Dynamics and control of the CO_2 level via a differential equation and alternative global emission strategies

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Abstract

The analysis in this paper shows that the fundamental theory of the CO_2 level in the atmosphere, under the influence of changing CO_2 emissions, can be modeled as a first order linear differential equation with a forcing function, describing industrial emissions.

Observations of the CO_2 level at the Mauna Loa CO_2 observatory and official statistics of global CO_2 emissions, from Edgar, the Joint Research Centre at the European Commission, are used to estimate all parameters of the forced CO_2 differential equation.

The estimated differential equation has a logical theoretical foundation and convincing statistical properties. It is used to reproduce the time path of the CO_2 data from Mauna Loa, from year 1990 to 2018, with very small errors. Furthermore, the differential equation shows that the global CO_2 level, without emissions, has a stable equilibrium at 280 ppm. This value has earlier been reported by IPCC as the pre-industrial CO_2 level.

The differential function is applied to derive four dynamic cases of the global CO_2 level, from the year 2020 until 2100, conditional on four different strategies concerning the development of global CO_2 emissions: i. Emissions continue to increase according to the trend during 1990 – 2018., ii. Emissions stay for ever at the 2020 level., iii. Emissions are reduced with a linear trend to become zero year 2100., iv. Emissions are reduced with a linear trend to become zero year 2050. In case i., the CO_2 level year 2100 will be 688 ppm. In cases ii. and iii., the CO_2 levels in 2100 will be 517 ppm and 389 respectively. In case iv., the CO_2 level in 2050 is 408 ppm and then rapidly falls.

Introduction

The global warming and CO_2 dynamics issue, for very good reasons, attracts considerable global interest. The climate of our planet is of key importance to all life. The author recommends the reader to study Ramade[4] in detail for a deep understanding of many of the connected issues and theories.

The first ambition is to understand the fundamental mechanisms of the dynamics of the CO_2 level of the atmosphere under the influence of global emissions.

We will investigate if it is possible to develop a theoretical mathematical model of the dynamics of CO_2. Such a model should be consistent with fundamental scientific principles. Furthermore, it should be possible to use the model to reproduce historical time series of empirical data. If such a model can be developed, it should be possible to use it also for predictions. Then, the most important application is to investigate how the global CO_2 level can be dynamically changed via different emissions strategies.

Statistics of the CO_2 level in the atmosphere and the global CO2 emissions

The CO_2 level of the atmosphere has been recorded since 1958, at the Mauna Loa observatory. See Tans and Keeling[6]. The statistical tables are well documented and freely available via the internet. In Figure 1., the annual mean values of CO_2 are shown. The web link connected to the reference provides access to all observations via a text file with instructions. In several cases, transformations between different physical units are necessary. O'Hara[3] includes the relevant conversion factors.

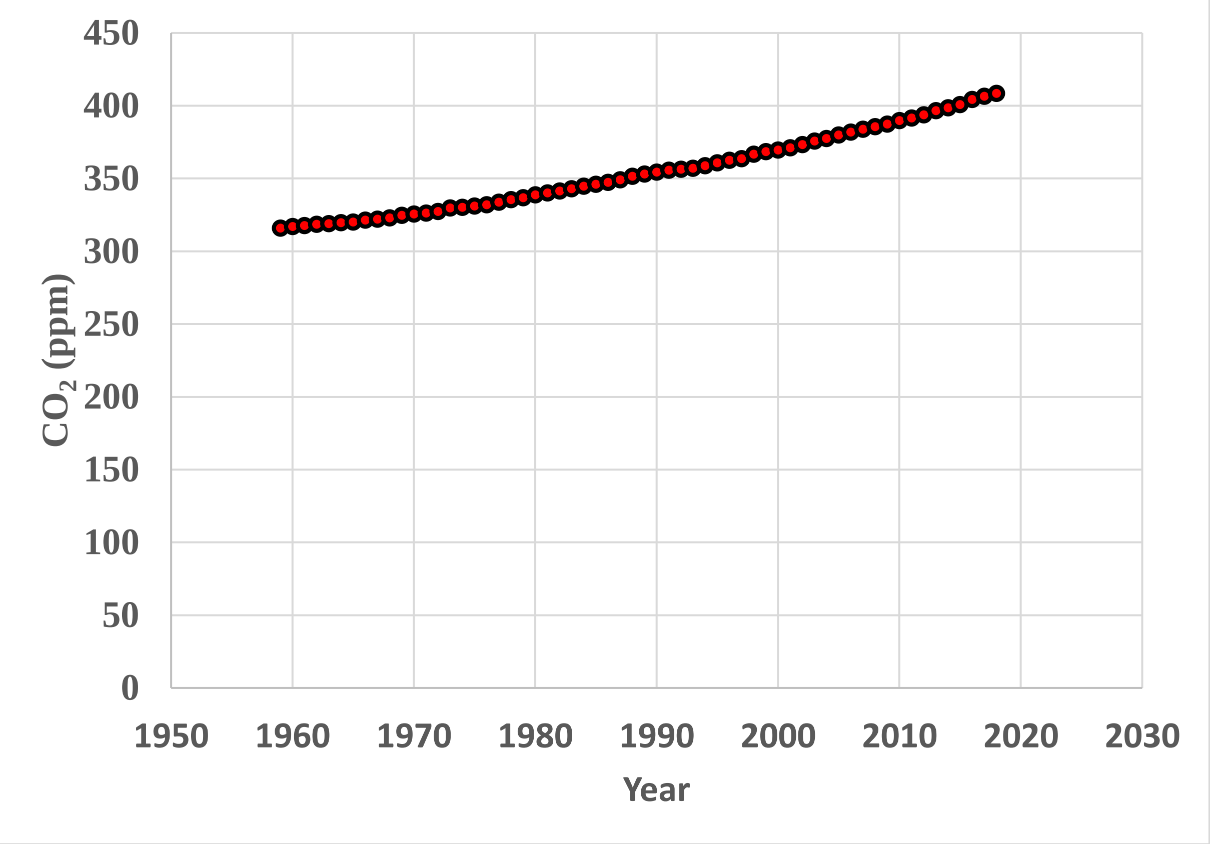

Figure 1.

