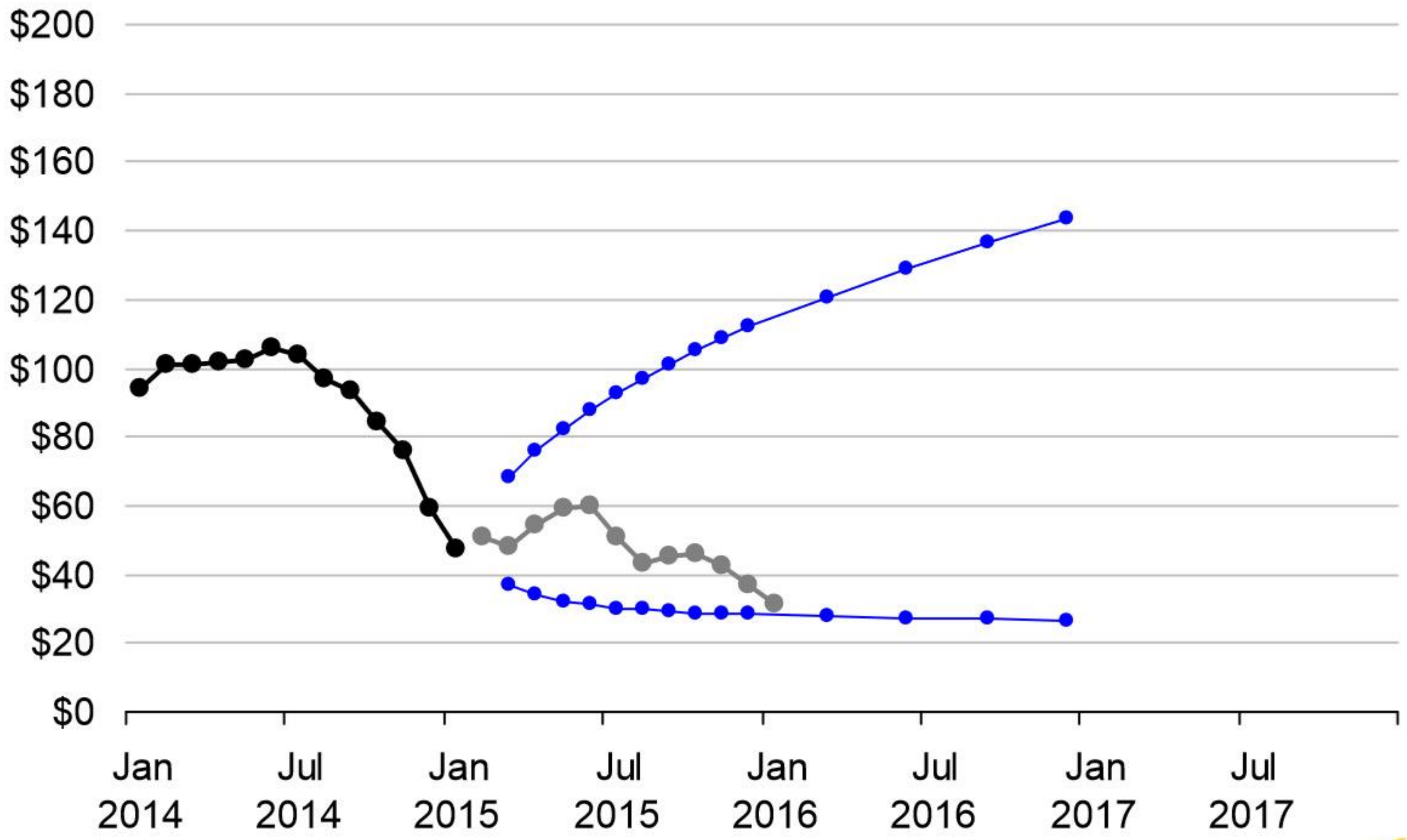
Short-Term Energy Outlook

West Texas Intermediate crude oil price
and NYMEX 95% confidence intervals

January 2015 – February 2016