CO_2 in the atmosphere, annual mean values, Mauna Loa, (ppm). Source: Tans and Keeling[6].

In Figure 2., we find observations of global CO_2 emissions from fossil fuels combustion and processes. These data come from European Commission[2]. The observations from 1990, 2000, 2010 and 2018 have been used in this analysis of this paper. There are two reasons for this:

First, emission data were only collected with ten year intervals during the early years. Second, sufficiently long time intervals are needed if we want to be able to estimate the changes of CO_2 in the atmosphere with sufficiently high precision.

In the estimations of a differential equation, the following three periods will be used: 1990 – 2000, 2000 – 2010 and 2010 – 2018. More details about these periods are found in Table 2.

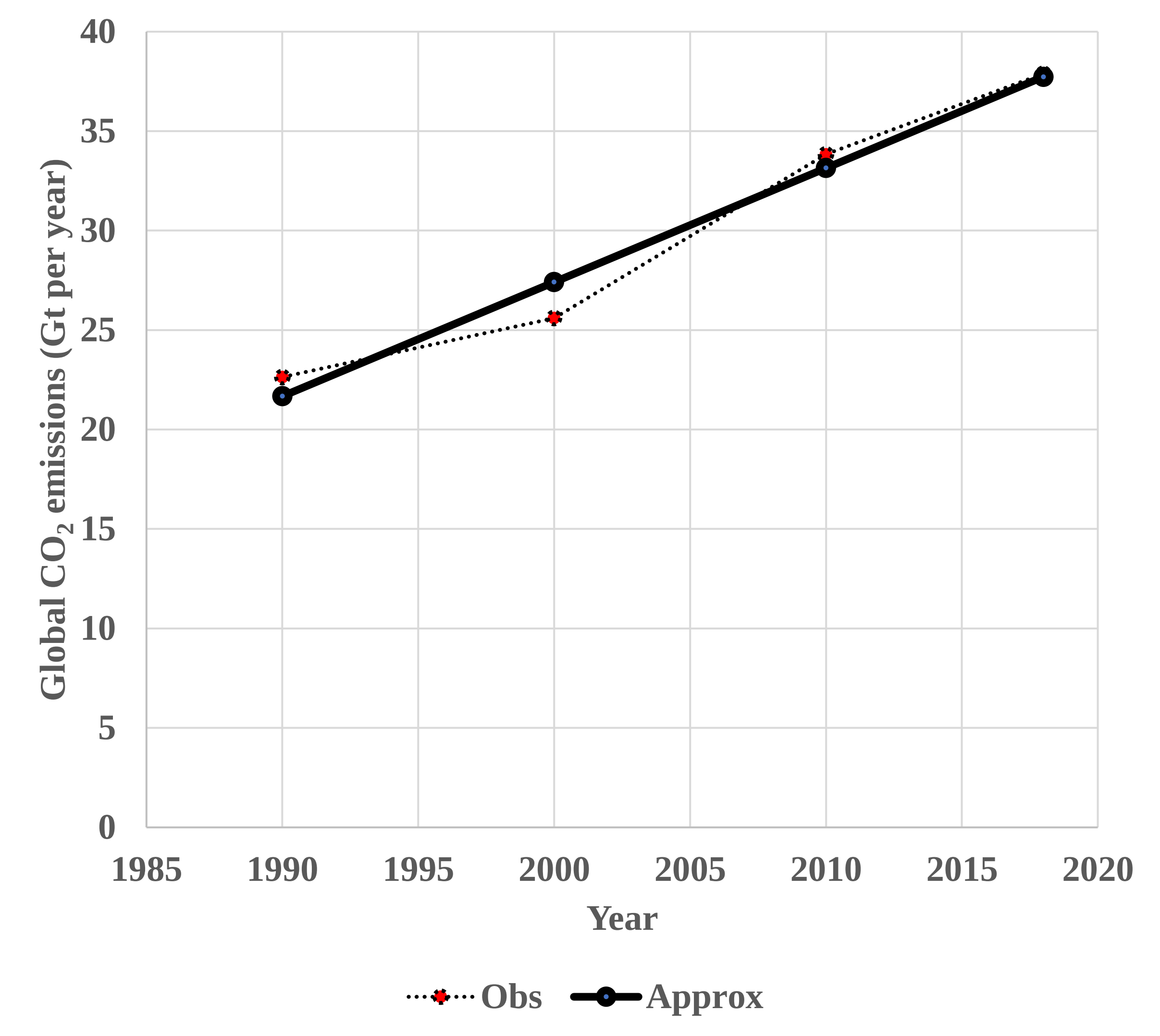

Figure 2.

Obs = Observations of global CO_2 emissions from fossil fuels combustion and processes. Source: European Commission[2]. Approx = Linear approximation via the least squares method, by the author of this paper. Compare equation (47).

Approx $= 21.672 + 0.57366(Year - 1990)$. $R \approx 0.984$.

The emission forced differential equation of the global CO_2 level

The general theory of differential equations can be studied in Braun[1].

Let us first consider the following differential equation. We will soon discover that it has to be adjusted in order to become relevant to the CO_2 problem.

$$\dot{x} = \frac{dx}{dt} = a_0 \tag{1}$$

$x = x(t)$ is the CO_2 level in the atmosphere as a function of time. $\dot{x} = \frac{dx}{dt}$ is the change per time unit, or the time derivative, of x. There are constant "natural" emissions, from the oceans, volcanoes and other parts of the natural environment, greater than zero. $a_0 > 0$. Hence, $\dot{x} = \frac{dx}{dt}$ would be strictly positive and x would increase over time, without bound, if nothing would stop that.

However, earlier CO_2 research has already shown that the CO_2 level has been stable during very long periods of time. Compare Ramade[4] and Solomon et al[5].

Let us assume that the oceans (and, to some degree, other parts of the natural environment) absorb a part of the CO_2 in the atmosphere. Let us also assume that the absorbtion is proportional to the CO_2 level in the atmosphere, x. This is a very reasonable assumption since the probability that a CO_2 molecule touches the surface of the sea is proportional to the CO_2 level in the atmosphere. Let the absorbtion be $-a_x x$. Then, we have this differential equation of global CO_2:

$$\dot{x} = \frac{dx}{dt} = a_0 + a_x x \tag{2}$$

Is there an equilibrium?

$$\dot{x} = \frac{dx}{dt} = a_0 + a_x x = 0 \tag{3}$$

Yes, there is one and only one equilbrium.

$$\left(\dot{x} = 0\right) \Rightarrow x = x_{eq} = \frac{-a_0}{a_x} > 0 \tag{4}$$

Is this equilibrium stable? Yes, if something disturbes x so that $x < x_{eq}$, then $\dot{x} > 0$, which means that x increases until $x = x_{eq}$. If $x > x_{eq}$, then $\dot{x} < 0$, and x decreases until $x = x_{eq}$.

According to earlier research, the pre-industrial equilibrium level of CO_2 was 280 ppm (parts per million). Compare the IPCC report by Solomon et al[5]. In this paper, we will find that the derived model confirms this finding. In other words, we will confirm that.

$$x_{eq} = \frac{-a_0}{a_x} \approx 280 \tag{5}$$

In order to determine the parameters of a function, it is necessary to have some variation in the data. In particular, when we want to determine the values of the parameters of the differential equation of x, we can not do this if $x = x_{eq}$ all the time. In this respect, it is useful to observe that the industrial emissions of CO_2 during the latest decades have created earlier not available variation in x. Let us regard global emissions of CO_2, after the industrial revolution, $\varphi(t)$, as a function of t. The emissions are added to the CO_2 in the atmosphere.

$$\dot{x} = a_0 + a_x x + \varphi(t) \qquad (6)$$

Now, since we have access to empirical data for $\left(\dot{x}, \varphi\right)$ in different time periods, we can estimate the parameters (a_0, a_x) via the ordinary least squares method (regression analysis) in the following way:

$$y(t) = \dot{x} - \varphi(t) = a_0 + a_x x(t) \qquad (7)$$

Table 1. includes the transformations of the available atmospheric CO_2 raw data to a time seriers of $\dot{x}$ that will be used in the analysis. In a similar way, in Table 2., the global emission data is developed to time series data for φ.

Table 1.

Atmospheric CO_2 data.

i (period)	t (year)	ψ_t (ppm)	Δx_i (ppm)	Δt (years)	x_i (ppm)	x_i (Gt CO_2)	$\dot{x}_i \approx \frac{\Delta x}{\Delta t}$ (ppm per year)	$\dot{x}_i \approx \frac{\Delta x}{\Delta t}$ (Gt CO_2 per year)
	1990	354.39						
1			15.16	10	361.97	2824.9	1.516	11.831
	2000	369.55						
2			20.35	10	379.725	2963.5	2.035	15.882
	2010	389.90						
3			18.62	8	399.21	3115.6	2.3275	18.165
	2018	408.52						

<u>Definitions in Table 1.:</u>

ψ_t = CO_2 in atmosphere, annual mean value of observations, Mauna Loa.

x_i = CO_2 in atmosphere, calculated mean value.

Gt denotes Giga tonnes and ppm denotes parts per million.

Table 2.

Atmospheric CO_2 data transformations.

i (period)	t (year)	γ_t (Gt CO_2)	φ_i (Gt CO_2 per year)	ϕ_i (Gt CO_2 per year)	ϕ_i (ppm per year)
	1990	22.637			
1			24.119	12.288	1.5745
	2000	25.601			
2			29.7185	13.8365	1.7729
	2010	33.836			
3			35.8615	17.6965	2.2675
	2018	37.887			

Definitions in Table 2.:

γ_t = Global total CO_2 emission, observation.

φ_i = Global total CO_2 emission, calculated mean value.

$$\phi_i = \varphi_i - \dot{x}_i$$

In different statistical sources and equations, the CO_2 of the atmosphere is given in different units. Following the principles by O'Hara[3], the following transformation rules have been applied:

1 ppm (CO_2) can be transformed to 2.13*3.664 = 7.80432 Gt CO_2. 1 ppm by volume of atmosphere CO_2 = 2.13 Gt C. 1 g C = 0.083 mole CO_2 = 3.664 g CO_2.

Now, the data series developed in Table 1. and Table 2. are used to produce the regression data set found in Table 3.

Table 3.

Regression data.

i	x_i (ppm)	y_i (Gt CO_2 per year)
1	361.97	-12.288
2	379.725	-13.8365
3	399.21	-17.6965

Definitions in Table 3:

$$y_i = -\phi_i = -\left(\varphi_i - \dot{x}_i\right)$$

Below, a very high level of detail in the calculations has been selected. The motivation is the following: The CO_2 dynamics and global warming issue is critical to the present global political debate. It is necessary that the reader can investigate and repeat all derivations without problems.

We want to determine the parameters (a_0, a_x) in this function:

$$y = a_0 + a_x x \tag{8}$$

We minimize the sum of squares of the residuals:

$$\min_{a_0, a_x} Z = \sum_{i=1}^{N} \left(y_i - a_0 - a_x x_i \right)^2 \tag{9}$$

These are the first order optimum conditions:

$$\begin{cases} \dfrac{dZ}{da_0} = \sum_{i=1}^{N} \left(2\left(y_i - a_0 - a_x x_i \right)(-1) \right) = 0 \\ \dfrac{dZ}{da_x} = \sum_{i=1}^{N} \left(2\left(y_i - a_0 - a_x x_i \right)(-x_i) \right) = 0 \end{cases} \tag{10}$$

They are further developed:

$$\begin{cases} \dfrac{dZ}{da_0} = 2\sum_{i=1}^{N} \left(\left(a_0 + a_x x_i - y_i \right) \right) = 0 \\ \dfrac{dZ}{da_x} = 2\sum_{i=1}^{N} \left(\left(a_0 x_i + a_x x_i^2 - x_i y_i \right) \right) = 0 \end{cases} \tag{11}$$