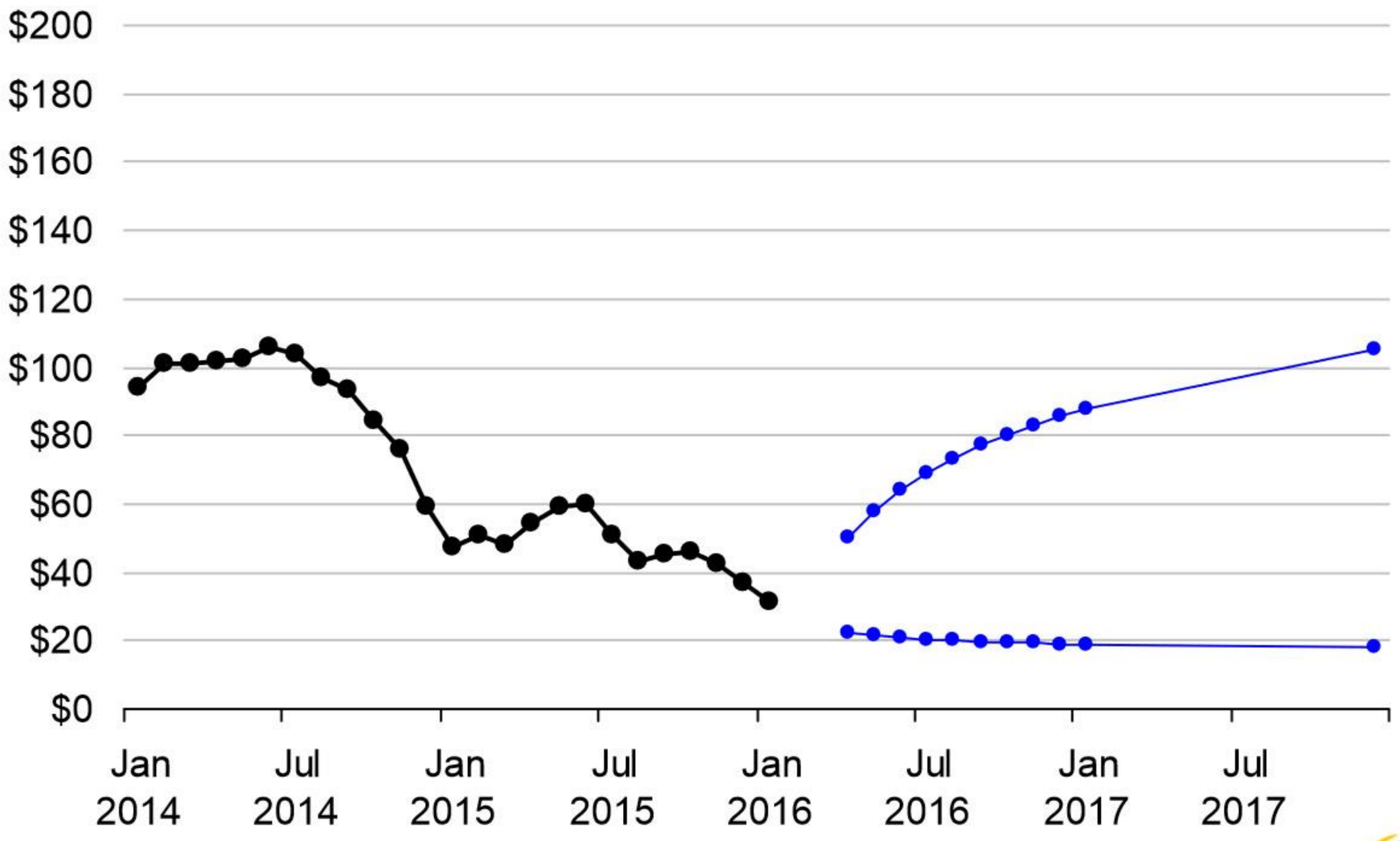
Obs!
The following two graphs are derived via lognormal price probability density function assumptions.
Comment by Peter Lohmander