We also want to investigate if the derived solution gives a unique minimum:

$$\frac{d^2 Z}{da_0^{\,2}} = 2\sum 1 = 2N > 0 \tag{12}$$

$$\frac{d^2 Z}{da_x^{\,2}} = 2\sum x^2 > 0 \tag{13}$$

$$\Phi = \begin{vmatrix} \dfrac{d^2 Z}{da_0^{\,2}} & \dfrac{d^2 Z}{da_0 da_x} \\ \dfrac{d^2 Z}{da_x da_0} & \dfrac{d^2 Z}{da_x^{\,2}} \end{vmatrix} = \begin{vmatrix} 2\sum 1 & 2\sum x \\ 2\sum x & 2\sum x^2 \end{vmatrix} \tag{14}$$

$$\Phi = 4\begin{vmatrix} N & \sum x \\ \sum x & \sum x^2 \end{vmatrix} = 4\left(N\sum x^2 - \left(\sum x\right)^2 \right) \tag{15}$$

(16)

$$\Phi = 4N^2\left(\frac{\sum x^2}{N} - \left(\frac{\sum x}{N}\right)^2\right)$$

$$\Phi = 4N^2\left(E\left[x^2\right] - E\left[x\right]^2\right) \tag{17}$$

$$\left(N > 2 \wedge Var(x) > 0\right) \Rightarrow \Phi > 0 \tag{18}$$

Hence, the second order conditions of a unique minimum are satisfied. The first order conditions give a unique minimum. The first order optimum conditions imply:

$$\begin{cases} (N)a_0 + \left(\sum x_i\right)a_x = \left(\sum y_i\right) \\ \left(\sum x_i\right)a_0 + \left(\sum x_i^2\right)a_x = \left(\sum x_i y_i\right) \end{cases} \tag{19}$$

The parameters can be determined from this simultaneous equation system:

$$\begin{bmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_x \end{bmatrix} = \begin{bmatrix} \sum y_i \\ \sum x_i y_i \end{bmatrix} \tag{20}$$

<u>Table 4.</u>

Parameter values.

N	3
$\sum x_i$	1140.905
$\sum x_i^2$	434581.9806
$\sum x_i y_i$	-16766.57209
$\sum y_i$	-43.821

The point (a_0, a_x) is determined via Cramers rule:

$$a_0 = \frac{\begin{vmatrix} \sum y_i & \sum x_i \\ \sum x_i y_i & \sum x_i^2 \end{vmatrix}}{\begin{vmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{vmatrix}} \approx \frac{85248.955}{2081.723} \approx 40.951 \tag{21}$$

$$a_x = \frac{\begin{vmatrix} N & \sum y_i \\ \sum x_i & \sum x_i y_i \end{vmatrix}}{\begin{vmatrix} N & \sum x_i \\ \sum x_i & \sum x_i^2 \end{vmatrix}} \approx \frac{-304.118}{2081.723} \approx -0.14609 \tag{22}$$

If we express $\dot{x}$ in the unit Gt CO_2 /year, and x in the unit ppm, we have this equation:

$$\dot{x} = 40.951 - 0.14609x \tag{23}$$

What is the equilibrium value of x, via the derived function, in case there are no emissions?

$$\dot{x} = \frac{dx}{dt} = a_0 + a_x x_{eq} = 0 \tag{24}$$

$$x_{eq} = \frac{-a_0}{a_x} \approx 280.31 \quad (ppm) \tag{25}$$

Note that this value confirms the ealier empirical finding by Solomon et al[5]. If we express $\dot{x}$ in the unit Gt CO_2 /year, and x in the unit Gt CO_2, we get the following differential equation. Note that the coefficient of x has been divided by 2.13*3.664, namely by 7.80432 :

$$\dot{x} = 40.951 - 0.0187191x \tag{26}$$

$$x_{eq} = \frac{-40.951}{-0.0187191} \approx 2187.66 \quad (Gt) \tag{27}$$

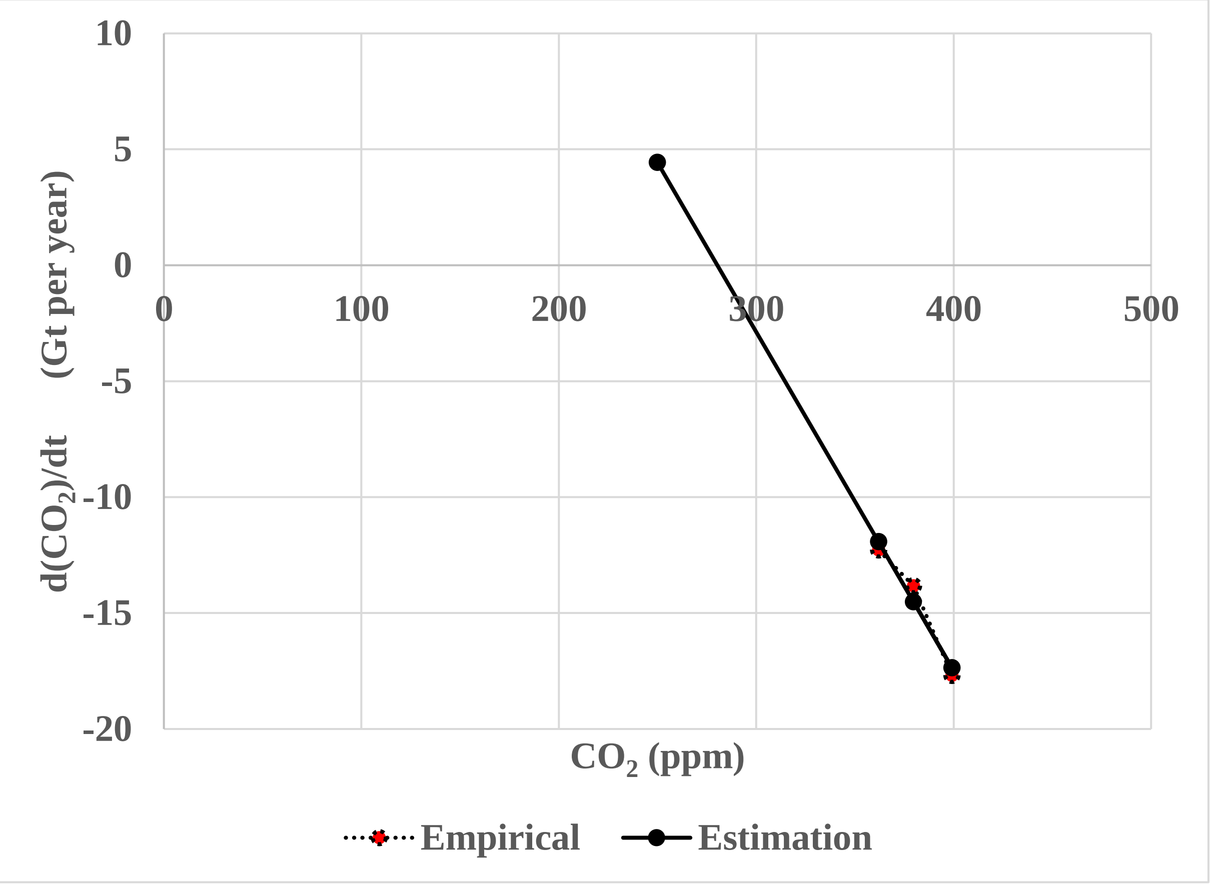

Figure 3.

Determination of the global CO_2 differential equation via the empirical observations of CO_2 from Mouna Loa and the empirical observations of global CO_2 emissions. The estimated equilibrium value of CO_2 is 280 ppm, in case the global emissions of CO_2 is zero. This confirms the earlier findings. Compare Solomon et al[5]. The estimated function is: 40.951 – 0.14609 * CO_2 (ppm). The multiple correlation coefficient R = 0.977. Since the number of observations is limited, more detailed regression statistics will not be given here.

Determination of the differential equation of CO_2 in the atmosphere under the influence of changing CO_2 emissions

Now, the complete differential equation will be determined, giving the dynamic development of the CO_2 level in the amosphere as a function of the development of the global emissions.

This is the differential equation in general form:

$$\dot{x} = a_0 + a_x x + \varphi(t) \tag{28}$$

We will consider the special case of emissions that grow with a linear trend, since that is supported by the available empirical data. (Note that the forcing function could be generalized to almost any form, if considered relevant.)

$$\varphi(t) = m_0 + m_1 t \tag{29}$$

The differential equation becomes:

$$\dot{x} - a_x x = a_0 + m_0 + m_1 t \tag{30}$$

Solution of the homogenous equation:

$$\dot{x}_h - a_x x_h = 0 \tag{31}$$

$$x_h = Ae^{st} \tag{32}$$

$$\dot{x}_h = sAe^{st} \tag{33}$$

$$(s - a_x)x_h = 0 \tag{34}$$

$$\left(x_h \neq 0\right) \Rightarrow s = a_x \tag{35}$$

$$x_h(t) = Ae^{a_x t} \tag{36}$$

Determination of the particular solution:

$$x_p = k_0 + k_1 t \tag{37}$$

$$\dot{x}_p - a_x x_p = a_0 + m_0 + m_1 t \tag{38}$$

$$k_1 - a_x \left(k_0 + k_1 t\right) = a_0 + m_0 + m_1 t \tag{39}$$

$$\begin{cases} k_1 - a_x k_0 = a_0 + m_0 \\ \quad -a_x k_1 = m_1 \end{cases} \tag{40}$$

$$\left(-a_x k_1 = m_1\right) \Rightarrow k_1 = \frac{-m_1}{a_x} \tag{41}$$

$$\left(k_1 - a_x k_0 = a_0 + m_0\right) \wedge \left(k_1 = \frac{-m_1}{a_x}\right) \Rightarrow \left(\frac{-m_1}{a_x} - a_x k_0 = a_0 + m_0\right) \tag{42}$$

$$k_0 = \frac{-\left(a_0 + m_0 + \frac{m_1}{a_x}\right)}{a_x} \tag{43}$$

Determination of $\varphi(t) = m_0 + m_1 t$

Now, in order to use the derived function for predictions, we estimate the parameters $\left(m_0, m_1\right)$. We follow the same procedure as in the earlier section of this paper.

Table 5.

Regression data.

j	Year	t	$\varphi(t)$ (Gt CO_2 per year)
1	1990	0	22.637
2	2000	10	25.601
3	2010	20	33.836
4	2018	28	37.887

Definitions in Table 5:

t = Year – 1990.

Table 6.

Parameter values.

N	4
$\sum t_j$	58
$\sum t_j^{\,2}$	1284
$\sum t_j \varphi_j$	1993.566
$\sum \varphi_j$	119.961

The parameters can be determined from this simultaneous equation system:

$$\begin{bmatrix} N & \sum t_j \\ \sum t_j & \sum t_j^{\,2} \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \end{bmatrix} = \begin{bmatrix} \sum \varphi_j \\ \sum t_j \varphi_j \end{bmatrix} \tag{44}$$

The point (m_0, m_1) is determined via Cramers rule:

$$m_0 = \frac{\begin{vmatrix} \sum \varphi_j & \sum t_j \\ \sum t_j \varphi_j & \sum t_j^{\,2} \end{vmatrix}}{\begin{vmatrix} N & \sum t_j \\ \sum t_j & \sum t_j^{\,2} \end{vmatrix}} \approx \frac{38403.096}{1772} \approx 21.672 \tag{45}$$

$$m_1 = \frac{\begin{vmatrix} N & \sum \varphi_j \\ \sum t_j & \sum t_j \varphi_j \end{vmatrix}}{\begin{vmatrix} N & \sum t_j \\ \sum t_j & \sum t_j^{\,2} \end{vmatrix}} \approx \frac{1016.526}{1772} \approx 0.57366 \tag{46}$$