Historical WTI price and 95% NYMEX Confidence Interval, January 2015



1

Historical WTI price and 95% NYMEX Confidence Interval, February 2016



14

eia

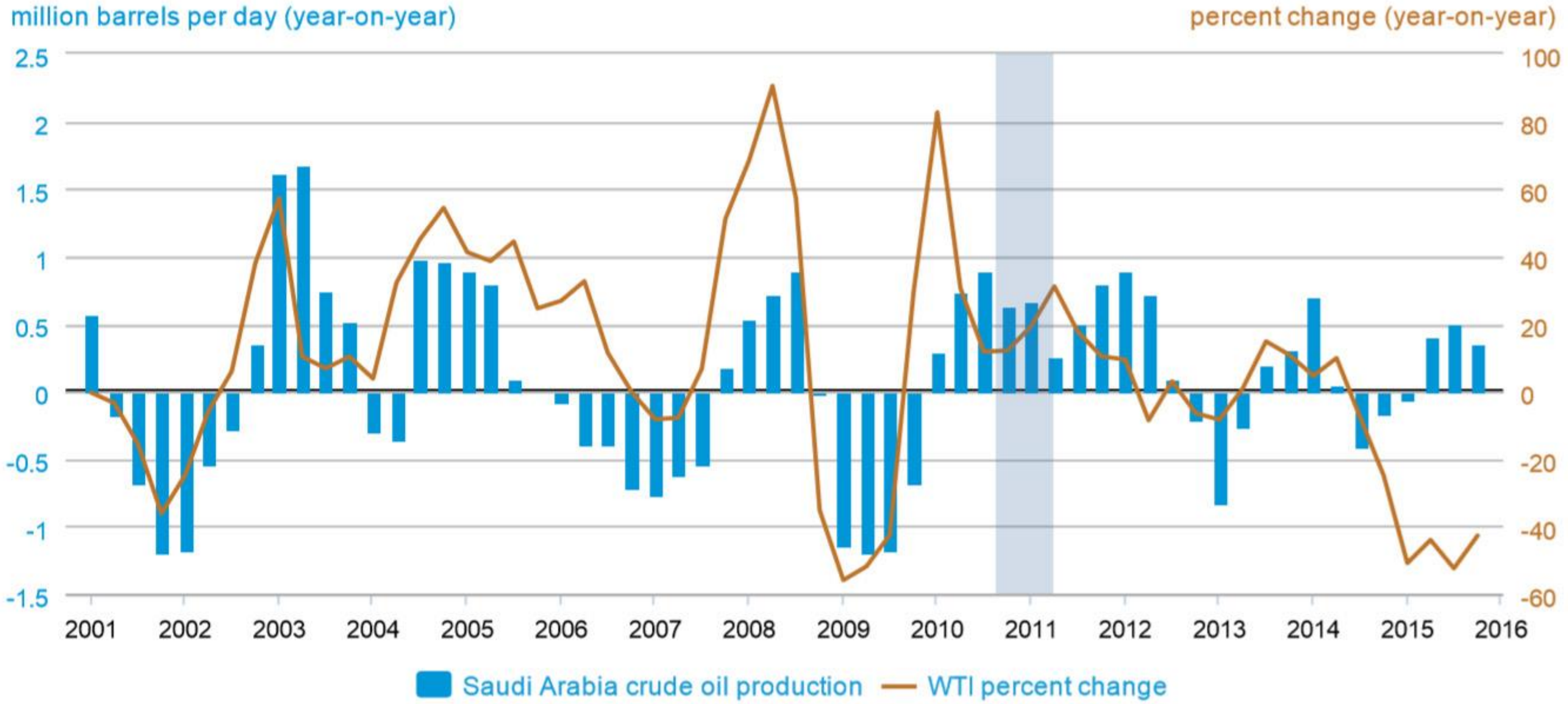
Please note that several alternative interpretations of the graph on the next slide can be made.

Peter Lohmander

Changes in Saudi Arabia crude oil production can affect oil prices

Changes in Saudi Arabia crude oil production and WTI crude oil prices

interactive



Source: U.S. Energy Information Administration, Thomson Reuters.
Updated: Quarterly | Last Updated: 02/09/2016

Download Data in CSV

https://www.eia.gov/finance/markets/supply-opec.cfm

Oil markets often respond to changing expectations of future supply and demand. This chart shows how projections of changes in Saudi Arabia crude oil production results in changes in WTI crude oil prices.

Energy prices may be strongly influenced (partly controlled) by large producers and cartels, since production levels influence the price level.

Under such conditions, optimal dynamic production plans have to be optimized using adaptive controls, derived via stochastic control theory and/or stochastic dynamic programming.

This paper presents alternative modelling options and a detailed analysis of a sample problem.

It is possible to use stochastic dynamic programming as a master problem in combination with detailed multidimensional solutions to production and logistics problems for each state and stage.

This can be done in finite time and also in infinite time, via stochastic dynamic programming in Markov chains with linear programming as a solution method.

A sample problem is defined as a stochastic dynamic programming problem for optimal adaptive control of energy production under the influence of market risk and energy reserve constraints.

$$f(t,s,m) = \max_{h \in H(t,s,m)} \left(\pi(h;t,s,m) + \sum_n \tau(n|m) f(t+1, s-h, n) \right)$$

$$\forall (t \le T, s, m)$$

$$f(T+1, s, m) = 0 \quad \forall (s, m)$$

```
DIM f(101, 100, 9), hopt(101, 100, 9), d(101)

FOR t = 0 TO 101
    d(t) = EXP(-.05 * t)
    FOR s = 0 TO 100
        FOR m = 1 TO 9
            f(t, s, m) = 0
            hopt(t, s, m) = 0
        NEXT m
    NEXT s
NEXT t
```

$$T = 10$$

$$0 \leq t \leq T$$

$$0 \leq s \leq 100$$

$$1 \leq m \leq 9$$

$$\tau(n|m) = \frac{1}{9} \; \forall (m, n)$$

$$P(h, m) = 30 + (m - 5) - h$$

```
FOR t = 10 TO 0 STEP -1
    FOR s = 0 TO 100
        FOR m = 1 TO 9
            FOR h = 0 TO s

                fnext = 0
                FOR n = 1 TO 9
                    fnext = fnext + 1 / 9 * f(t + 1, s - h, n)
                NEXT n

                fev = d(t) * (30 + (m - 5) - h) * h + fnext

                IF fev > f(t, s, m) THEN hopt(t, s, m) = h
                IF fev > f(t, s, m) THEN f(t, s, m) = fev

            NEXT h
        NEXT m
    NEXT s
NEXT t
```