(The multiple correlation coefficient: R = 0.984)

$$\varphi(t) = 21.672 + 0.57366\, t \tag{47}$$

$$k_1 = \frac{-m_1}{a_x} = \frac{-0.57366}{-0.0187191} \approx 30.646 \tag{48}$$

$$k_0 = \frac{-\left(a_0 + m_0 + \frac{m_1}{a_x}\right)}{a_x} = \frac{-(40.951 + 21.672 - 30.646)}{-0.0187191} \approx 1708.27 \tag{49}$$

$$x(t) = Ae^{-0.0187191t} + 1708.27 + 30.646\,t \tag{50}$$

$$x(0) = A + 1708.27 \tag{51}$$

$$A = x(0) - 1708.27 \tag{52}$$

$$A = 354.39 \cdot 2.13 \cdot 3.664 - 1708.27 \tag{53}$$

$$A \approx 1057.52 \tag{54}$$

$$x(t) = 1057.52e^{-0.0187191t} + 1708.27 + 30.646\ t \qquad (Gt) \tag{55}$$

If the function is divided by (2.13*3.664), the unit becomes ppm.

$$x(t) = 135.50e^{-0.0187191t} + 218.89 + 3.927\ t \qquad (ppm) \tag{56}$$

In Figure 4., we find that the estimated function can reproduce the CO_2 observations from Mauna Loa extremely well. Most years, during the period 1990 to 2018, the deviations are less than 1 ppm.

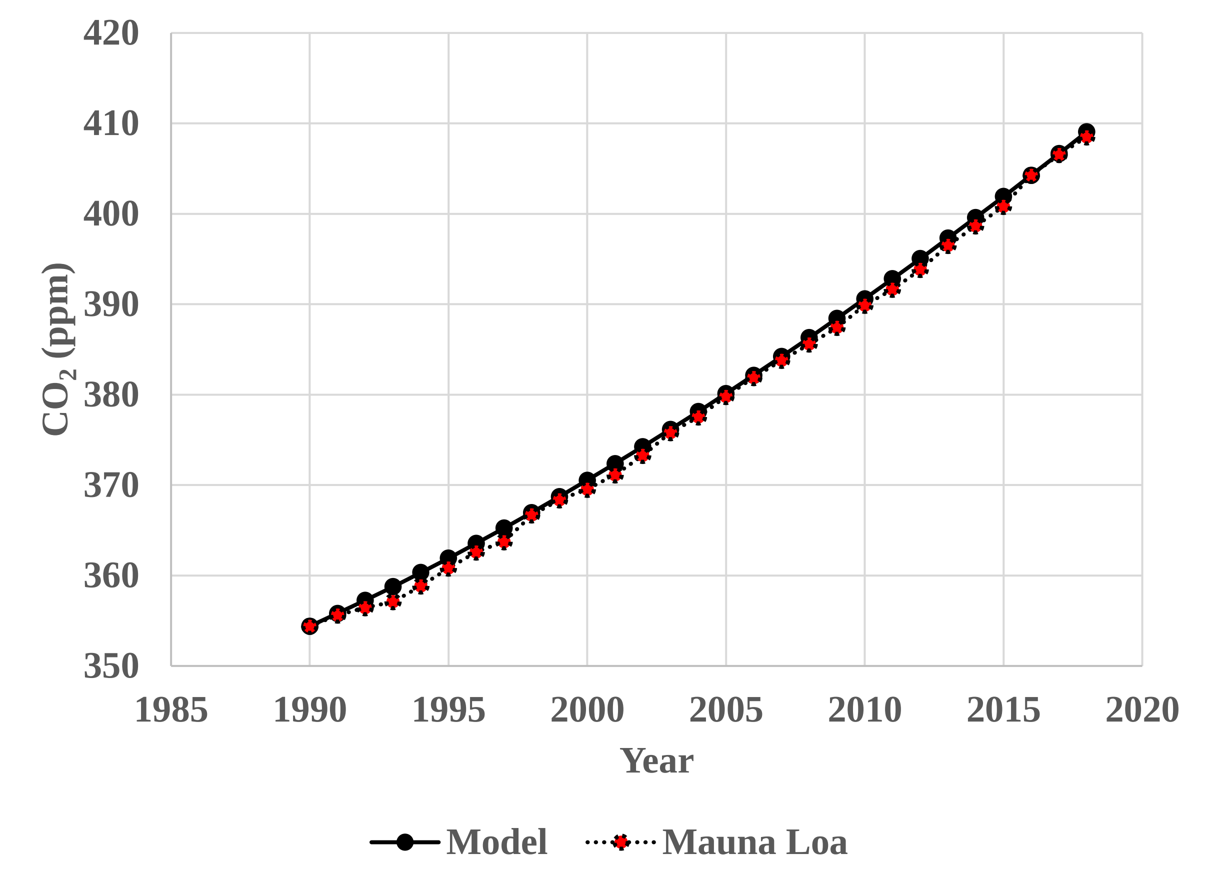

Figure 4.

Mauna Loa = CO_2 observations from 1990 to 2018. Model = CO_2 prediction model. The empirical CO_2 observations from Mauna Loa, compare Figure 1., and the prediction according to the derived differential equation model are almost identical. The graph was derived with the following equation:

$$x(t) = 135.50e^{-0.0187191t} + 218.89 + 3.927t \qquad (ppm).$$

Predictions into the future

Now, the estimated differential equation will be used to predict the future development of the CO_2 level, conditional on the following four alternative global emission strategies:

Cont: During the period 2020 to 2100, the emissions continue to increase according to the trend estimated during the period 1990 to 2018.

Lev 2020: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions stay at that level until 2100.

Stop 2100: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions are reduced with a constant amount each year, such that the emissions are zero in 2100.

Stop 2050: The emissions 2020 are estimated from the trend 1990 to 2018. Then, the emissions are reduced with a constant amount each year, such that the emissions are zero in 2050.

In Figure 5., we see the graphs of the four emission scenarios and in Table 7. we find more details about the four scenarios.

Table 7.

Parameter values for predictions.

Alternative	Year when t=0	x(0)_ppm	a0	ax	m0	m1
Cont	1990	354,39	40,951	-0,01872	21,672	0,57366
Lev 2020	2020	413,96911	40,951	-0,01872	38,8818	0
Stop 2100	2020	413,96911	40,951	-0,01872	38,8818	-0,48602
Stop 2050	2020	413,96911	40,951	-0,01872	38,8818	-1,29606

The general principles derived and described in the earlier sections of this paper have been used to derive the equations of the CO_2 level that are consistent with the four different emission scenarios. The parameters are presented in Table 8., for the unit ppm, and in Table 9., for the unit Gt.

Table 8.

Parameter values for predictions.

Alternative	k0 (Gt)	k1 (Gt)	A (Gt)
Cont	1708,271011	30,64570412	1057,501954
Lev 2020	4264,777687	0	-1034,030282
Stop 2100	5651,809577	-25,96398865	-2421,062173
Stop 2050	7963,529394	-69,23730308	-4732,781989

Table 9.

Parameter values for predictions.

Alternative	k0 (ppm)	k1 (ppm)	A (ppm)
Cont	218,8878738	3,926761604	135,5021262
Lev 2020	546,4637133	0	-132,4946033
Stop 2100	724,1898816	-3,326873918	-310,2207716
Stop 2050	1020,400162	-8,871663781	-606,4310522

Results and discussion

The developed model will now be used to investigate the dynamic effects of four different alternative scenarios for the future development of global CO_2 emissions, during the time interval 2020 to 2100. In Figure 5., we find the four emission scenarios. The predictions of the future CO_2 level, conditional on the different emission strategies, are found in Figure 6. The predictions function, (57) is used. Then, t is defined according to the information in Table 7. and the parameter values A, k_0, k_1 from Table 9. are used.

$$x(t) = Ae^{-0.0187191t} + k_0 + k_1 \ t \qquad (ppm) \qquad (57)$$

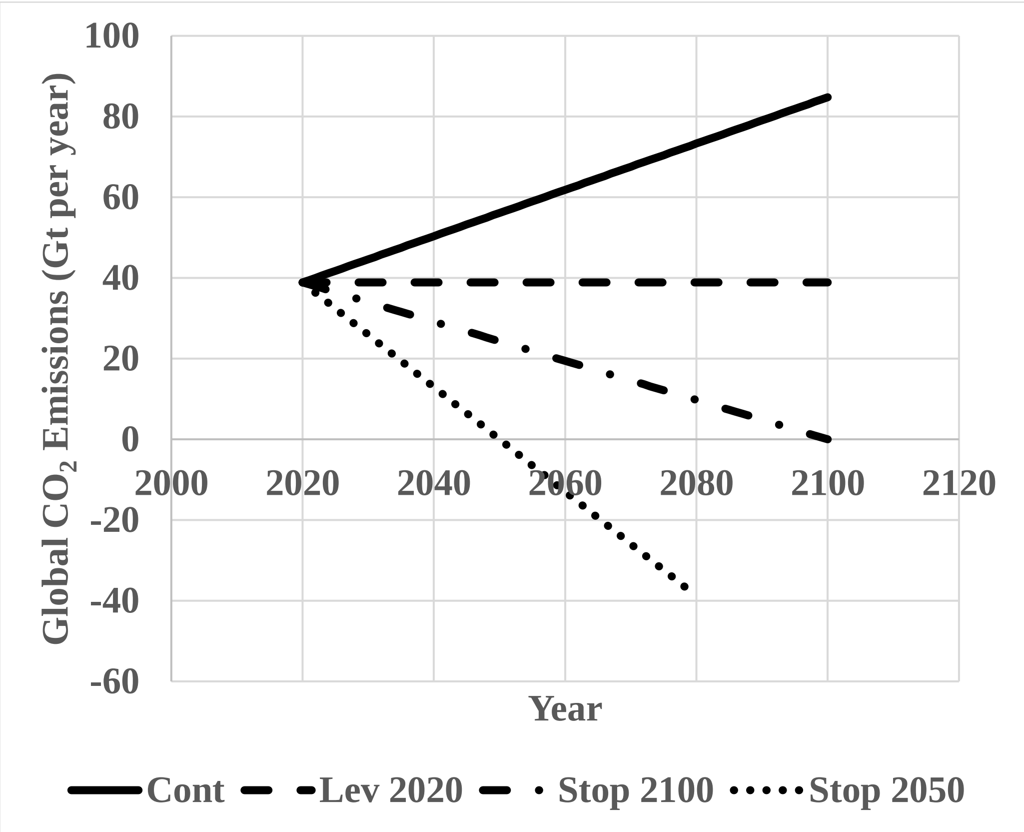

Figure 5.

Four different alternative scenarios for the future development of global CO_2 emissions, during the time interval 2020 to 2100. The emission level 2020 is estimated via the linear approximation based on data from the time interval 1990 to 2018. The scenarios are used to predict the future development of CO_2 in the atmosphere. Compare Figure 6. **Cont** = The emissions continue to develop according to the trend during 1990 to 2018. **Lev 2020** = The emissions stay, for ever, at the level of 2020. **Stop 2100** = The emissions are reduced with the same amount each year, during the time interval 2020 until 2100. Then, the total emission is zero. **Stop 2050** = The emissions are reduced with the same amount each year, during the time interval 2020 until 2050. (Observation: The negative emissions after 2050 are technically possible but not necessarily optimal and relevant.)