General results concerning how the optimal adaptive production decisions and expected resource values are affected by increasing risk in the energy markets are reported.

The main results are presented in connection to the present modelling results.

Table 1 – Optimal extraction table for t = 0.

We find that the optimal extraction level is an increasing function of the market state and of the size of the remaining reserve.

	Reserve										
Market	0	10	20	30	40	50	60	70	80	90	100
m = 1	0	1	3	4	4	5	6	6	7	8	9
m = 2	0	2	3	4	5	6	6	7	8	8	9
m = 3	0	2	4	5	5	6	7	7	8	9	10
m = 4	0	3	4	5	6	6	7	8	9	9	10
m = 5	0	3	4	5	6	7	8	8	9	10	10
m = 6	0	4	5	6	7	7	8	9	10	10	11
m = 7	0	4	5	6	7	8	9	9	10	11	11
m = 8	0	4	6	7	8	8	9	10	10	11	12
m = 9	0	5	6	7	8	9	10	10	11	12	12

Table 2 – Optimal expected present value table for t = 0.

The optimal expected present value is an increasing function of the market state and of the size of the remaining reserve.

Market	Reserve 0	10	20	30	40	50	60	70	80	90	100
m = 1	0	252	470	667	846	1012	1163	1300	1423	1532	1627
m = 2	0	254	473	671	851	1017	1169	1307	1430	1540	1636
m = 3	0	256	476	675	856	1023	1175	1314	1438	1549	1645
m = 4	0	258	480	680	862	1029	1182	1321	1446	1558	1655
m = 5	0	261	484	685	868	1036	1190	1329	1455	1567	1665
m = 6	0	264	489	691	874	1043	1198	1338	1464	1577	1676
m = 7	0	268	494	697	881	1051	1206	1347	1474	1588	1687
m = 8	0	272	500	703	889	1059	1215	1357	1484	1599	1699
m = 9	0	277	506	710	897	1067	1224	1367	1495	1610	1711

Optimal decisions in period 0 and 5:

- Table 3 contains similar information as Table 1. However, in Table 3, we have reached period 5.
- The **general tendencies are the same in period 0 and period 5, but in period 5 we should extract more** of the resource than in period 0, in case the market state and the reserve level are the same.
- This is understandable, since in period 5, the **number of remaining periods is lower.**
- Hence, the **probability** that we will be able to sell the resource at a much higher price in the future is reduced.
- Furthermore, if we plan to strongly increase extraction in the near future, this will reduce the **price level** very much.
- For these reasons, it is better to extract more of the resource in period 5, even if the prices are not very good and even if we have not a very large remaining resource.

Table 3 – Optimal extraction table for t = 5.

Market	Reserve 0	10	20	30	40	50	60	70	80	90	100
m = 1	0	2	3	5	6	7	9	10	12	13	13
m = 2	0	2	3	5	6	8	9	11	12	13	13
m = 3	0	2	4	5	7	8	10	11	13	14	14
m = 4	0	3	4	6	7	9	10	12	13	14	14
m = 5	0	3	5	6	8	9	11	12	14	15	15
m = 6	0	4	5	7	8	10	11	12	14	15	15
m = 7	0	4	6	7	9	10	11	13	14	16	16
m = 8	0	4	6	7	9	10	12	13	15	16	16
m = 9	0	5	6	8	9	11	12	14	15	17	17

Optimal expected present values in period 0 and 5:

- Table 4 corresponds to Table 2, but we have now reached period 5.
- The **optimal expected present values are still increasing functions of the market state and the size of the remaining reserve, but all values are lower** than the corresponding values in period 0.
- There are several reasons for this:
- **Discounting** during five years reduces all profits.
- Furthermore, since a lower number of periods remain, the **number of options to extract during very good market states** has decreased.
- Finally, the remaining reserve has to be distributed over a lower number of periods, which gives **more negative effects on the price level**.

Table 4 – Optimal expected present value table for t = 5.

Market	Reserve 0	10	20	30	40	50	60	70	80	90	100
m = 1	0	196	363	506	628	726	801	853	883	893	893
m = 2	0	197	365	510	632	732	808	862	893	903	903
m = 3	0	199	368	514	637	738	816	870	902	913	914
m = 4	0	201	371	519	643	744	823	879	912	924	925
m = 5	0	203	375	523	649	751	831	888	923	936	936
m = 6	0	206	379	528	655	759	840	898	933	947	948
m = 7	0	209	383	534	661	766	848	908	944	959	961
m = 8	0	212	387	539	668	774	858	918	956	972	973
m = 9	0	215	392	545	675	783	867	929	967	984	986

Optimal oil industry management


















Inclusion of:

Linear and quadratic programming sub problems

$$f(t,s,m) = \max_{h \in H(t,s,m)} \left(\begin{array}{l} \max \pi(x_1,.,x_n;h,t,s,m) \\ s.t.. \\ a_{11}x_1 + ... + \alpha_{1n}x_n \le C_1 \\ ... \\ a_{m1}x_1 + ... + \alpha_{mn}x_n \le C_m \end{array} + \sum_n \tau(n|m) f(t+1, s-h, n) \right)$$

File = PL_OptOil_cases 160219, **OptOil Cases**, ***Peter Lohmander*** 2016-02-19

```
! OptOil 160219
  Peter Lohmander ;

model:

sets:
time/1..7/: year,d;
stock/1..11/:;
market/1..9/:;
sm(stock, market)::;
tsm(time, stock, market): f, hopt, prof;
endsets

submodel initial:
@for(time(t): year(t)=t-1;
     d(t)=@exp(-.05*(t-1))
    );
@for(tsm(t,s,m):
     f(t,s,m) = 0;
     hopt(t,s,m)=0;
     prof(t,s,m)=0;
    );
endsubmodel
```

```
submodel loc1:
max = locobj;
locobj = dloc*p*h + fnext;
h = hloc;
p = 30+(mloc-5)-h;
endsubmodel

submodel loc2:
max = locobj;
locobj = dloc*(p1*x1 + p2*x2) + fnext;
x1+x2 <= hloc;
x1 <= 2;
x2 <= 1;
p1 = 30+(mloc-5)-x1;
p2 = 26-0.5*x2;
endsubmodel
```

```
calc:
@set( 'TERSEO', 2);
@solve(initial);

  TMAX = 7;
  tloc = TMAX;

  @while(tloc #GE#2:
         tloc = tloc-1;
         @write(@newline(1));

            @for(stock(s):
            hmax = s-1;

               @for(market(m):
                    m1 = m;
                    h=-1;

                    @while(h #LT# hmax:
                           h = h+1;

                               fnext = 0;
                               m2 = 0;
                               @while(m2 #LT# 9:
                                      m2 = m2 + 1;
                                      fnext = fnext + 1/9*f(tloc+1, s-h, m2);
                                     );

                                   hloc = h;
                                   mloc = m;
                                   dloc = d(tloc);

                                   @solve(loc2);

                                   @ifc(locobj #GT# f(tloc,s,m):
                                           f(tloc,s,m) = locobj;
                                        hopt(tloc,s,m) = hloc);
                            );
                     );
                );
         );
```

The MAIN STDP LOOP

Since the format of this section is very small, the next pages include the details in large format.

```
calc:
@set( 'TERSEO', 2);
@solve(initial);

  TMAX = 7;
  tloc = TMAX;

  @while(tloc #GE#2:
         tloc = tloc-1;
         @write(@newline(1));

            @for(stock(s):
            hmax = s-1;
```

```
@for(market(m):
                    m1 = m;
                    h=-1;

                    @while(h #LT# hmax:
                           h = h+1;
```

```
fnext = 0;

   m2 = 0;
   @while(m2 #LT# 9:
          m2 = m2 + 1;
          fnext = fnext + 1/9*f(tloc+1, s-h, m2);
          );
```

```
        hloc = h;
        mloc = m;
        dloc = d(tloc);

         @solve(loc2);

         @ifc(locobj #GT# f(tloc,s,m):

              f(tloc,s,m) = locobj;
               hopt(tloc,s,m) = hloc);
              );
       );
    );
);
```

```
@write (@NEWLINE(1));
@write ( 'OptOil by Peter Lohmander 2016-02-19', @Newline(1));
@write ( @newline(2),'Optimal expected present values', @Newline(2));
@for(time(t2):
     @write ( @newline(1), 'Year = ', year(t2), @Newline(1));
     @write ('Reserve =        ');
     @writefor(stock(s):@format(s-1, '5.0f'));
     @write(@newline(2));
     @for(market(m):
          @write('Market = ', @format(m, '3.0f'));
          @write('    ');
          @writefor(stock(s): @format(f(t2,s,m), '5.0f'));
          @write(@newline(1));
          );
     @write(@newline(1));
     );
```

```
@write ( @newline(2),'Optimal extraction levels', @Newline(2));
@for(time(t2):
     @write (@newline(1), 'Year = ', year(t2), @Newline(1));
     @write ('Reserve =        ');
     @writefor(stock(s):@format(s-1, '5.0f'));
     @write(@newline(2));
     @for(market(m):
          @write('Market = ', @format(m, '3.0f'));
          @write('    ');
          @writefor(stock(s): @format(hopt(t2,s,m), '5.0f'));
          @write(@newline(1));
          );
     @write(@newline(1));
     );



endcalc
```