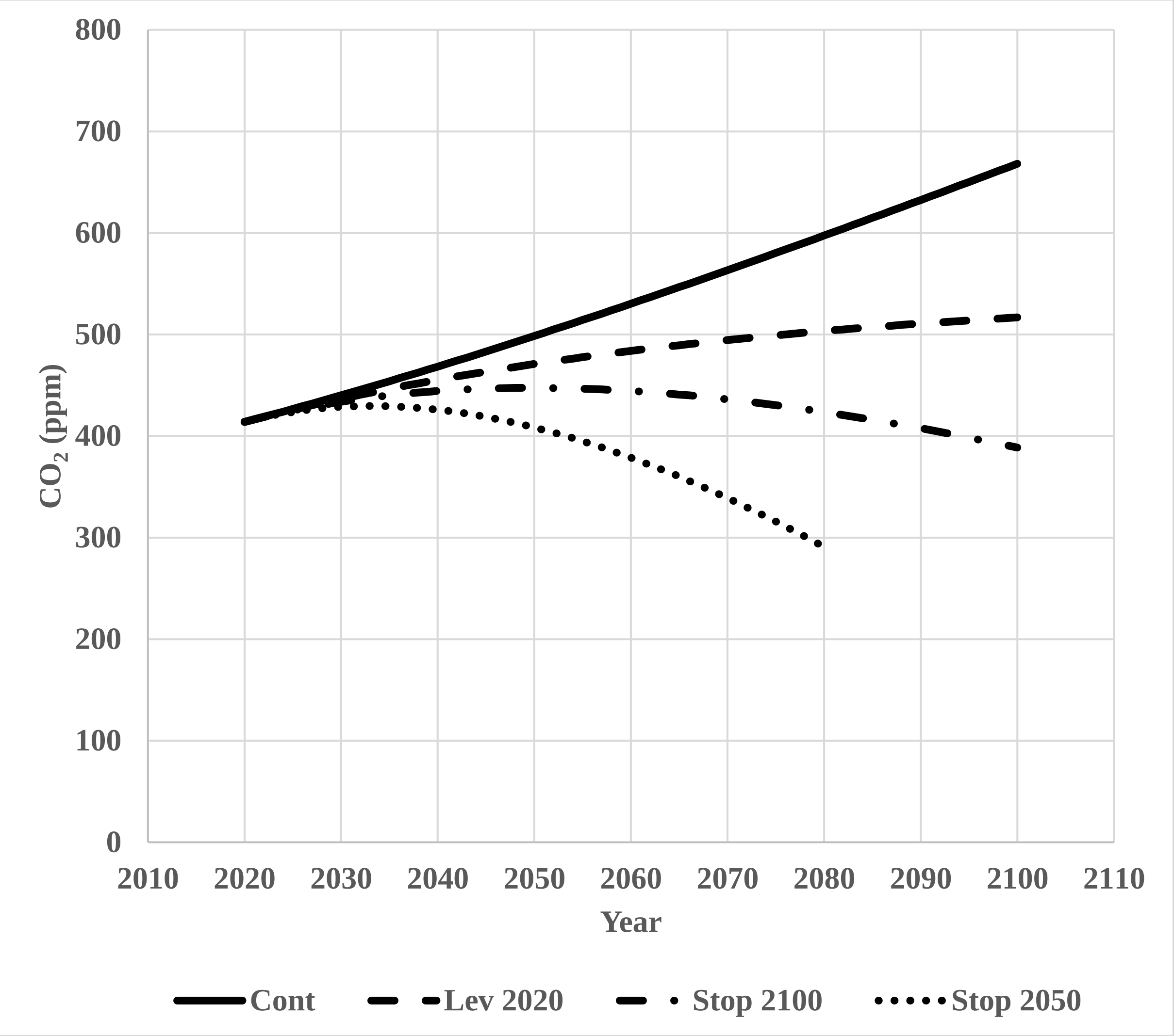

Figure 6.

Four different alternative scenarios for the future development of CO_2 level in the atmosphere, during the time interval 2020 to 2100. The scenarios are conditional on the global emission scenarios found in Figure 5. The emission level 2020 is estimated via the linear approximation based on data from the time interval 1990 to 2018.

Cont = The emissions continue to develop according to the trend during 1990 to 2018.
Lev 2020 = The emissions stay, for ever, at the level of 2020.
Stop 2100 = The emissions are reduced with the same amount each year, during the time interval 2020 until 2100. Then, the total emission is zero.
Stop 2050 = The emissions are reduced with the same amount each year, during the time interval 2020 until 2050. After 2050, the net emission is strictly negative and follows the same trend as before 2050. (Observation: The negative emissions after 2050 contribute to the dramatic fall of the CO_2 level after 2050 in this scenario. If the emissions would be zero after 2050, the CO_2 level would converge to the pre-industrial level of 280 ppm. Alternative scenarios may easily be constructed.)

Conclusions

Now, it is possible to understand the fundamental mechanisms of the dynamics of the CO_2 level of the atmosphere, under the influence of global emissions.

A theoretical mathematical model of the dynamics of CO_2 has been developed. This model is consistent with fundamental scientific principles. Furthermore, we can use the model to reproduce historical time series of empirical data. We can even use the model to calculate the pre-industrial level of CO_2 and discover that the calculated equilibrium value is consistent with earlier research findings. The model can also be used for preditictions. We have investigated how the global CO_2 level can be dynamically changed via different emissions strategies. Detailed predictions of possible future developments have been produced and described.

The CO_2 and global warming topic is central to the present global political agenda. It is necessary to create a fundamental understanding of the principles and methods that can be used to handle the problems and to stabilize our global climate. The model developed in this paper can hopefully make it possible for a large part of the human population to really understand how the CO_2 dynamics and emissions are connected. Without this fundamental understanding, it is difficult to convince critical persons that large investments in emission reductions may be necessary in order to stabilize the global climate.

The model developed in this paper should be possible to understand, investigate and to reproduce, in every detail by every person that has a PhD or masters degree in engineering, mathematics, mathematical statistics or mathematical economics. Earlier models presented on similar topics are not presented with all the details. Completeness and transparancy are necessary for complete understanding and acceptance.

According to the Occams razor, a scientific model should not be more complicated than necessary. In this paper, a differential equation is developed that is *only* based on very fundamental principles from physical science and mathematics. Two highly reliable sources of empirical data have been used to estimate the parameters. In the analysis, we have seen that a first order differential equation with emission forcing has been able to explain the development of the dynamics of the CO_2 level in the atmosphere, with very high precision. Furthermore, the function shows that the CO_2 equilibrium level, before the industrial revolution, should be 280 ppm, which confirms earlier empirical research. According to the opinion of the author, it is hardly possible to develop a more simple scientific model that explains the CO_2 dynamics in a better way.

Finally, the author hopes that the new model will be used to optimize and control global emission reductions, in order to give our planet the optimal climate.

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