CASE 1.

Price is stochastic and exogenous.

$p = 30+(m-5)$

p is not affected by the production level.
There is only one product: Crude oil.
There are no production constraints.

Optimal decisions:

If the price is above the "reservation price", you instantly extract everything.

In the final period, you extract everything, irrespective of the price level.

Optimal extraction levels

Year = 0

	Reserve =	0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	0	0	0	0	0	0	0	0	0	0
Market =	2	0	0	0	0	0	0	0	0	0	0	0
Market =	3	0	0	0	0	0	0	0	0	0	0	0
Market =	4	0	0	0	0	0	0	0	0	0	0	0
Market =	5	0	1	2	3	4	5	6	7	8	9	10
Market =	6	0	1	2	3	4	5	6	7	8	9	10
Market =	7	0	1	2	3	4	5	6	7	8	9	10
Market =	8	0	1	2	3	4	5	6	7	8	9	10
Market =	9	0	1	2	3	4	5	6	7	8	9	10

Year = 5

Reserve =	0	1	2	3	4	5	6	7	8	9	10
Market = 1	0	1	2	3	4	5	6	7	8	9	10
Market = 2	0	1	2	3	4	5	6	7	8	9	10
Market = 3	0	1	2	3	4	5	6	7	8	9	10
Market = 4	0	1	2	3	4	5	6	7	8	9	10
Market = 5	0	1	2	3	4	5	6	7	8	9	10
Market = 6	0	1	2	3	4	5	6	7	8	9	10
Market = 7	0	1	2	3	4	5	6	7	8	9	10
Market = 8	0	1	2	3	4	5	6	7	8	9	10
Market = 9	0	1	2	3	4	5	6	7	8	9	10

Optimal expected present values

Year = 0

Reserve =	0	1	2	3	4	5	6	7	8	9	10
Market = 1	0	29	59	88	117	146	176	205	234	263	293
Market = 2	0	29	59	88	117	146	176	205	234	263	293
Market = 3	0	29	59	88	117	146	176	205	234	263	293
Market = 4	0	29	59	88	117	146	176	205	234	263	293
Market = 5	0	30	60	90	120	150	180	210	240	270	300
Market = 6	0	31	62	93	124	155	186	217	248	279	310
Market = 7	0	32	64	96	128	160	192	224	256	288	320
Market = 8	0	33	66	99	132	165	198	231	264	297	330
Market = 9	0	34	68	102	136	170	204	238	272	306	340

CASE 2.

Price is stochastic and partly endogenous.

p = 30+(m–5)–h

There is only one product: Crude oil.
There are no production constraints.

Optimal decisions:

The optimal extraction level is an increasing function of the state of the market and of the size of the remaining reserve.

In the final period, you extract everything, irrespective of the price level.

Optimal extraction levels

Year = 0

		0	1	2	3	4	5	6	7	8	9	10
Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	0	0	0	0	1	1	1	1	1	2
Market =	2	0	0	0	0	1	1	1	1	2	2	2
Market =	3	0	0	0	1	1	1	2	2	2	2	2
Market =	4	0	0	1	1	1	2	2	2	2	3	3
Market =	5	0	1	1	1	2	2	2	3	3	3	3
Market =	6	0	1	1	2	2	2	3	3	3	3	4
Market =	7	0	1	2	2	2	3	3	3	4	4	4
Market =	8	0	1	2	2	3	3	3	4	4	4	4
Market =	9	0	1	2	3	3	3	4	4	4	5	5

Optimal expected present values

Year = 0

	Reserve =	0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	28	55	81	107	132	157	181	205	228	251
Market =	2	0	28	55	81	107	133	158	182	206	230	253
Market =	3	0	28	55	82	108	134	159	184	208	232	255
Market =	4	0	28	56	83	109	135	161	186	210	234	258
Market =	5	0	29	57	84	111	137	163	188	213	237	261
Market =	6	0	30	58	86	113	139	165	191	216	240	264
Market =	7	0	31	60	88	115	142	168	194	219	244	268
Market =	8	0	32	62	90	118	145	171	197	223	248	272
Market =	9	0	33	64	93	121	148	175	201	227	252	277

CASE 3.

```
submodel loc2:
max = locobj;
locobj = d*(p1*x1 + p2*x2) + fnext;
x1+x2 <= h;
p1 = 30+(m-5)-x1;
p2 = 26-0.0*x2;
endsubmodel
```

There are two products, x1 (Crude oil) and x2 (refined).

p1 is highly dependent on the stochastic market and very volume sensitive.

p2 is constant and insensitive to production volume.

There are no capacity limitations.

<u>Optimal decisions (in Case 3):</u>

If the remaining reserve level is very high, you should extract large quantitites and produce large volumes of x2.

If the remaining reserve level is low, you should use most of it for x1, waiting for good levels of p1.

In the final period, you extract everything, irrespective of price.

Optimal extraction levels

Year = 0

Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	0	0	0	1	2	3	4	5	6	7
Market =	2	0	0	0	1	1	2	3	4	5	6	7
Market =	3	0	0	0	1	1	2	3	4	5	6	7
Market =	4	0	0	1	1	2	2	3	4	5	6	7
Market =	5	0	1	1	1	2	2	3	4	5	6	7
Market =	6	0	1	1	2	2	3	3	4	5	6	7
Market =	7	0	1	2	2	2	3	3	4	5	6	7
Market =	8	0	1	2	2	3	3	4	4	5	6	7
Market =	9	0	1	2	3	3	3	4	4	5	6	7

Optimal expected present values

Year = 0

Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	28	55	81	107	133	159	185	211	237	263
Market =	2	0	28	55	82	108	134	160	186	212	238	264
Market =	3	0	28	55	82	108	134	160	186	212	238	264
Market =	4	0	28	56	83	110	136	162	188	214	240	266
Market =	5	0	29	57	84	111	137	163	189	215	241	267
Market =	6	0	30	58	86	113	140	166	192	218	244	270
Market =	7	0	31	60	88	115	142	168	194	220	246	272
Market =	8	0	32	62	90	118	145	172	198	224	250	276
Market =	9	0	33	64	93	121	148	175	201	227	253	279

CASE 4.

```
submodel loc2:
max = locobj;
locobj = d*(p1*x1 + p2*x2) + fnext;
x1+x2 <= h;
p1 = 30+(m-5)-x1;
p2 = 26-0.5*x2;
endsubmodel
```

There are two products, x1 (Crude oil) and x2 (refined).
p1 is highly dependent on the stochastic market and very volume sensitive.
p2 is not stochastic but sensitive to production volume.
There are no capacity limitations.

Optimal decisions (in Case 4):

If the remaining reserve level is very high, you should produce rather high volumes of x2.

If the remaining reserve level is low, you should use most of it for x1, waiting for good levels of p1.

In the final period, you extract everything, irrespective of price.

Optimal extraction levels

Year = 0

Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	0	0	0	1	1	2	2	2	3	3
Market =	2	0	0	0	1	1	1	2	2	3	3	3
Market =	3	0	0	0	1	1	2	2	3	3	3	4
Market =	4	0	0	1	1	2	2	2	3	3	4	4
Market =	5	0	1	1	1	2	2	3	3	4	4	4
Market =	6	0	1	1	2	2	3	3	3	4	4	5
Market =	7	0	1	2	2	2	3	3	4	4	5	5
Market =	8	0	1	2	2	3	3	4	4	4	5	5
Market =	9	0	1	2	3	3	3	4	4	5	5	6

Optimal expected present values

Year = 0

		0	1	2	3	4	5	6	7	8	9	10
Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	28	55	81	107	133	158	183	207	231	256
Market =	2	0	28	55	82	108	133	158	183	208	233	257
Market =	3	0	28	55	82	108	134	160	185	210	234	258
Market =	4	0	28	56	83	110	136	161	186	211	236	261
Market =	5	0	29	57	84	111	137	163	189	214	239	263
Market =	6	0	30	58	86	113	140	166	191	216	241	266
Market =	7	0	31	60	88	115	142	168	194	220	245	270
Market =	8	0	32	62	90	118	145	172	198	223	248	273
Market =	9	0	33	64	93	121	148	175	201	227	253	278

CASE 5.

```
submodel loc2:
max = locobj;
locobj = d*(p1*x1 + p2*x2) + fnext;
x1+x2 <= h;
x1 <= 2;
x2 <= 1;
p1 = 30+(m-5)-x1;
p2 = 26-0.5*x2;
endsubmodel
```

There are two products, x1 (Crude oil) and x2 (refined).
p1 is highly dependent on the stochastic market and very volume sensitive.
p2 is not stochastic but sensitive to production volume.
The production levels of of x1 and x2 have capacity constraints.

Optimal decisions (in Case 5):

If the remaining reserve level is very high, you should produce x2 at the capacity limit.

If the remaining reserve level is low, you should use most of it for x1, waiting for good levels of p1.

In the final period, you extract as much as possible, considering the capacity limitations, irrespective of price.

In periods close to the final period, the expected optimal present values are strongly reduced by the production capacity limitations, in case the remaining reserve is large.

Optimal extraction levels

Year = 0

Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	0	0	0	1	1	2	2	2	2	2
Market =	2	0	0	0	1	1	2	2	2	2	3	3
Market =	3	0	0	0	1	1	2	2	3	3	3	3
Market =	4	0	0	1	1	2	2	3	3	3	3	3
Market =	5	0	1	1	1	2	2	3	3	3	3	3
Market =	6	0	1	1	2	2	2	3	3	3	3	3
Market =	7	0	1	2	2	2	2	3	3	3	3	3
Market =	8	0	1	2	2	2	2	3	3	3	3	3
Market =	9	0	1	2	2	2	2	3	3	3	3	3

Year = 5

		0	1	2	3	4	5	6	7	8	9	10
Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	1	2	3	3	3	3	3	3	3	3
Market =	2	0	1	2	3	3	3	3	3	3	3	3
Market =	3	0	1	2	3	3	3	3	3	3	3	3
Market =	4	0	1	2	3	3	3	3	3	3	3	3
Market =	5	0	1	2	3	3	3	3	3	3	3	3
Market =	6	0	1	2	3	3	3	3	3	3	3	3
Market =	7	0	1	2	3	3	3	3	3	3	3	3
Market =	8	0	1	2	3	3	3	3	3	3	3	3
Market =	9	0	1	2	3	3	3	3	3	3	3	3

Optimal expected present values

Year = 0

		0	1	2	3	4	5	6	7	8	9	10
Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	28	55	81	107	132	157	182	206	229	253
Market =	2	0	28	55	82	108	133	158	183	207	231	254
Market =	3	0	28	55	82	108	134	159	184	209	233	256
Market =	4	0	28	56	83	110	136	161	186	211	235	258
Market =	5	0	29	57	84	111	137	163	188	213	237	260
Market =	6	0	30	58	86	113	139	165	190	215	239	262
Market =	7	0	31	60	88	115	141	167	192	217	241	264
Market =	8	0	32	62	90	117	143	169	194	219	243	266
Market =	9	0	33	64	92	119	145	171	196	221	245	268

Year = 5

Reserve =		0	1	2	3	4	5	6	7	8	9	10
Market =	1	0	20	39	57	57	57	57	57	57	57	57
Market =	2	0	20	40	59	59	59	59	59	59	59	59
Market =	3	0	21	41	60	60	60	60	60	60	60	60
Market =	4	0	22	42	62	62	62	62	62	62	62	62
Market =	5	0	23	44	63	63	63	63	63	63	63	63
Market =	6	0	23	45	65	65	65	65	65	65	65	65
Market =	7	0	24	47	67	67	67	67	67	67	67	67
Market =	8	0	25	48	68	68	68	68	68	68	68	68
Market =	9	0	26	50	70	70	70	70	70	70	70	70

General conclusions

- Stochastic dynamic programming is a fantastic method that can be used in many highly relevant problems, in particular with stochastic market prices and adaptive production decisions.
- A common opinion is however that the “curse of dimensionality” makes it impossible to handle relevant real world problems with this method. For this reason, several attempts have been made to develop alternative approaches.
- In the light of this situation, the author suggests the use of stochastic dynamic programming as a master problem combined with linear programming or quadratic programming solutions to optimal decisions at each stage and state.
- This makes it possible to keep the real stochastic structure of the problem and to optimize the relevant adaptive production level decisions.
- At the same time, any level of detail can be handled in the many production and logistics oriented decisions at lower levels.
- These approaches are found here: Lohmander [5] and [9].

Suggestions for future research

- Use the presented approach to optimization. Peter Lohmander is personally interested to cooperate with these projects:
- For alternative cases, with consideration of market structure, in particular concerning coordination within OPEC and other market actors:
- Develop a complete optimization model for management of oil and other relevant resources, using relevant price processes for crude oil and refined products, local data for oil fields, oil extraction equipment, logistics, refining and other relevant data.
- Determine optimal investments in refining capacities.
- Determine optimal investments in pipelines, other infrastructure and oil tankers.
- Determine optimal management of the oil industry.

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